Matching at LO and NLO

Introduction to QCD - Lecture 4

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P. Skands (CERN)

The Problem

Lecture 2 : Matrix elements are correct

When all jets are hard and there are no hierarchies

(single-scale problem = small corner of phase space, but an important one!)

But they are unpredictive for strongly ordered emissions

Lecture 3 : Parton Showers are correct

When all emissions are (successively) strongly ordered

(= dominant QCD structures)

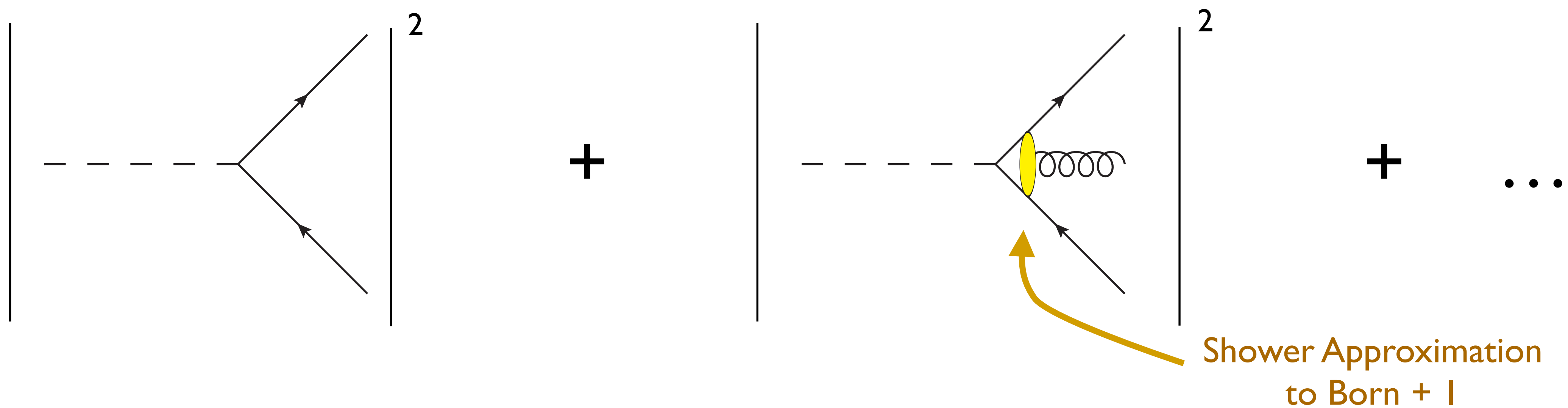
But they are unpredictive for hard jets

Often too soft (but not guaranteed! Can also err by being too hard!)

ME-PS matching → ONE calculation to rule them all

Example: $H^0 \to b\bar{b}$

Born + Shower



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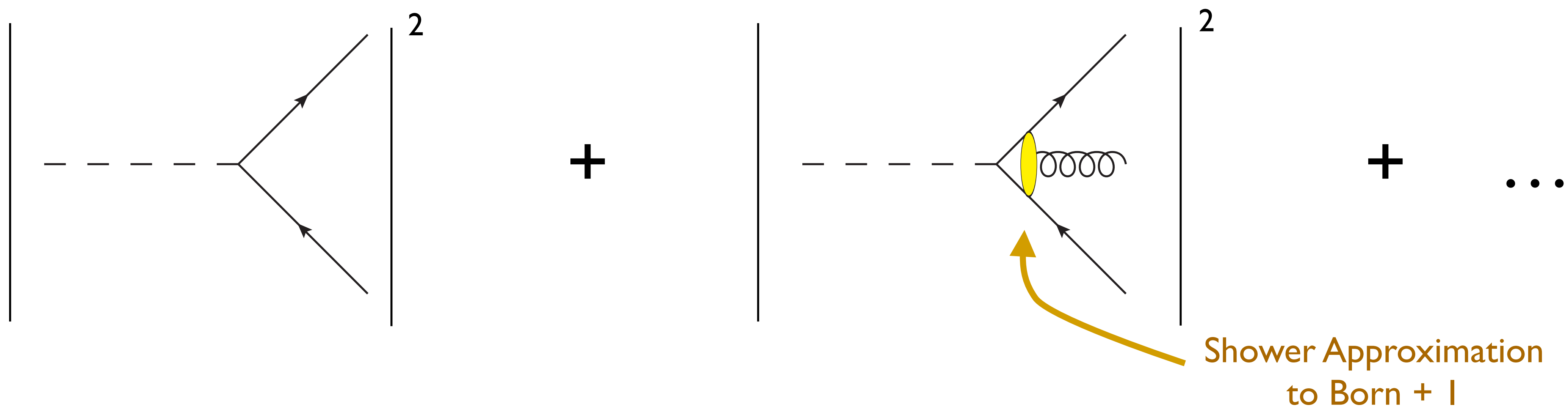
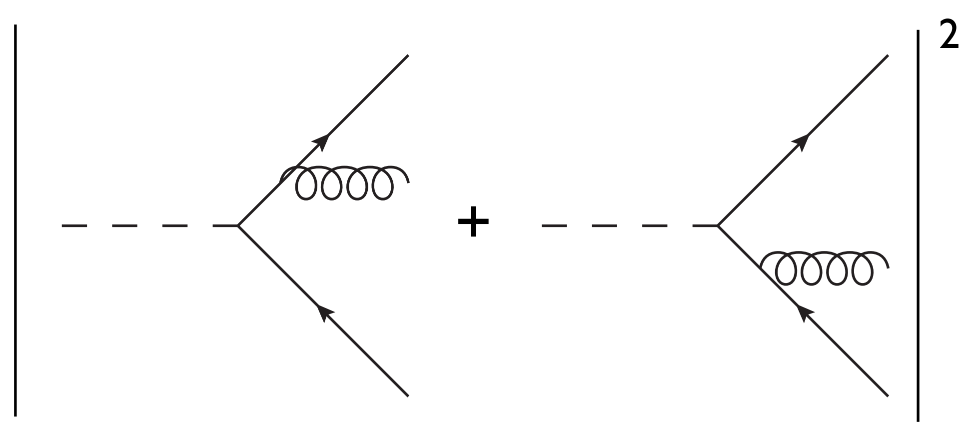
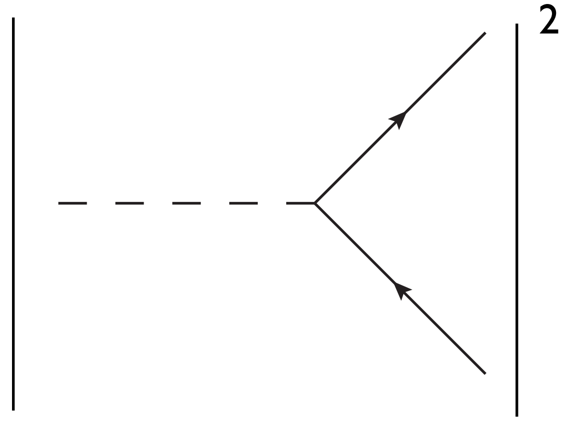


Born + 1 @ LO



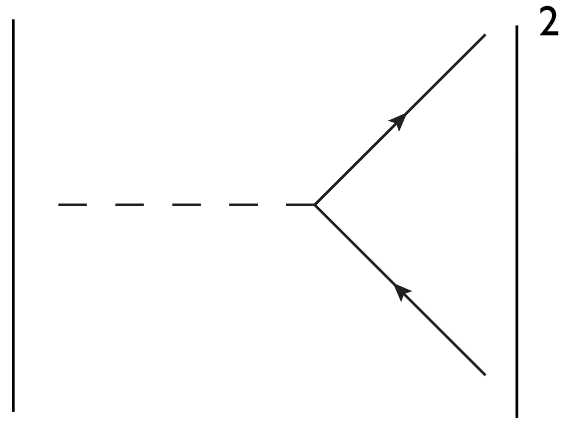
Example: $H^0 \to b\bar{b}$

Born + Shower



$$\left|\ \cdots\ \right|^2 \left(1 + g_s^2\, 2C_F \left[\frac{2s_{ik}}{s_{ij}s_{jk}} + \frac{1}{s_{IK}} \left(\frac{s_{ij}}{s_{jk}} + \frac{s_{jk}}{s_{ij}} \right) \right] + \ldots \right)$$

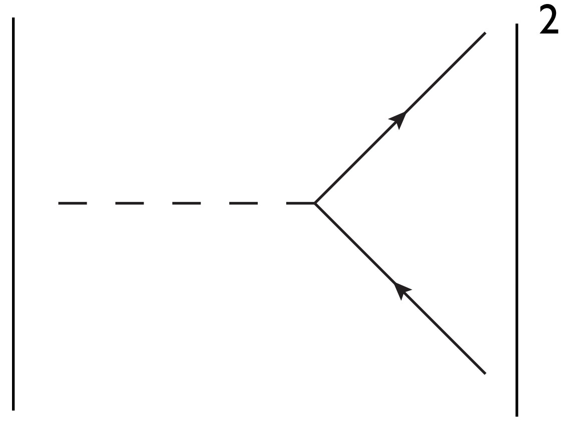
Born + 1 @ LO



$$\left|\ \cdots\ \right|^2 \left(g_s^2\, 2C_F \left[\frac{2s_{ik}}{s_{ij}s_{jk}} + \frac{1}{s_{IK}} \left(\frac{s_{ij}}{s_{jk}} + \frac{s_{jk}}{s_{ij}} + 2 \right) \right] \right)$$

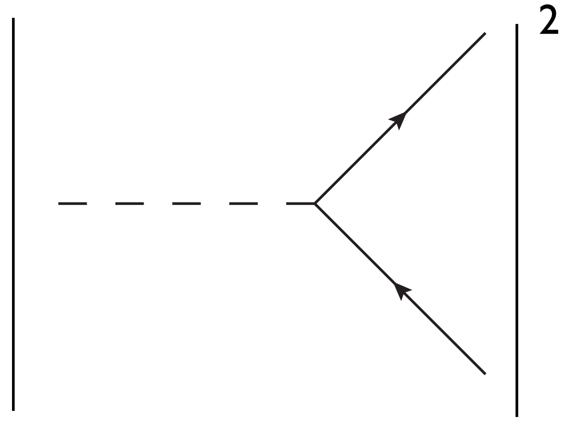
Example: $H^0 \to b\bar{b}$

Born + Shower



$$\left(1 + g_s^2\, 2C_F \left[\frac{2s_{ik}}{s_{ij}s_{jk}} + \frac{1}{s_{IK}} \left(\frac{s_{ij}}{s_{jk}} + \frac{s_{jk}}{s_{ij}} \right) \right] + \ldots \right)$$

Born + 1 @ LO



$$\left(g_s^2\, 2C_F \left[\frac{2s_{ik}}{s_{ij}s_{jk}} + \frac{1}{s_{IK}} \left(\frac{s_{ij}}{s_{jk}} + \frac{s_{jk}}{s_{ij}} + 2 \right) \right] \right)$$

Total Overkill to add these two. All I really need is just that **+2** …

Adding Calculations

Born × Shower

(see lecture 3)

$X^{(2)}$	$X+1^{(2)}$	…		
$X^{(1)}$	$X+1^{(1)}$	$X+2^{(1)}$	$X+3^{(1)}$	…
Born	$X+1^{(0)}$	$X+2^{(0)}$	$X+3^{(0)}$	…

… Fixed-Order Matrix Element

… Shower Approximation

X+1 @ LO

(with p_T cutoff, see lecture 2)

$X+1^{(2)}$	…		
$X+1^{(1)}$	$X+2^{(1)}$	$X+3^{(1)}$	…
$X+1^{(0)}$	$X+2^{(0)}$	$X+3^{(0)}$	…

… Fixed-Order ME above p_T cut & nothing below

Adding Calculations

Born × Shower
(see lecture 3)

