Perturbative Evolution

$|M_H^{(0)}|^2$

Factorization Scale

Hadronization

Boost 2011

VINCIA
Peter Skands (CERN)
\[ |M_H^{(0)}|^2 \]

Perturbative Evolution

Factorization Scale

Hadronization

Parton Showers
Leading Log
Leading Color

Boost 2011

Peter Skands (CERN)
2\text{Re} \left[ M_H^{(1)} M_H^{(0)*} \right]

\begin{align*}
|M_H^{(0)}|^2 \\
|M_H^{(0)}|^2 \\
|M_H^{(0)}|^2 \\
|M_H^{(0)}|^2
\end{align*}

\begin{align*}
|M_{H+2}^{(0)}|^2 |M_{H+3}^{(0)}|^2 \\
|M_{H+1}^{(0)}|^2 \\
\end{align*}
Perturbative Evolution

| $M_{H+2}^{(0)}|^2 |M_{H+3}^{(0)}|^2$ |
| $|M_{H+1}^{(0)}|^2$ |

$2 \text{Re} \left[ M_{H}^{(1)} M_{H}^{(0)*} \right]$  

$2 \text{Re} \left[ M_{H+1}^{(1)} M_{H+1}^{(0)*} \right]$  

$2 \text{Re} \left[ M_{H+2}^{(1)} M_{H+2}^{(0)*} \right]$  

Factorization Scale

Hadronization

Parton Showers

Leading Log

Leading Color

Higher-Order

Singular Structures

Boost 2011
Peter Skands (CERN)

Factorization Scale

HADRONIZATION

Perturbative Evolution

2Re \[ \left[ M_{H+1}^{(1)} M_{H+1}^{(0)*} \right] \]

2Re \[ \left[ M_{H+2}^{(1)} M_{H+2}^{(0)*} \right] \]

\[
\frac{|M_{H+2}^{(0)}|^2}{|M_{H+3}^{(0)}|^2} \cdot \frac{|M_{H+1}^{(0)}|^2}{|M_{H+1}^{(0)}|^2}
\]

Parton Showers

Leading Log

Leading Color

Higher-Order Singular Structures

PYTHIA

Collider

Observables

Confrontation

with Data

Factorization Scale

Hadronization
Why?

- Jet Substructure
- Underlying Event & Jet Calibration
- Hadronization
- Structure of QCD
Why?

Jet Substructure

Underlying Event & Jet Calibration

Large \( m \) = subleading

Hadronization

Structure of QCD
Why?

Jet Substructure

Underlying Event & Jet Calibration

Broad Jet with little Underlying Event

VS.

Narrow jet with more Underlying Event

Large $m = \text{subleading}$

Hadronization

Structure of QCD
Why?

Jet Substructure

- Large $m$ = subleading

Underlying Event & Jet Calibration

- Broad Jet with little Underlying Event vs. Narrow Jet with more Underlying Event

Hadronization

- Matrix Element → Parton Shower
- Hadronization
- Factorization Scale

Better control of perturbative part → better constraints on non-perturbative part

Structure of QCD
Why?

Jet Substructure

Underlying Event & Jet Calibration

Large $m$ = subleading

Broad Jet with little Underlying Event

Narrow jet with more Underlying Event

Hadronization

Structure of QCD

Better control of perturbative part → better constraints on non-perturbative part
VINCIA

What is it?

Plug-in to PYTHIA 8 (http://projects.hepforge.org/vinia)

What does it do?

“Matched Markov antenna showers”

Improved parton showers
+ Re-interprets tree-level matrix elements as $2 \rightarrow n$ antenna functions
+ Extends matching to soft region (no “matching scale”)

Extensive (and automated) uncertainty estimates

Systematic variations of shower functions, evolution variables, $\mu_R$, etc.
→ A vector of output weights for each event (central value = unity = unweighted)

Who is doing it?