$X^{(2)}$	$X+1^{(2)}$	…		
$X^{(1)}$	$X+1^{(1)}$	$X+2^{(1)}$	$X+3^{(1)}$	…
Born	$X+1^{(0)}$	$X+2^{(0)}$	$X+3^{(0)}$	…

… Fixed-Order Matrix Element

… Shower Approximation

X+1 @ LO × Shower
(with p_T cutoff, see lecture 2)

	$X+1^{(2)}$	…		
	$X+1^{(1)}$	$X+2^{(1)}$	$X+3^{(1)}$	…
	$X+1^{(0)}$	$X+2^{(0)}$	$X+3^{(0)}$	…

… Fixed-Order ME above p_T cut & nothing below

… Shower approximation above p_T cut & nothing below

→ Double Counting

Born × Shower + (X+1) × shower

Double Counting of terms present in both expansions

$X^{(2)}$	$X+1^{(2)}$	…		
$X^{(1)}$	$X+1^{(1)}$	$X+2^{(1)}$	$X+3^{(1)}$	…
Born	$X+1^{(0)}$	$X+2^{(0)}$	$X+3^{(0)}$	…

Worse than useless

… Fixed-Order Matrix Element

… Shower Approximation

… Double counting above p_T cut & shower approximation below

Interpretation

- A (Complete Idiot's) Solution – Combine
  1. $[X]_{ME}$ + showering
  2. $[X + 1 \text{ jet}]_{ME}$ + showering
  3. …

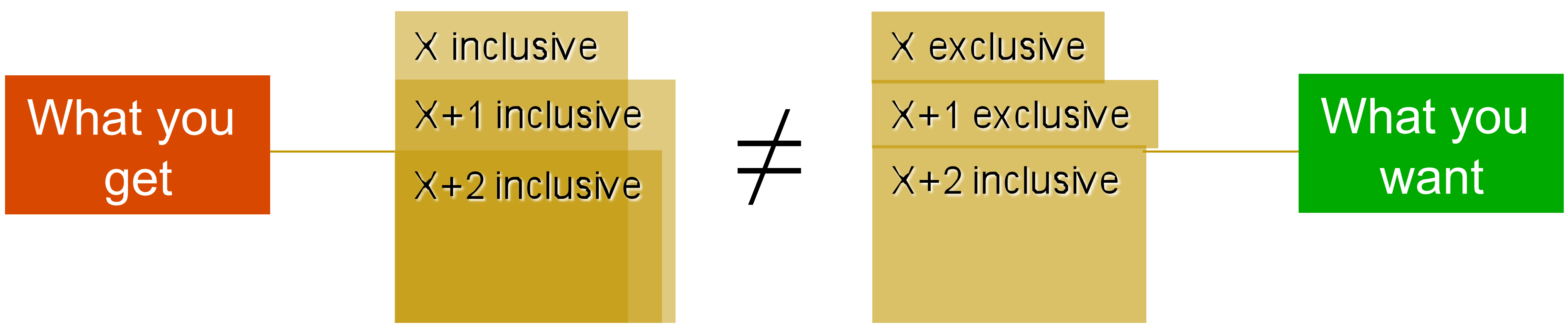
Run generator for X (+ shower)

Run generator for X+1 (+ shower)

Run generator for … (+ shower)

Combine everything into one sample

- Doesn't work
  - [X] + shower is inclusive
  - [X+1] + shower is also inclusive



Overlapping "bins"

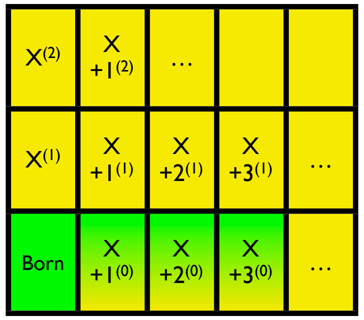
One sample

Cures

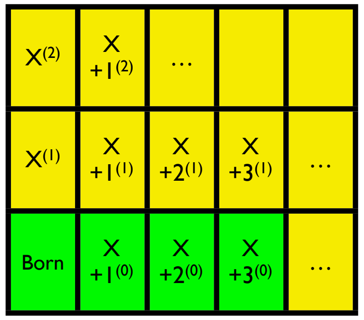
Cures

Tree-Level Matrix Elements

- PHASE-SPACE SLICING *(a.k.a. CKKW, MLM, …)*



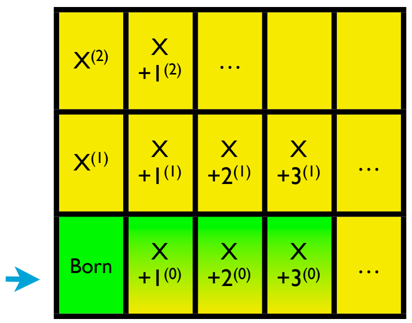
- UNITARITY *(a.k.a. merging, PYTHIA, VINCIA, …)*



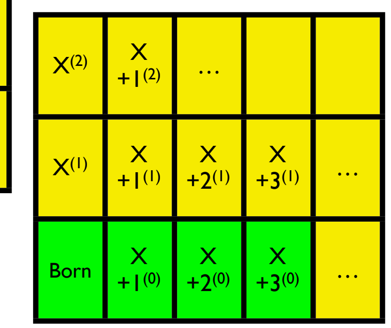
Cures

Tree-Level Matrix Elements

PHASE-SPACE SLICING *(a.k.a. CKKW, MLM, …)*



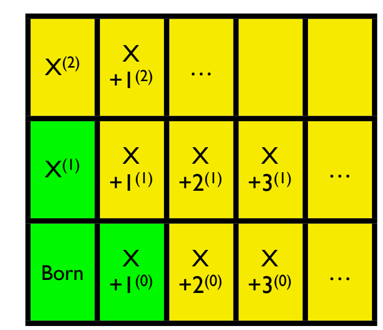
UNITARITY *(a.k.a. merging, PYTHIA,VINCIA, …)*



NLO Matrix Elements

SUBTRACTION *(a.k.a. MC@NLO)*

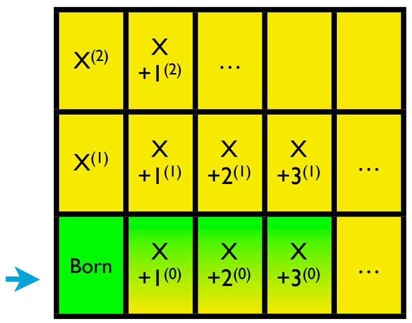
UNITARITY + SUBTRACTION *(a.k.a. POWHEG,VINCIA)*



Cures

Tree-Level Matrix Elements

PHASE-SPACE SLICING *(a.k.a. CKKW, MLM, …)*



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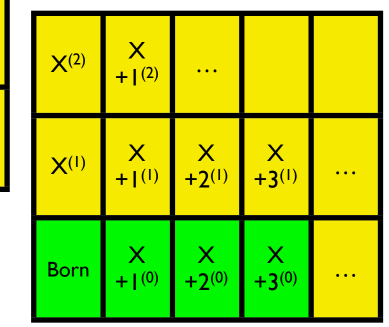

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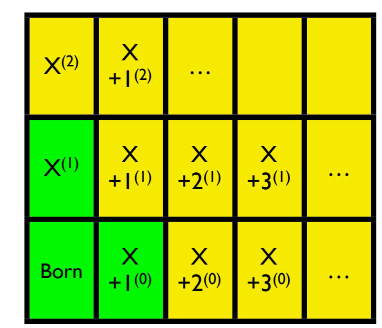
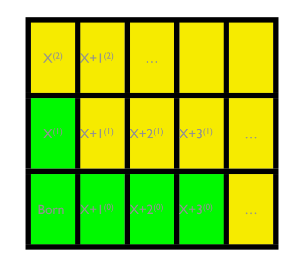
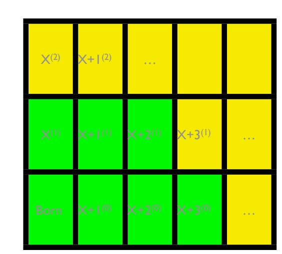


+ WORK IN PROGRESS …

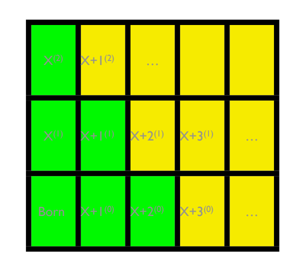
NLO + multileg tree-level matrix elements



NLO multileg matching



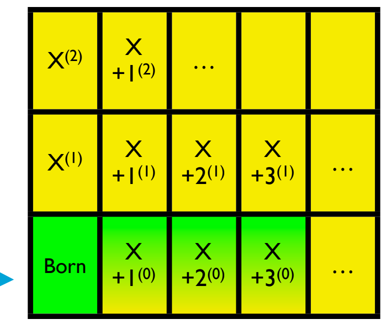
Matching at NNLO



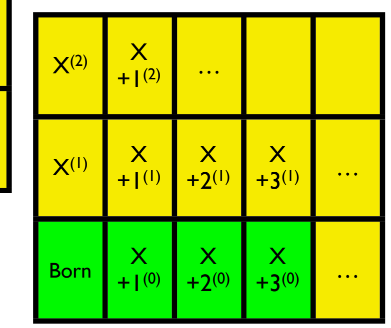
Cures

Tree-Level Matrix Elements

PHASE-SPACE SLICING *(a.k.a. CKKW, MLM, …)*



UNITARITY *(a.k.a. merging, PYTHIA, VINCIA, …)*



NLO Matrix Elements

SUBTRACTION *(a.k.a. MC@NLO)*

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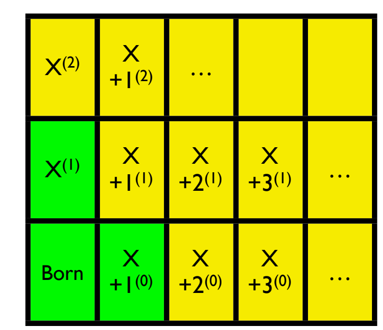

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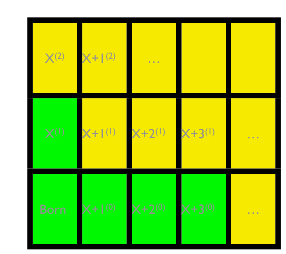
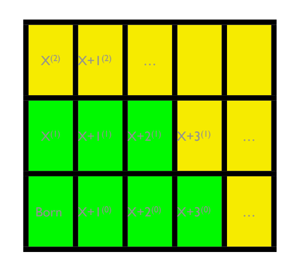
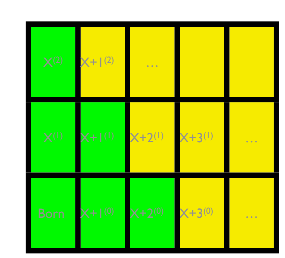


NLO multileg matching



Matching at NNLO



QCD

Lecture IV

$X^{(2)}$	$X+1^{(2)}$	...		
$X^{(1)}$	$X+1^{(1)}$	$X+2^{(1)}$	$X+3^{(1)}$	...
Born	$X+1^{(0)}$	$X+2^{(0)}$	$X+3^{(0)}$	...

Phase-Space Slicing

Matching to Tree-Level Matrix Elements

A.K.A. CKKW, CKKW-L, MLM

Phase Space Slicing

(with "matching scale")

Born × Shower

+ shower veto above p_T

$X^{(2)}$	$X+1^{(2)}$	…		
$X^{(1)}$	$X+1^{(1)}$	$X+2^{(1)}$	$X+3^{(1)}$	…
Born	$X+1^{(0)}$	$X+2^{(0)}$	$X+3^{(0)}$	…

X+1 @ LO × Shower

with 1 jet above p_T

	$X+1^{(2)}$	…		
	$X+1^{(1)}$	$X+2^{(1)}$	$X+3^{(1)}$	…
	$X+1^{(0)}$	$X+2^{(0)}$	$X+3^{(0)}$	…

… Fixed-Order Matrix Element

… Shower Approximation

… Fixed-Order ME above p_T cut & nothing below

Phase Space Slicing

(with "matching scale")

Born × Shower **+** **X+1 @ LO × Shower**

+ shower veto above p_T with 1 jet above p_T

X+1 now correct in both soft and hard limits

$X^{(2)}$	$X+1^{(2)}$	…		
$X^{(1)}$	$X+1^{(1)}$	$X+2^{(1)}$	$X+3^{(1)}$	…
Born	$X+1^{(0)}$	$X+2^{(0)}$	$X+3^{(0)}$	…

- … Fixed-Order Matrix Element
- … Shower Approximation
- … Fixed-Order ME above p_T cut & nothing below
- … Fixed-Order ME above p_T cut & Shower Approximation below

Multi-Leg Slicing

(a.k.a. CKKW or MLM matching)

CKKW: Catani, Krauss, Kuhn, Webber, JHEP 0111:063,2001.

MLM: Michelangelo L Mangano

Keep going

Veto all shower emissions above "matching scale"

Except for the highest-multiplicity matrix element (not competing with anyone)

Multileg Tree-level matching:

$X^{(2)}$	$X+1^{(2)}$	…		
$X^{(1)}$	$X+1^{(1)}$	$X+2^{(1)}$	$X+3^{(1)}$	…
Born	$X+1^{(0)}$	$X+2^{(0)}$	$X+3^{(0)}$	…

Precision:

LO: when all jets hard

Still LL: for soft emissions

Classic Example

W + Jets

Number of jets in pp→W+X at the LHC

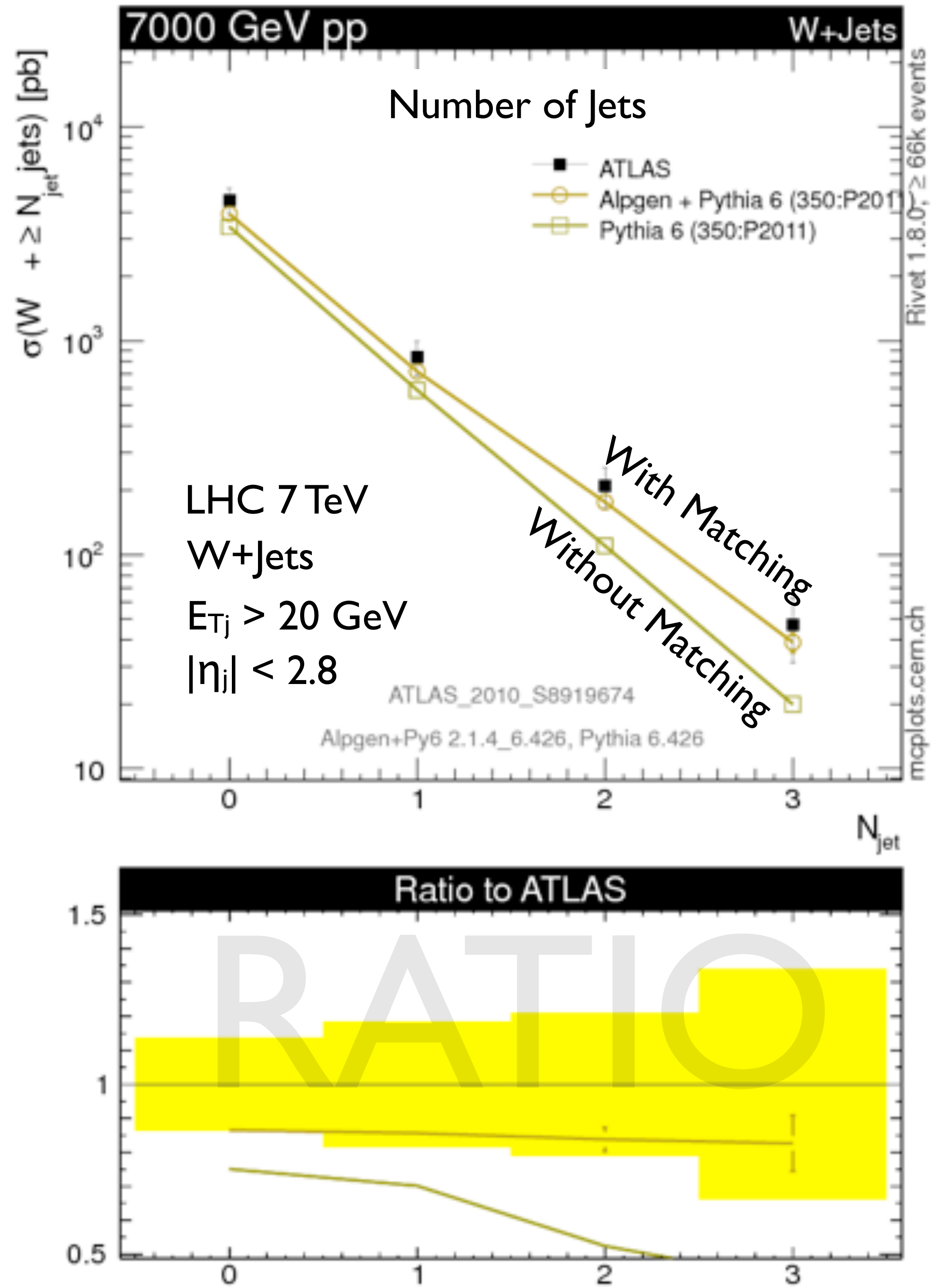
From 0 (W inclusive) to W+3 jets

PYTHIA includes matching up to W+1 jet + shower

With ALPGEN, also the LO matrix elements for 2 and 3 jets are included

But Normalization still only LO

mcplots.cern.ch



Classic Example

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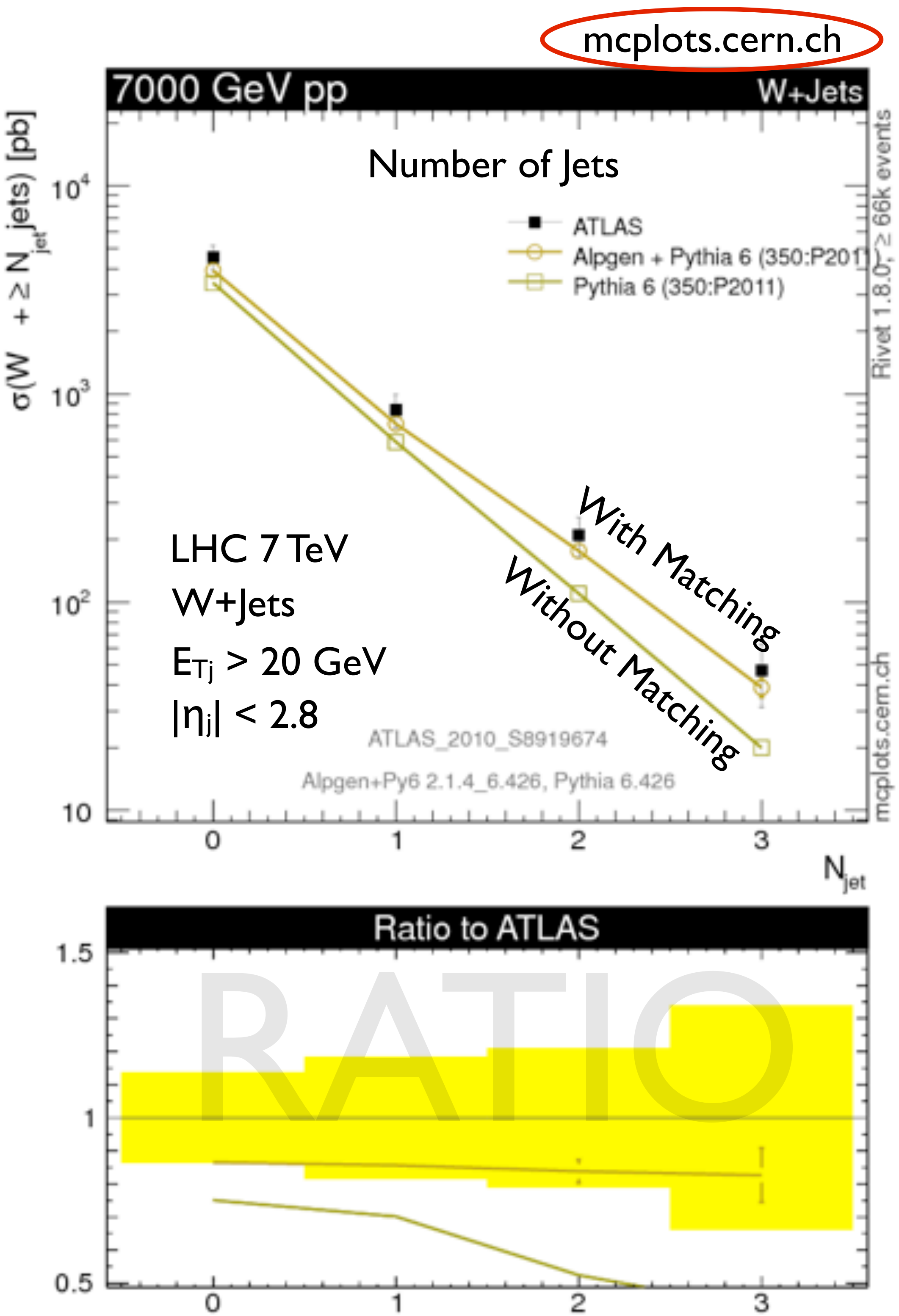


Slicing: Some Subtleties

Choice of slicing scale (=matching scale)

Fixed order must still be reliable when regulated with this scale

→ matching scale should never be chosen more than ~ one order of magnitude below hard scale.

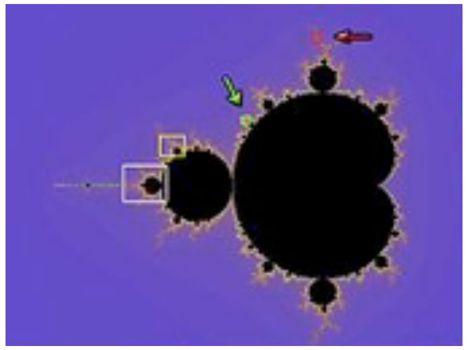
Precision still "only" Leading Order

Choice of Renormalization Scale

We already saw this can be very important (and tricky) in multi-scale problems.