GEEKS: Giele, Kosower, Skands + Gehrmann-de-Ridder & Ritzmann (mass effects), Lopez-Villarejo (“sector showers”), Hartgring & Laenen (NLO multileg)
pQCD with Markov Chains

**Starting Point:** reformulate perturbative series as Markov Chain

~ all-orders parton shower with all-orders matrix-element corrections

**For Each “Evolution Step”** = increase in parton multiplicity (on-shell)

Cover all of phase space with (large) trial overestimate = “approximate”

Compute the physical evolution probability using...

Matched = Approximate

\[
\frac{\text{Exact}}{\text{Approximate}}
\]

→ Must be able to compute both numerator and denominator
**Starting Point:** reformulate perturbative series as Markov Chain

~ all-orders parton shower with all-orders matrix-element corrections

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\[
\text{Exact} \quad \frac{\text{Approximate}}{\text{Approximate}}
\]

→ Must be able to compute both numerator and denominator

_E.g., get from MadGraph_

Already widely used at first order:

E.g., by PYTHIA for mass and ME corrections, and by POWHEG for virtual ones

Also similar to GenEva?
pQCD with Markov Chains

**Starting Point:** reformulate perturbative series as Markov Chain

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**For Each “Evolution Step”** = increase in parton multiplicity (on-shell)

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Compute the physical evolution probability using ...

![Diagram](image)

- Matched = Approximate \[\text{Exact} \quad \text{Approximate}\]
- \[\rightarrow \text{Must be able to compute both numerator and denominator}\]

**Unitarity** \[\rightarrow \text{No need to impose “matching scale”}\] (Matching corrections applied directly to Markov chain as it evolves

\[\rightarrow \text{self-regulating} \rightarrow \text{can be applied over all of phase space, also inside jets}\]

E.g., get from MadGraph

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→ Must be able to compute both numerator and denominator

**Unitarity** → No need to impose “matching scale” (Matching corrections applied directly to Markov chain as it evolves

→ self-regulating → can be applied over all of phase space, also inside jets)

→ One single unweighted event sample (Effectively, n-parton samples use parton shower itself as phase space generator = highly efficient “multi-channel” integration → speed gains expected, + unitarity → unit-weights)
The Denominator

Number of Histories:

Existing parton showers are not really Markov Chains

Further evolution (restart scale) depends on which branching happened last \( \rightarrow \) proliferation of terms

Number of histories contributing to \( n^{\text{th}} \) branching \( \sim 2^n n! \)

\[
\begin{align*}
\sim & + + + \\
\left( \sim + \right) & \rightarrow 2 \text{ terms} \\
\end{align*}
\]
The Denominator

Number of Histories:

Existing parton showers are \textit{not} really Markov Chains

Further evolution (restart scale) depends on which branching happened last $\rightarrow$ proliferation of terms

Number of histories contributing to $n^{th}$ branching $\propto 2^n n!$

Parton- or Catani-Seymour Shower:
- After 2 branchings: 8 terms
- After 3 branchings: 48 terms
- After 4 branchings: 384 terms

\begin{align*}
\sim & \quad + \quad + \quad + \\
(\sim & \quad + \quad ) & \quad j = 1 \\
& \quad \rightarrow 2 \text{ terms} \\
& \quad j = 2 \\
& \quad \rightarrow 4 \text{ terms}
\end{align*}
Matched Markovian Antenna Showers

**Parton and CS showers**: $2^n n!$
One term per parton (two for gluons)

**Antenna showers**: $2^n n! \rightarrow n!$
One term per parton pair
Matched Markovian Antenna Showers

**Parton and CS showers:** $2^n n!$
One term per parton (two for gluons)

**Antenna showers:** $2^n n! \rightarrow n!$
One term per parton pair

+ Change “shower restart” to Markov criterion:
  Given an $n$-parton configuration, “ordering” scale is
  
  $$Q_{\text{ord}} = \min(Q_{E1}, Q_{E2}, ..., Q_{En})$$

  Unique restart scale, independently of how it was produced

+ Matching: $n! \rightarrow n$
  Given an $n$-parton configuration, its phase space weight is:
  
  $$|M_n|^2 : \text{Unique weight, independently of how it was produced}$$
Parton and CS showers: \(2^n n!\)
One term per parton (two for gluons)

Antenna showers: \(2^n n! \rightarrow n!\)
One term per parton pair

+ **Change “shower restart” to Markov criterion:**

Given an \(n\)-parton configuration, “ordering” scale is

\[ Q_{\text{ord}} = \min(Q_{E1}, Q_{E2}, ..., Q_{En}) \]

Unique restart scale, independently of how it was produced

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Given an \(n\)-parton configuration, its phase space weight is:

\[ |M_n|^2 : \text{Unique weight, independently of how it was produced} \]

**Matched Markovian Antenna Shower:**
- After 2 branchings: 2 terms
- After 3 branchings: 3 terms
- After 4 branchings: 4 terms

**Parton- or Catani-Seymour Shower:**
- After 2 branchings: 8 terms
- After 3 branchings: 48 terms
- After 4 branchings: 384 terms
Approximations

Distribution of $\log_{10}(\text{PS}_{\text{LO}}/\text{ME}_{\text{LO}})$ (inverse ~ matching coefficient)

- **$Z \rightarrow 4$** (second order)
  - Vincia 1.025 + MadGraph 4.426
  - Matched to $Z \rightarrow 3$
  - Strong Ordering

- **$Z \rightarrow 5$** (third order)
  - Vincia 1.025 + MadGraph 4.426
  - Matched to $Z \rightarrow 3$
  - Strong Ordering

- **$Z \rightarrow 6$** (fourth order)
  - Vincia 1.025 + MadGraph 4.426
  - Matched to $Z \rightarrow 3$
  - Strong Ordering
Better Approximations

Distribution of $\log_{10}(\text{PS}_{\text{LO}}/\text{ME}_{\text{LO}})$ (inverse ~ matching coefficient)

For $Z \rightarrow 4$
- Vincia 1.025 + MadGraph 4.426
- Matched to $Z \rightarrow 3$
- Strong Ordering

For $Z \rightarrow 5$
- Vincia 1.025 + MadGraph 4.426
- Matched to $Z \rightarrow 3$
- Strong Ordering

For $Z \rightarrow 6$
- Vincia 1.025 + MadGraph 4.426
- Matched to $Z \rightarrow 3$
- Strong Ordering

Leading Order, Leading Color, Flat phase-space scan, over all of phase space (no matching scale)

For $Z \rightarrow 4$
- Vincia 1.025 + MadGraph 4.426
- Matched to $Z \rightarrow 3$
- Smooth Ordering

For $Z \rightarrow 5$
- Vincia 1.025 + MadGraph 4.426
- Matched to $Z \rightarrow 3$
- Smooth Ordering

For $Z \rightarrow 6$
- Vincia 1.025 + MadGraph 4.426
- Matched to $Z \rightarrow 3$
- Smooth Ordering

GEEKS (Giele, Kosower, Skands): arXiv:1102.2126
+ Matching (+ full colour)

→ A very good all-orders starting point

GEEKS (Giele, Kosower, Skands): arXiv:1102.2126
### (Speed)