Caution advised (see also supplementary slides & lecture notes)

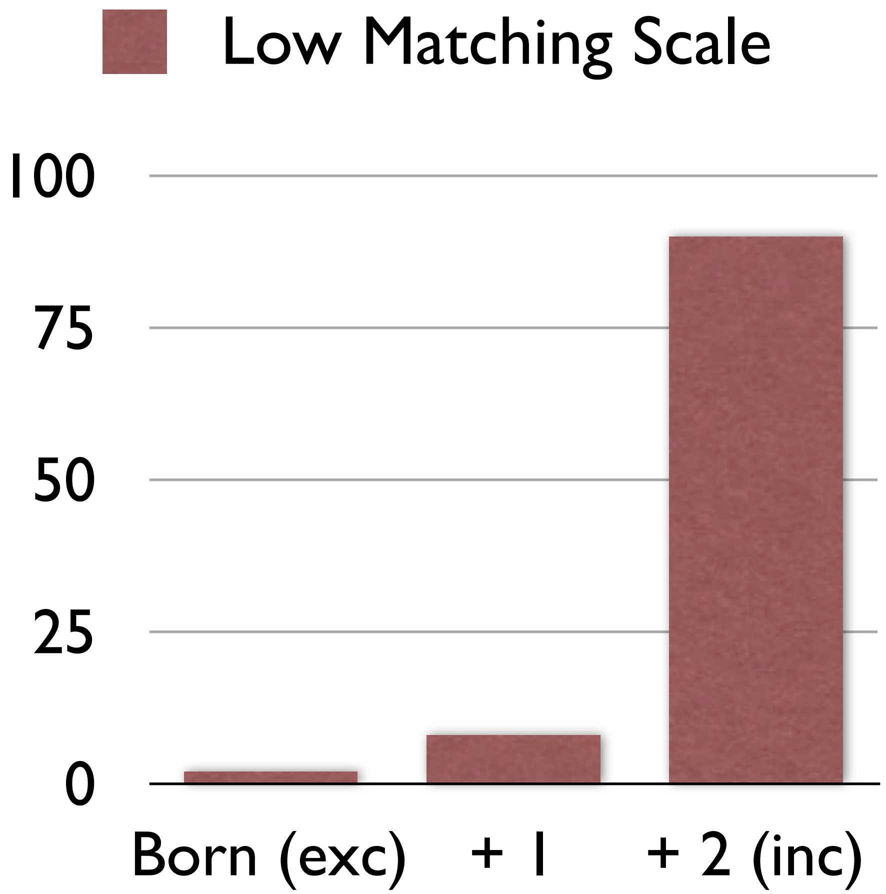
Choice of Matching Scale



Reminder: in perturbative region, QCD is approximately ***scale invariant***

→ A scale of 20 GeV for a W boson becomes 40 GeV for something weighing $2M_W$, etc … (+ adjust for C_A/C_F if g-initiated)

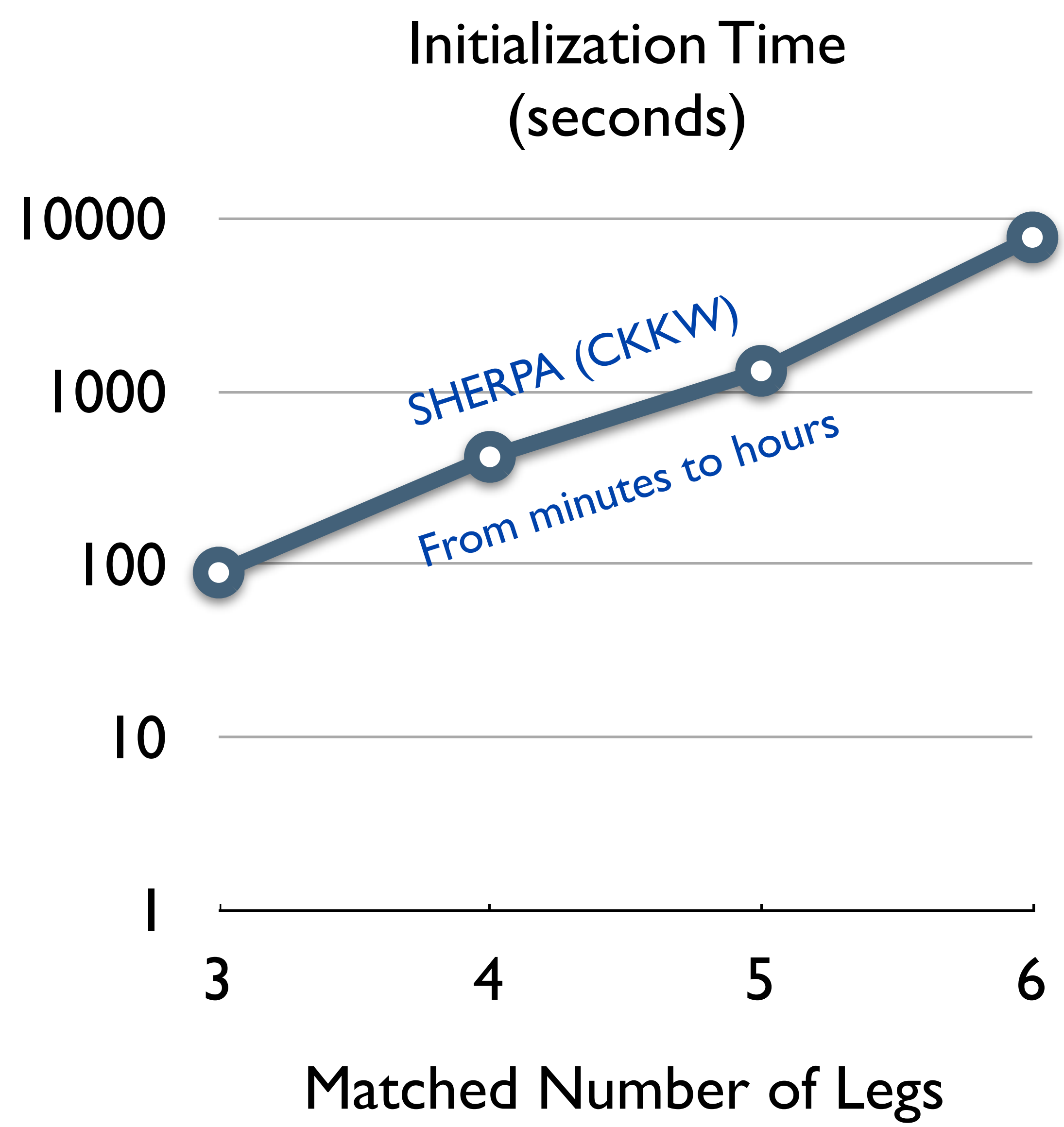
→ The matching scale should be written as a **ratio** (Bjorken scaling)
Using a too low matching scale → everything just becomes highest ME

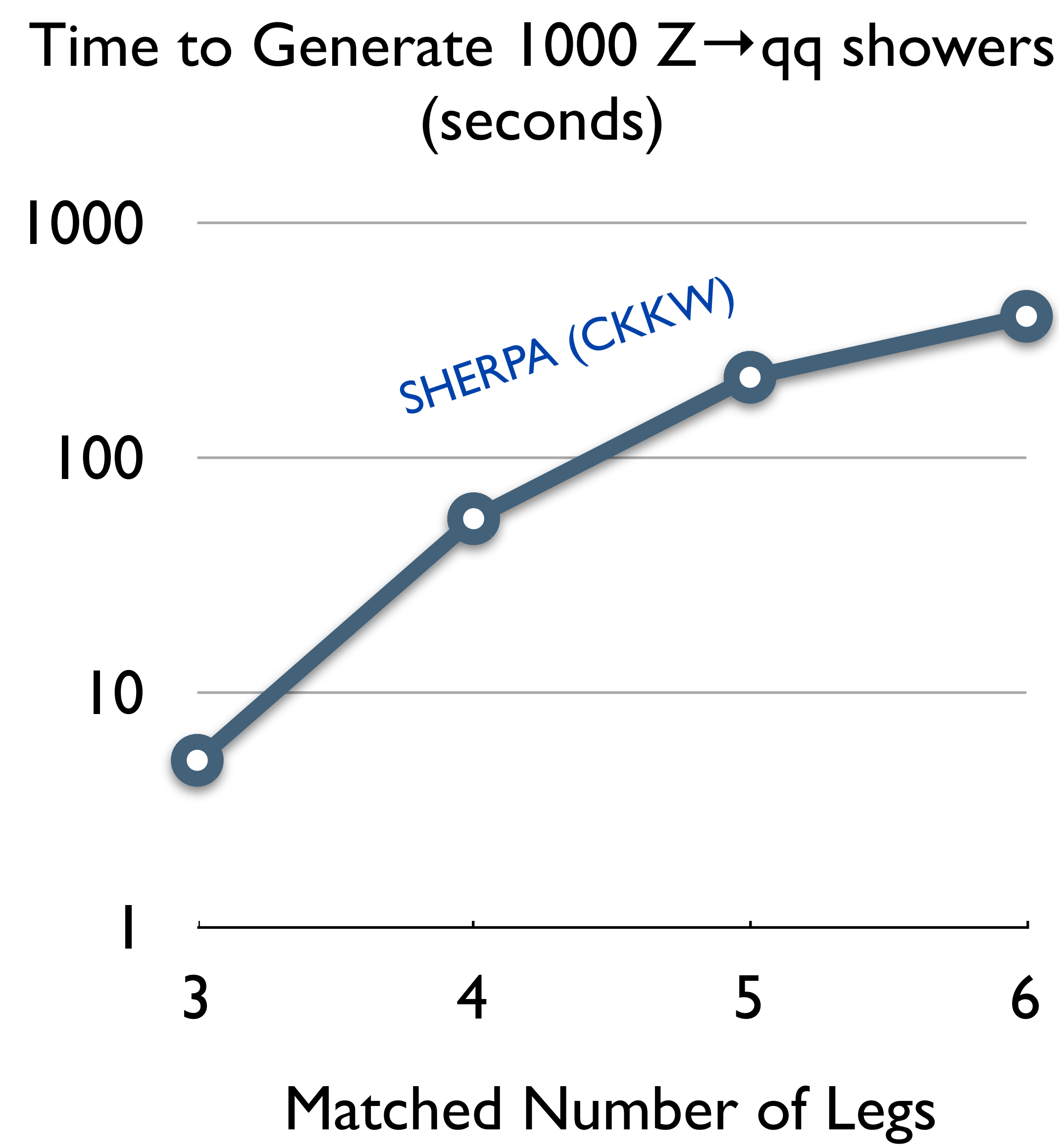


Caveat emptor: showers generally do not include helicity correlations

Phase-Space Slicing: SPEED

Here's what it costs





Z→qq *(q=udscb)* + shower. Matched and unweighted. Hadronization off

gfortran/g++ with gcc v.4.4 -O2 on single 3.06 GHz processor with 4GB memory

Generator Versions: Pythia 6.425 *(Perugia 2011 tune)*, Pythia 8.150, Sherpa 1.3.0, Vincia 1.026 *(without uncertainty bands, NLL/NLC=OFF)*

$X^{(2)}$	$X+1^{(2)}$	…		
$X^{(1)}$	$X+1^{(1)}$	$X+2^{(1)}$	$X+3^{(1)}$	…
Born	$X+1^{(0)}$	$X+2^{(0)}$	$X+3^{(0)}$	…