**Matched through:**

<table>
<thead>
<tr>
<th>Generator</th>
<th>Z→3</th>
<th>Z→4</th>
<th>Z→5</th>
<th>Z→6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pythia 6</td>
<td>0.19</td>
<td></td>
<td></td>
<td>ms/event</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$Z \rightarrow qq +$ shower. Matched and unweighted. <strong>Hadronization off</strong></td>
</tr>
<tr>
<td>Pythia 8</td>
<td>0.20</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(initialization time = zero)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vincia</td>
<td>0.24</td>
<td>0.62</td>
<td>5.60</td>
<td>112.50</td>
</tr>
<tr>
<td></td>
<td>(initialization time = zero)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sherpa</td>
<td>5.15*</td>
<td>53.00*</td>
<td>220.00*</td>
<td>400.00*</td>
</tr>
<tr>
<td></td>
<td>(Q(_{match} = 5) GeV)</td>
<td>90,000 ms</td>
<td>420,000 ms</td>
<td>1,320,000 ms</td>
</tr>
<tr>
<td></td>
<td>* + initialization time</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Generator Versions: Pythia 6.425 *(with Perugia 2011 tune)*, Pythia 8.150, Sherpa 1.3.0 *(not including initialization)*, Vincia 1.026 *(NLL,NLC, and uncertainties OFF)*

(+ working with J. Lopez-Villarejo at CERN to further increase multi-parton matching speed)
Uncertainties
Uncertainty Variations

A result is only as good as its uncertainty

Normal procedure:

*Run MC $2N+1$ times (for central + $N$ up/down variations)*

- Takes $2N+1$ times as long
- + uncorrelated statistical fluctuations
A result is only as good as its uncertainty

Normal procedure:

Run MC $2N+1$ times (for central + $N$ up/down variations)

- Takes $2N+1$ times as long
- + uncorrelated statistical fluctuations

Automate and do everything in one run

VINCIA: all events have weight = 1

Compute unitary alternative weights on the fly

→ sets of alternative weights representing variations (all with $<w>=1$)

Same events, so only have to be hadronized/detector-simulated ONCE!

MC with Automatic Uncertainty Bands

GEEKS (Giele, Kosower, Skands): arXiv:1102.2126
**Uncertainties**

For each branching, recompute weight for:
- Different renormalization scales
- Different antenna functions
- Different ordering criteria
- Different subleading-colour treatments

<table>
<thead>
<tr>
<th></th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal</td>
<td>1</td>
</tr>
<tr>
<td>Variation</td>
<td>( P_2 = \frac{\alpha_s a_2}{\alpha_s a_1} P_1 )</td>
</tr>
</tbody>
</table>
For each branching, recompute weight for:

- Different renormalization scales
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<td>Variation</td>
<td>$P_2 = \frac{\alpha_s a_2}{\alpha_s a_1} P_1$</td>
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</tbody>
</table>

+ Unitarity

For each failed branching:

$$P_{2;\text{no}} = 1 - P_2 = 1 - \frac{\alpha_s a_2}{\alpha_s a_1} P_1$$
For each branching, recompute weight for:
- Different renormalization scales
- Different antenna functions
- Different ordering criteria
- Different subleading-colour treatments

+ **Matching**

Differences explicitly matched out
(Up to matched orders)
(Can in principle also include variations of matching scheme...)

+ **Unitarity**

For each failed branching:
\[
P_{2;\text{no}} = 1 - P_2 = 1 - \frac{\alpha_s a_2}{\alpha_s a_1} P_1
\]
Automatic Uncertainties

Vincia:uncertaintyBands = on

Variation of renormalization scale (no matching)
**Automatic Uncertainties**

Vincia:uncertaintyBands = on

**Variation of “finite terms”** (no matching)
Putting it Together

VinciaMatching:order = 0

Vincia 1.025 + Pythia 8.150

Data from Phys.Rept. 399 (2004) 71

VinciaMatching:order = 3

Vincia 1.025 + MadGraph 4.426 + Pythia 8.150

Data from Phys.Rept. 399 (2004) 71
**VINCIA STATUS**

**Plug-in to PYTHIA 8**

**Stable and reliable for Final-State Jets** *(E.g., LEP)*

**Automatic matching and uncertainty bands**

**Improvements in shower** *(smooth ordering, NLC, matching, …)*

**Paper on mass effects ~ ready** *(with A. Gehrmann-de-Ridder & M. Ritzmann)*

---

**Next steps**

**Multi-leg one-loop matching** *(with L. Hartgring & E. Laenen, NIKHEF)*

**“Sector Showers”** *(with J. Lopez-Villarejo, CERN)*

→ **Initial-State Showers** *(with W. Giele, D. Kosower)*
VINCI\textsc{a Status}

\textbf{Next steps}

\textbf{Multi-leg one-loop matching}
(with L. Hartgring & E. Laenen, NIKHEF)

\textbf{“Sector Showers”}
(with J. Lopez-Villarejo, CERN)

→ \textbf{Initial-State Showers}
(with W. Giele, D. Kosower)
Backup Slides
pQCD as Markov Chain

Start from Born Level:

\[
\left. \frac{d\sigma_H}{d\Omega} \right|_{\text{Born}} = \int d\Phi_H \ |M_H^{(0)}|^2 \delta(\mathcal{O} - \mathcal{O}(\{p\}_H))
\]

H = Arbitrary hard process
pQCD as Markov Chain

Start from Born Level:

\[ \frac{d\sigma_H}{d\Omega} \bigg|_{\text{Born}} = \int d\Phi_H \ |M_H^{(0)}|^2 \delta(\mathcal{O} - \mathcal{O}(\{p\}_H)) \]

H = Arbitrary hard process

Insert Evolution Operator, \( S \):

\[ \frac{d\sigma_H}{d\Omega} \bigg|_{S} = \int d\Phi_H \ |M_H^{(0)}|^2 S(\{p\}_H, \mathcal{O}) \]

Think: starting a shower off an incoming on-shell momentum configuration
Postpone evaluating observable until shower “finished”
The Evolution Operator

**Depends on Evolution Scale:** \( Q_E \)

\[
S(\{p\}_H, s, Q_E^2, O) = \Delta(\{p\}_H, s, Q_E^2) \delta(O - O(\{p\}_H)) + \sum \int^{s}_{Q_E^2} \frac{d\Phi_{H+1}^{[r]}}{d\Phi_H} S_r \Delta(\{p\}_H, s, Q_{H+1}^2) S(\{p\}_{H+1}, Q_{H+1}^2, Q_E^2, O)
\]

**Legend:**

\( \Delta \) represents *no-evolution* probability (Sudakov): conserves probability = preserves event weights

\( S_r = \text{Emission probability} \) (partitioned among radiators \( r \))

According to *best known approximation* to \(|H+1|^2\) (e.g., ME or LL shower)
The Evolution Operator

**Depends on Evolution Scale :** \( Q_E \)

\[
S(\{p\}_H, s, Q_E^2, \mathcal{O}) = \frac{\Delta(\{p\}_H, s, Q_E^2) \delta (\mathcal{O} - \mathcal{O}(\{p\}_H))}{H + 0 \text{ exclusive above } Q_E} \\
+ \sum_r \int_{Q_E^2}^{s} \frac{d\Phi_H^{[r]}}{d\Phi_H} S_r \Delta(\{p\}_H, s, Q_{H+1}^2) S(\{p\}_{H+1}, Q_{H+1}^2, Q_E^2, \mathcal{O})
\]

\( H + 1 \text{ inclusive above } Q_E \)

**Legend:**

\( \Delta \) represents *no-evolution* probability (Sudakov): conserves probability = preserves event weights

\( S_r = \text{Emission probability} \) (partitioned among radiators \( r \))

*According to best known approximation to \(|H+1|^2\)* (e.g., ME or LL shower)
(Expand S to First Order)

Equivalent to Sjöstrand/POWHEG

\[ S^{(1)}(p_H, s, Q_E^2, \mathcal{O}) = \left( 1 + K_H^{(1)} - \int_{Q_E^2}^{s} \frac{d\Phi_{H+1}}{d\Phi_H} \frac{|M_{H+1}^{(0)}|^2}{|M_H^{(0)}|^2} \right) \delta(\mathcal{O} - \mathcal{O}(p_H)) \]

\[ + \int_{Q_E^2}^{s} \frac{d\Phi_{H+1}}{d\Phi_H} \frac{|M_{H+1}^{(0)}|^2}{|M_H^{(0)}|^2} \delta(\mathcal{O} - \mathcal{O}(p_{H+1})) . \]