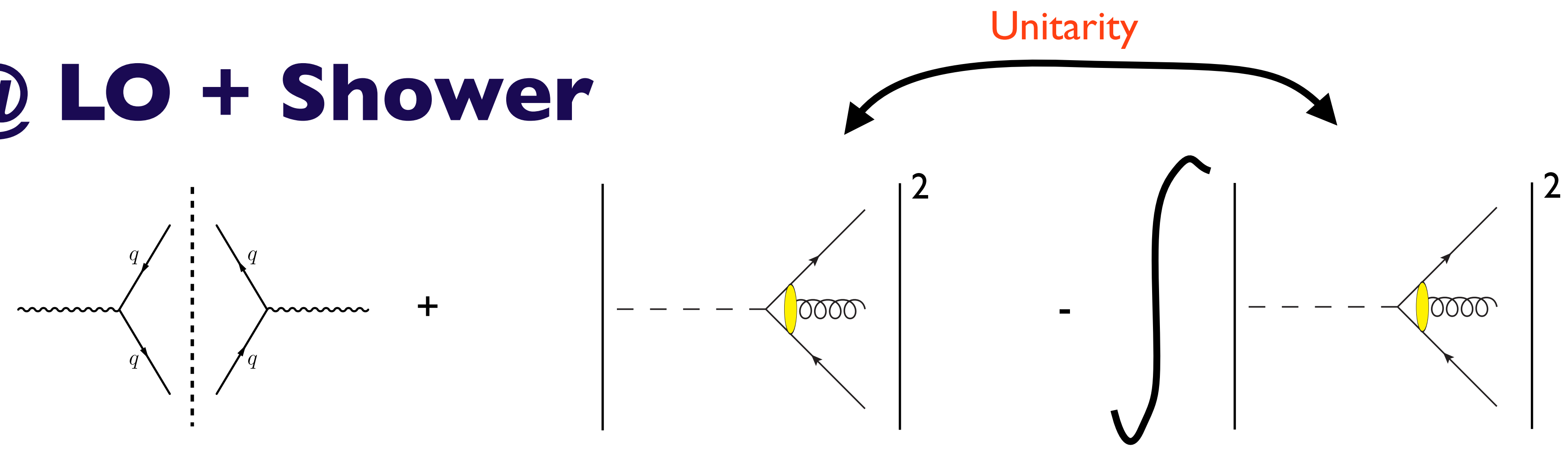
Subtraction

Matching to Born+NLO Matrix Elements

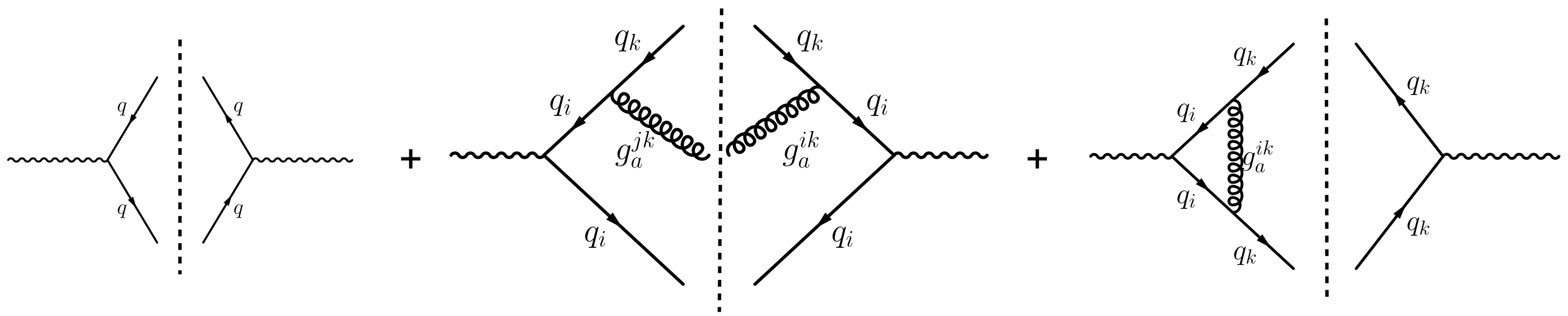
A.K.A. MC@NLO, POWHEG, VINCIA$_{\text{[incl X+n @ LO]}}$

Showers vs NLO

X @ LO + Shower

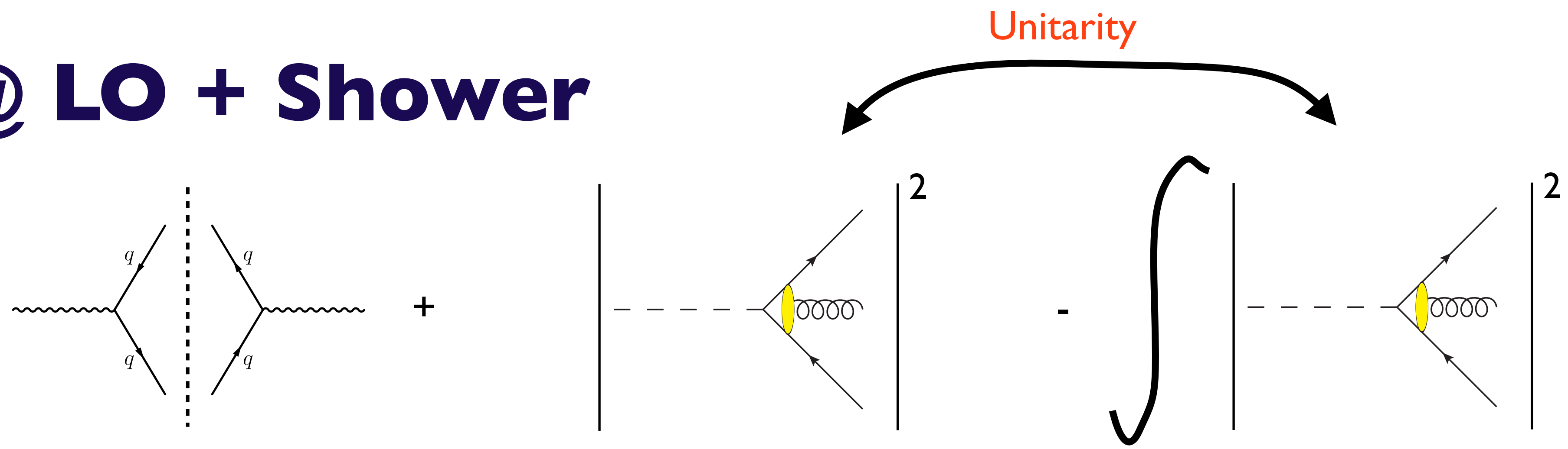


X @ NLO

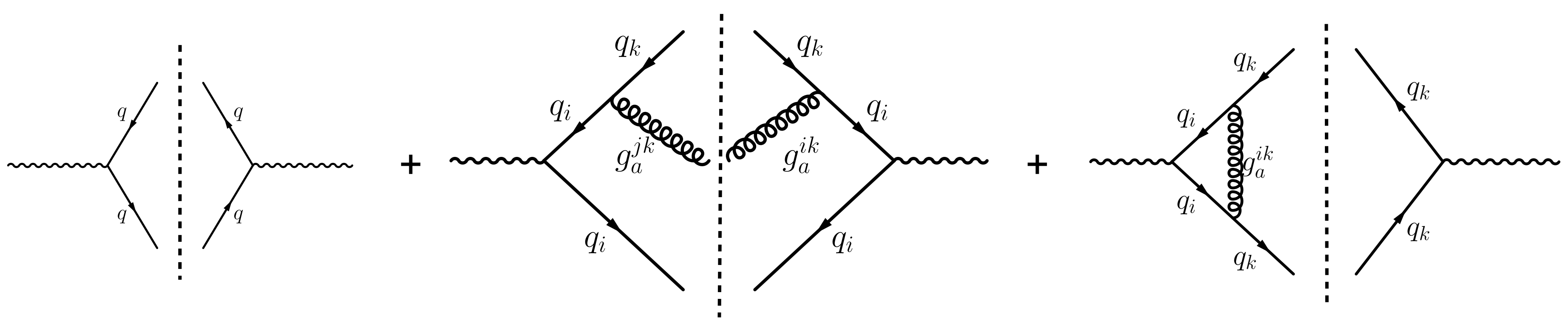


Showers vs NLO

X @ LO + Shower



X @ NLO



MC@NLO : Subtraction

LO × Shower

$X^{(2)}$	$X+1^{(2)}$	…		
$X^{(1)}$	$X+1^{(1)}$	$X+2^{(1)}$	$X+3^{(1)}$	…
Born	$X+1^{(0)}$	$X+2^{(0)}$	$X+3^{(0)}$	…

NLO

$X^{(2)}$	$X+1^{(2)}$	…		
$X^{(1)}$	$X+1^{(1)}$	$X+2^{(1)}$	$X+3^{(1)}$	…
Born	$X+1^{(0)}$	$X+2^{(0)}$	$X+3^{(0)}$	…

… Fixed-Order Matrix Element

… Shower Approximation

MC@NLO : Subtraction

Born × Shower

$X^{(2)}$	$X+1^{(2)}$	…		
$X^{(1)}$	$X+1^{(1)}$	$X+2^{(1)}$	$X+3^{(1)}$	…
Born	$X+1^{(0)}$	$X+2^{(0)}$	$X+3^{(0)}$	…

… Fixed-Order Matrix Element

… Shower Approximation

NLO - Shower$_{NLO}$

$X^{(2)}$	$X+1^{(2)}$	…		
$X^{(1)}$	$X+1^{(1)}$	$X+2^{(1)}$	$X+3^{(1)}$	…
Born	$X+1^{(0)}$	$X+2^{(0)}$	$X+3^{(0)}$	…

Expand shower approximation to NLO analytically, then subtract:

… Fixed-Order ME minus Shower Approximation (NOTE: can be < 0!)

MC@NLO : Subtraction

Born × Shower

$X^{(2)}$	$X+1^{(2)}$	…		
$X^{(1)}$	$X+1^{(1)}$	$X+2^{(1)}$	$X+3^{(1)}$	…
Born	$X+1^{(0)}$	$X+2^{(0)}$	$X+3^{(0)}$	…

… Fixed-Order Matrix Element

… Shower Approximation

(NLO - Shower$_{NLO}$) × Shower

$X^{(1)}$	$X^{(1)}$	…		
$X^{(1)}$	$X^{(1)}$	$X^{(1)}$	$X^{(1)}$	…
Born	$X+1^{(0)}$	$X^{(1)}$	$X^{(1)}$	…

… Fixed-Order ME minus Shower Approximation (NOTE: can be < 0!)

… Subleading corrections generated by shower off subtracted ME

MC@NLO : Subtraction

Combine → MC@NLO

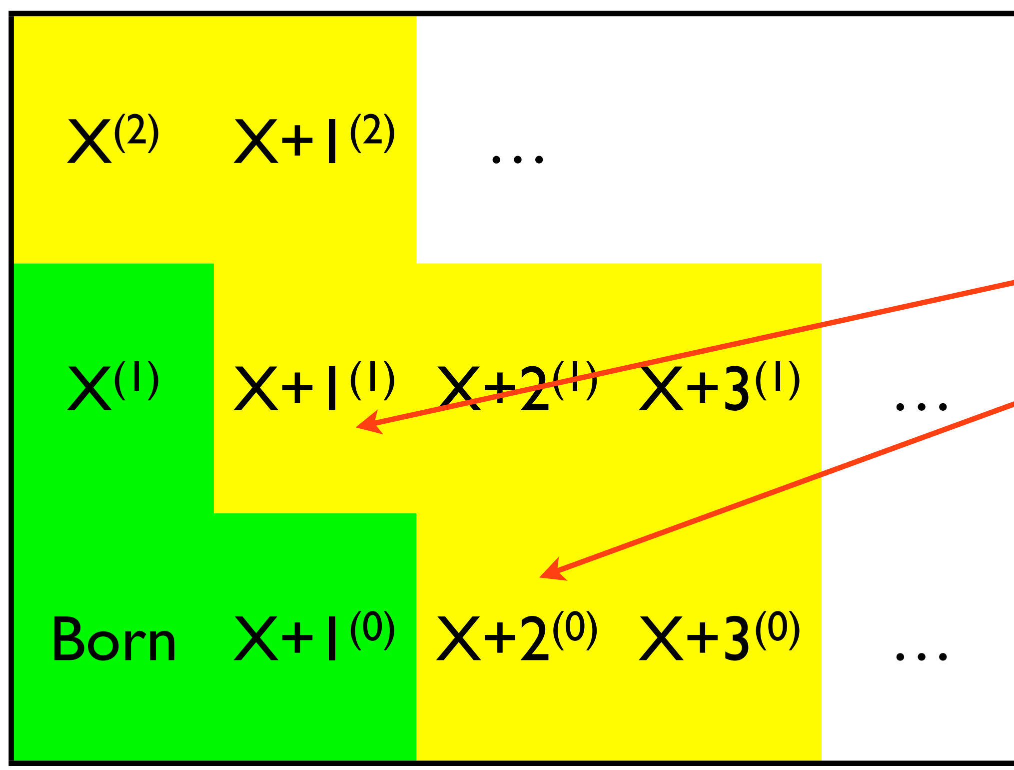
Frixione, Webber, JHEP 0206 (2002) 029

Consistent NLO + parton shower (though correction events can have w<0)