Virtual Correction (NLO normalization)

\[ \frac{2 \text{Re}[M_H^{(0)} M_H^{(1)*}]}{|M_H^{(0)}|^2} = K_H^{(1)} - \int_0^s \frac{d\Phi_{H+1}}{d\Phi_H} \frac{|M_{H+1}^{(0)}|^2}{|M_H^{(0)}|^2} \]

\[ \frac{a}{\epsilon^2} + \frac{b}{\epsilon} + c + \mathcal{O}(\epsilon) \]

\[ \frac{a}{\epsilon^2} + \frac{b}{\epsilon} + c' + \mathcal{O}(\epsilon) \]
(Expand S to First Order)

**Equivalent to Sjöstrand/POWHEG**

\[
S^{(1)}(\{p\}_H, s, Q^2_E, \mathcal{O}) = \left( 1 + K^{(1)}_H \right) - \int_{Q^2_E}^s \frac{\text{d} \Phi_{H+1}}{\text{d} \Phi_H} \frac{|M^{(0)}_{H+1}|^2}{|M^{(0)}_H|^2} \delta(\mathcal{O} - \mathcal{O}(\{p\}_H)) \\
+ \int_{Q^2_E}^s \frac{\text{d} \Phi_{H+1}}{\text{d} \Phi_H} \frac{|M^{(0)}_{H+1}|^2}{|M^{(0)}_H|^2} \delta(\mathcal{O} - \mathcal{O}(\{p\}_{H+1}))
\]

**Virtual Correction (NLO normalization)**

\[
\frac{2\text{Re}[M^{(0)}_H M^{(1)*}_H]}{|M^{(0)}_H|^2} = K^{(1)}_H - \int_0^s \frac{\text{d} \Phi_{H+1}}{\text{d} \Phi_H} \frac{|M^{(0)}_{H+1}|^2}{|M^{(0)}_H|^2} \\
\frac{a}{\epsilon^2} + \frac{b}{\epsilon} + c + \mathcal{O}(\epsilon)
\]

\[
\frac{a}{\epsilon^2} + \frac{b}{\epsilon} + c' + \mathcal{O}(\epsilon)
\]
Generate Trials without imposing strong ordering

At each step, each dipole allowed to fill its entire phase space

Overcounting removed by matching
(revert to strong ordering beyond matched multiplicities)
Generate Trials without imposing strong ordering

At each step, each dipole allowed to fill its entire phase space

Overcounting removed by matching

+ smooth ordering beyond matched multiplicities

\[
\frac{p_1^2}{\hat{p}_1^2 + p_{LL}^2} \quad \text{last branching}
\]

\[
\frac{p_1^2}{\hat{p}_1^2} \quad \text{current branching}
\]
Isolate double-collinear region:

\[ Z \rightarrow 4 : [q,g,g,q\bar{g}] \text{ with } m_{gg} = m_Z \]
LEP event shapes

PYTHIA 8 already doing a very good job
VINCI A adds uncertainty bands + can look at more exclusive observables?
Multijet resolution scales

\[ y_{45} = \text{scale at which 5}\textsuperscript{th} \text{ jet becomes resolved} \sim \text{“scale of 5}\textsuperscript{th} \text{ jet”} \]
Interesting to look at more exclusive observables, but which ones?

**4-jet angles**
Sensitive to polarization effects

**Good News**
VINCI A is doing reliably well
Non-trivial verification that shower+matching is working, etc.

**Higher-order matching needed?**
PYTHIA 8 already doing a very good job on these observables

4-Jet Angles