Recently, has been almost fully automated in aMC@NLO

Frederix, Frixione, Hirschi, Maltoni, Pittau, Torrielli, JHEP 1202 (2012) 048

NLO: for X inclusive
LO for X+1
LL: for everything else



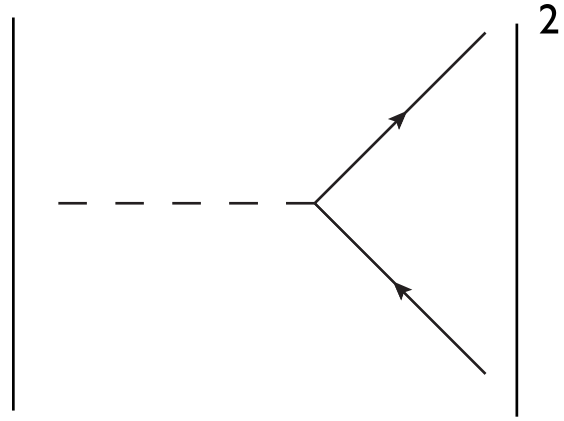
Note 1: NOT NLO for X+1

Note 2: Multijet tree-level matching still superior for X+2

w < 0 are a problem because they kill efficiency:
E.g, 1000 positive-weight - 999 negative-weight → statistical precision of 1 event, for 2000 generated

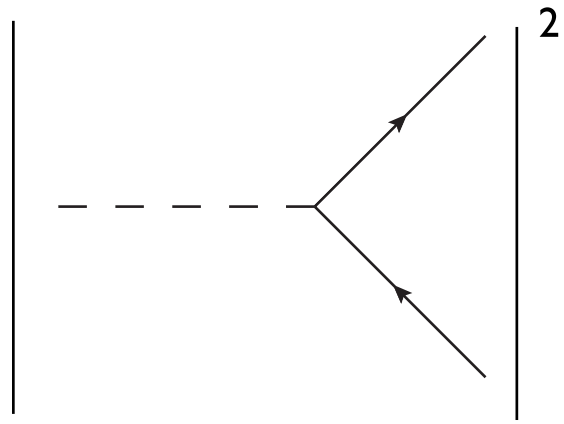
POWHEG/PYTHIA/VINCIA

Born × Shower



$$\left(1 + g_s^2\, 2C_F \left[\frac{2 s_{ik}}{s_{ij} s_{jk}} + \frac{1}{s_{IK}} \left(\frac{s_{ij}}{s_{jk}} + \frac{s_{jk}}{s_{ij}} \right) \right] + \ldots \right)$$

Born + 1 @ LO



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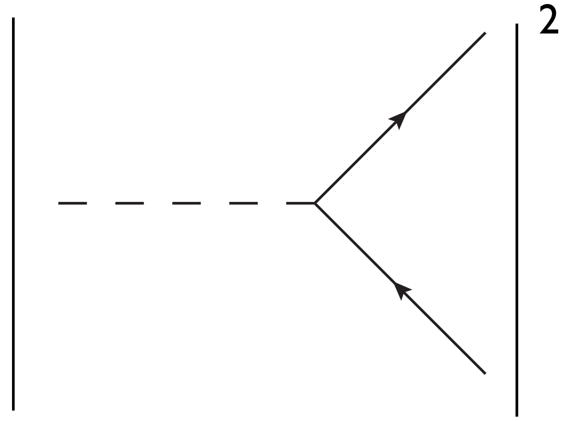
POWHEG/PYTHIA/VINCIA

Born × Shower



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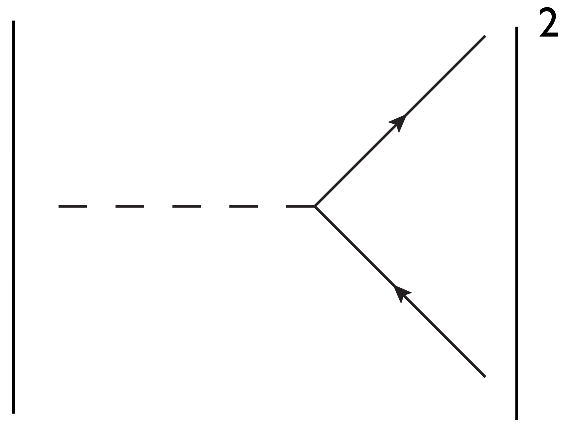

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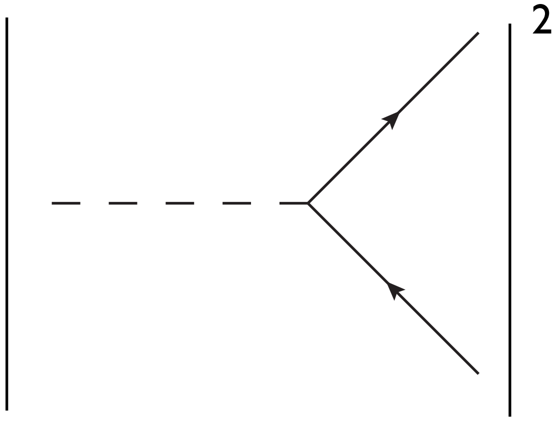
POWHEG/PYTHIA/VINCIA

Bengtsson, Sjöstrand, PLB 185 (1987) 435



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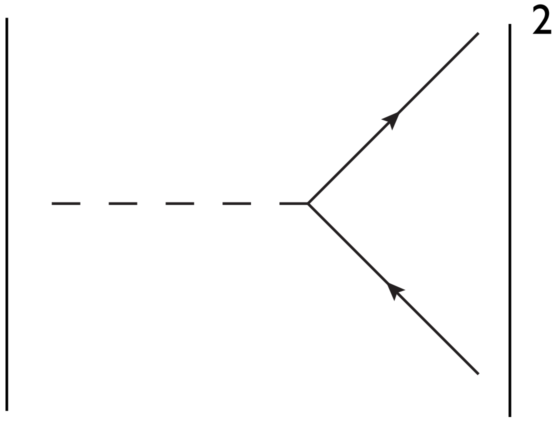

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Born × First-Order Corrected Shower

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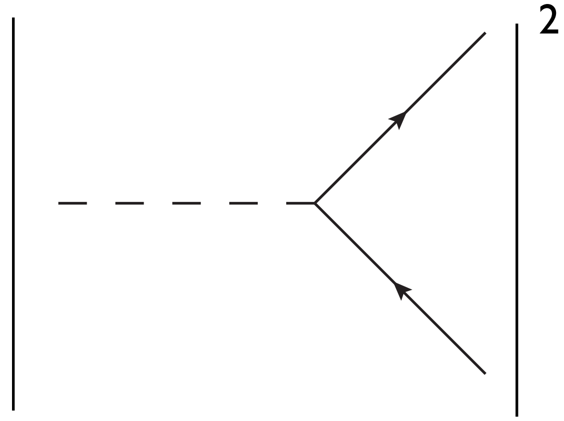

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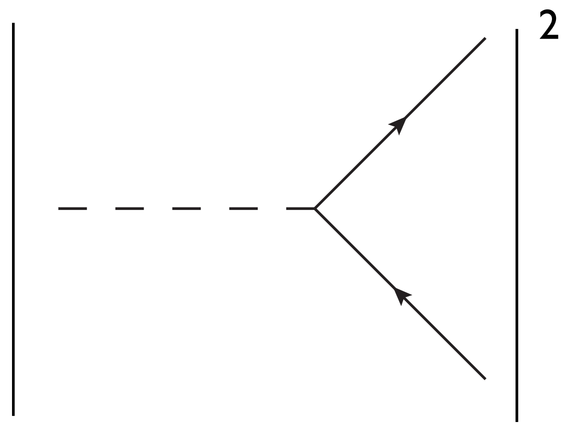

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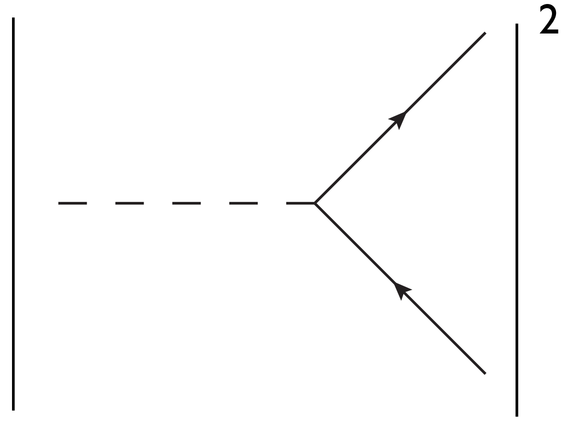

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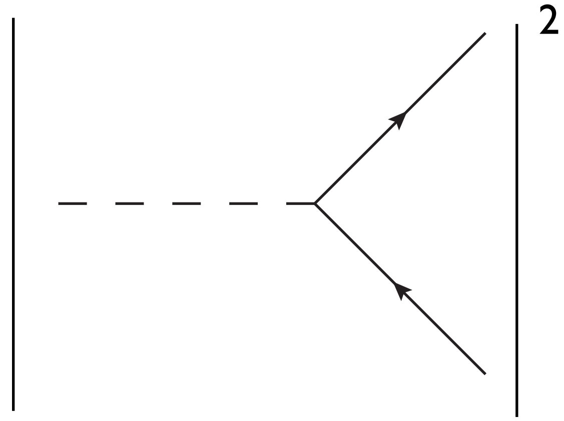

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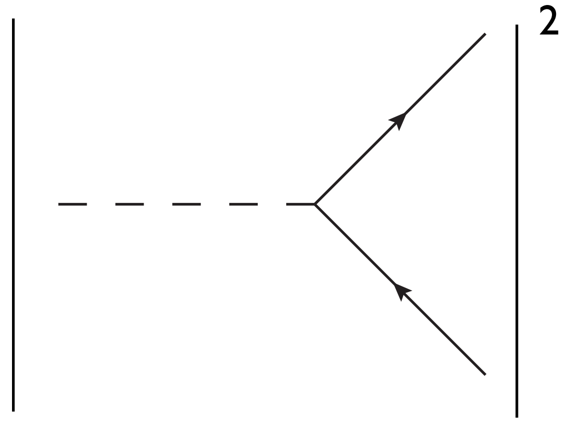
Bengtsson, Sjöstrand, PLB 185 (1987) 435



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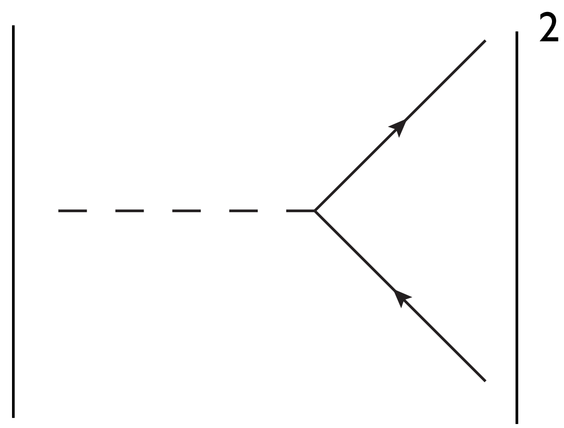

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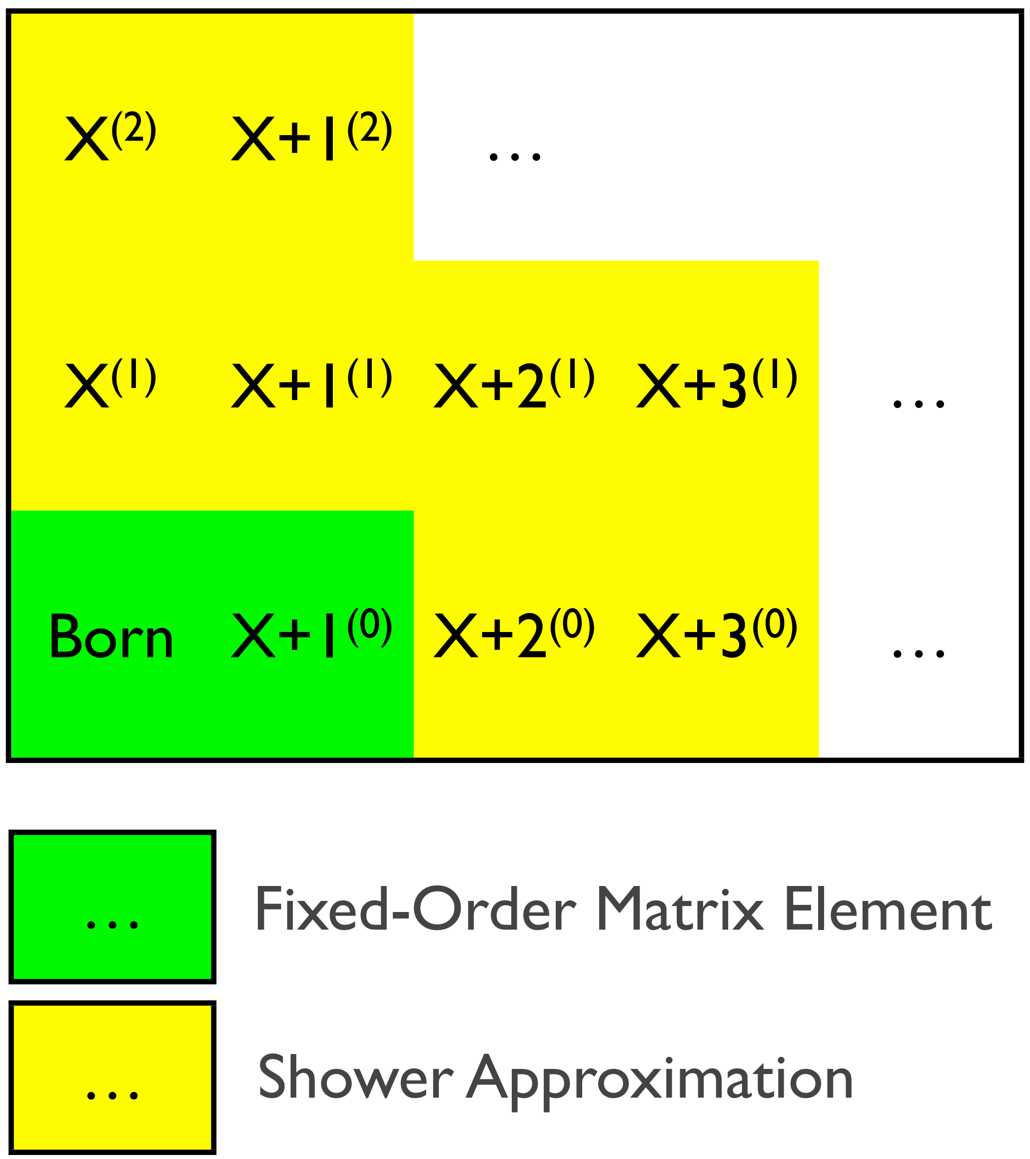
POWHEG

Combine w subtracted NLO → POWHEG

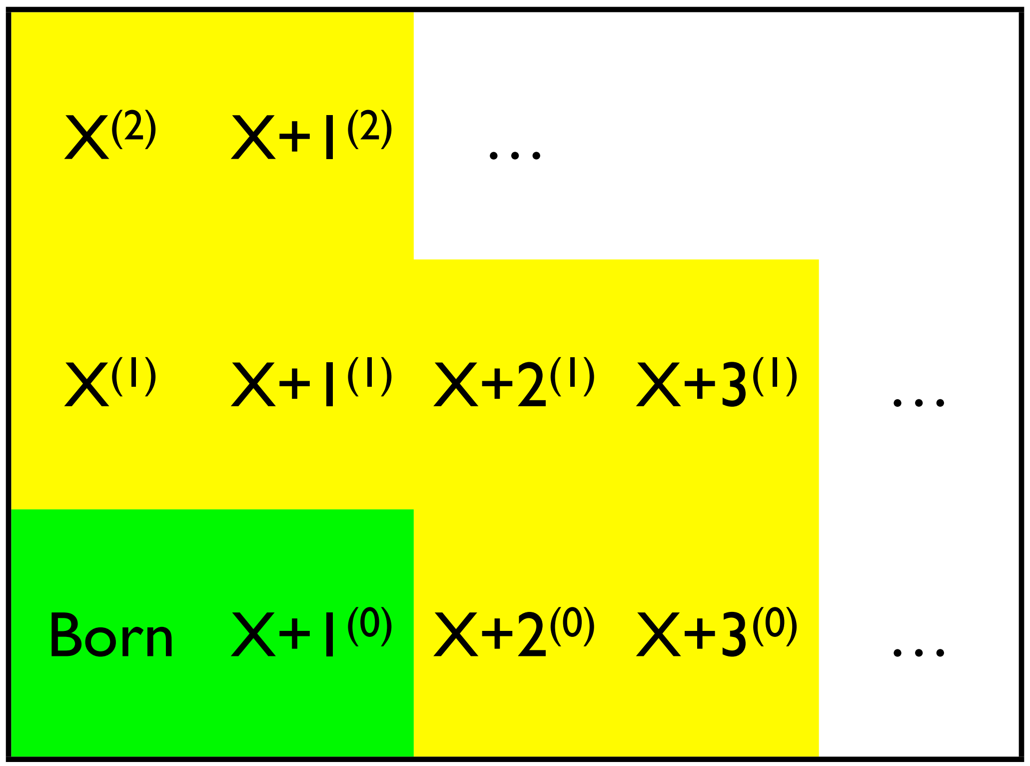
Nason, JHEP 0411 (2004) 040



Use exact (process-dependent) splitting function for first splitting(s)

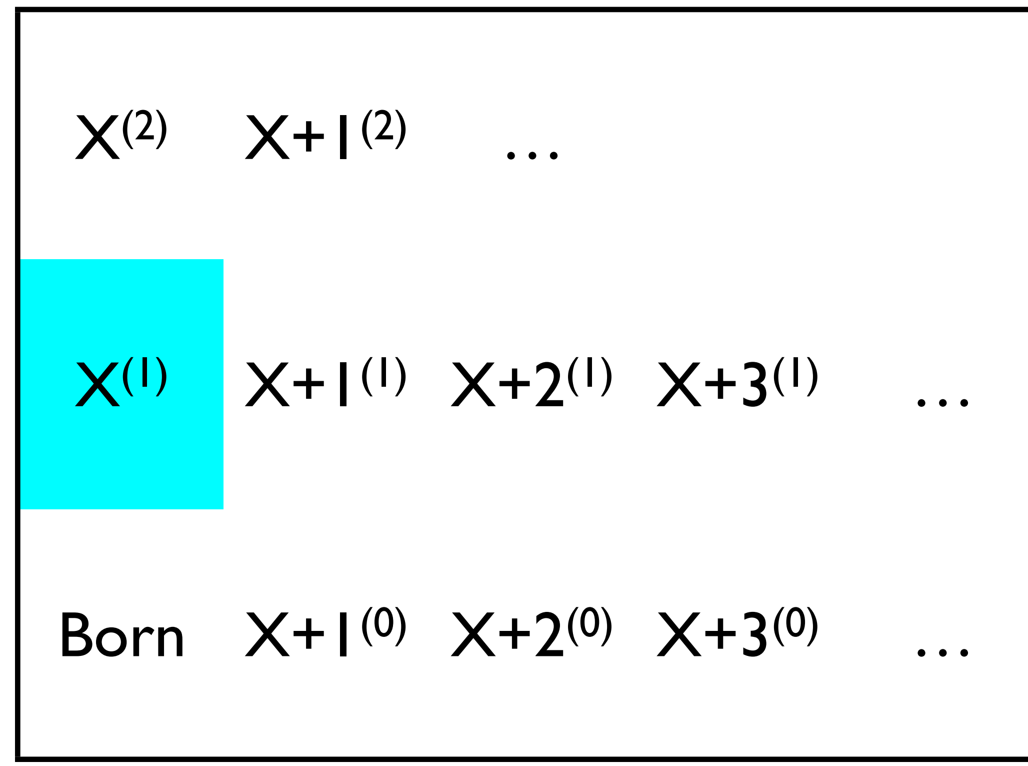
Combine w subtracted NLO → POWHEG

Nason, JHEP 0411 (2004) 040



… Fixed-Order Matrix Element

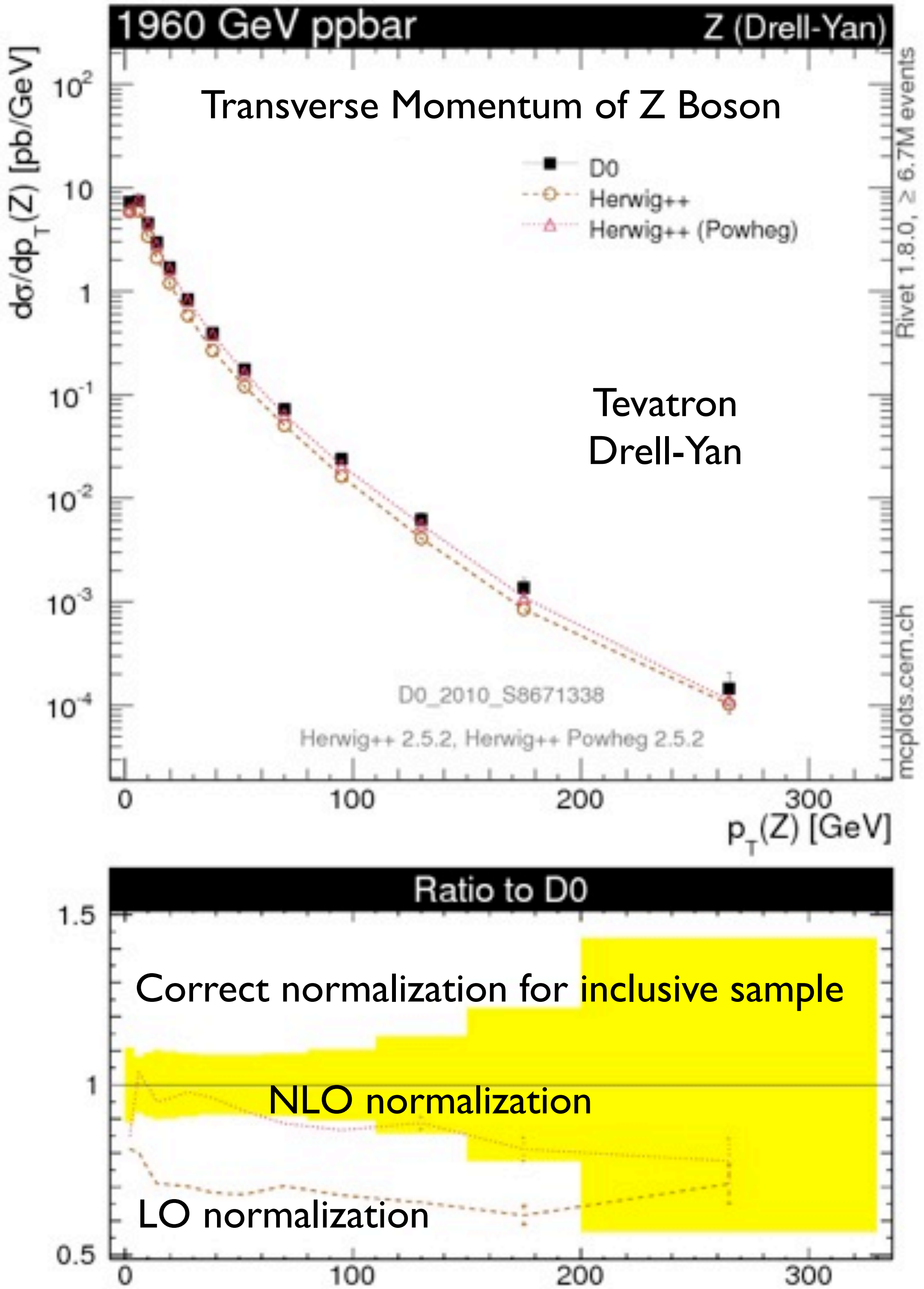
… Shower Approximation



Use exact (process-dependent) splitting function for first splitting(s)

… Fixed-Order ME minus Shower Approximation (usually positive)

Classic Example



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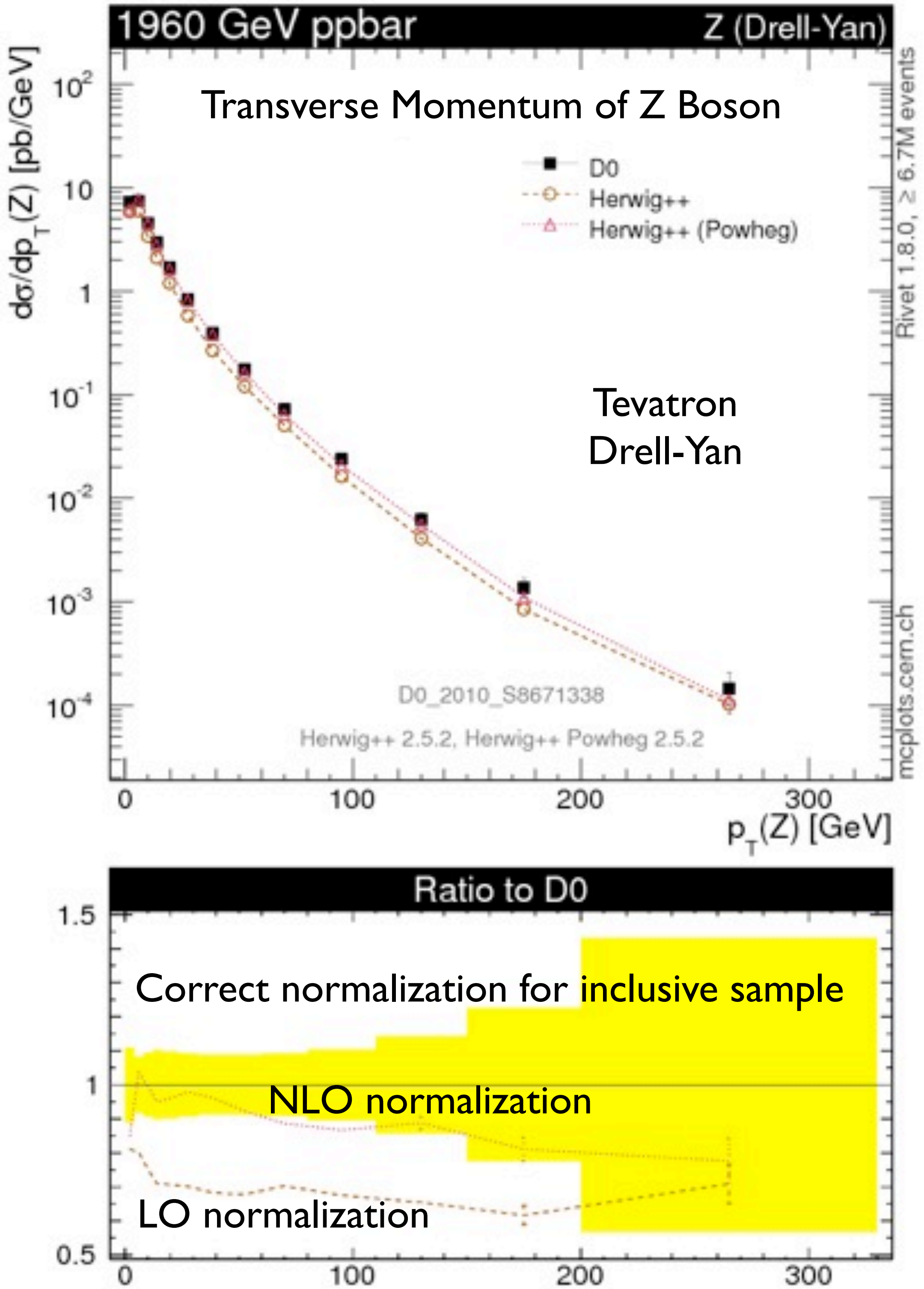

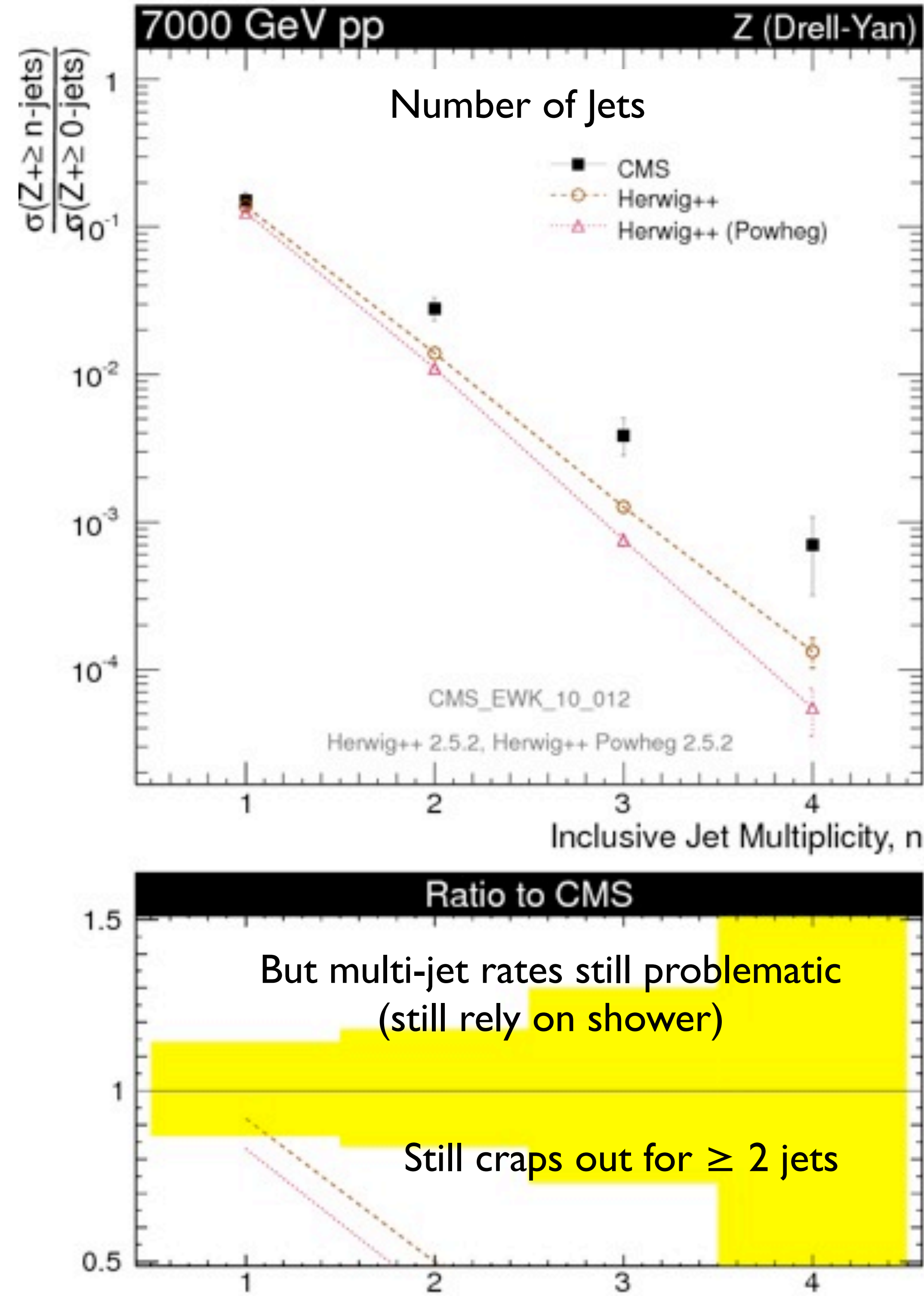

The Problem

Tree-level matching (slicing: CKKW, MLM)

Good for generating Born + several hard jets + shower

But normalization remains LO

NLO matching (MC@NLO or POWHEG)

Good for generating NLO Born + shower

But only has LO precision for Born + 1 jet

Remains pure shower for Born + more jets

ME-PS matching → ONE calculation to rule them all? Things got better, but still have to choose :(

The Best of Both?

Ideal:

Generate entire perturbative series

Use all available NLO amplitudes

When you run out of NLO amplitudes, use LO ones

When you run out of LO amplitudes, use pure shower

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Yes!

Use parton shower algorithm as phase-space generator

Knows about singular structure of QCD, so gets dominant approximately right

Use exact amplitudes as radiation kernels

Until you run out of amplitudes

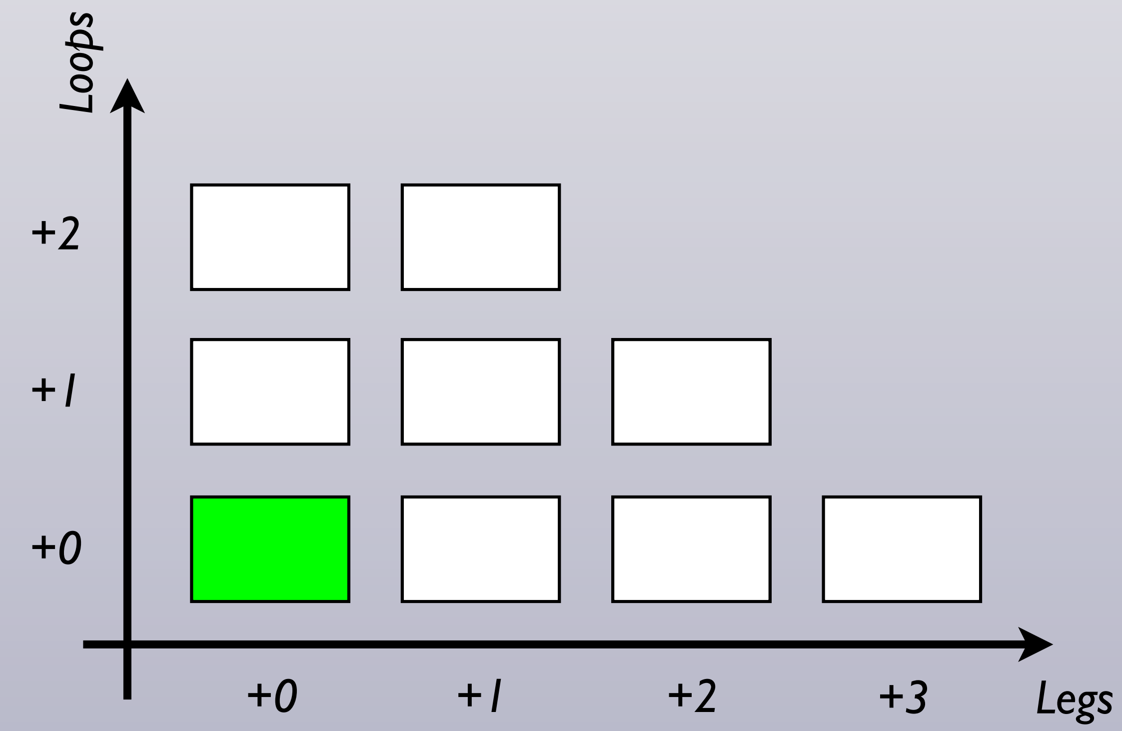
Giele, Kosower, PS, PRD 84 (2011) 054003
Lopez-Villarejo, PS, JHEP 1111 (2011) 150

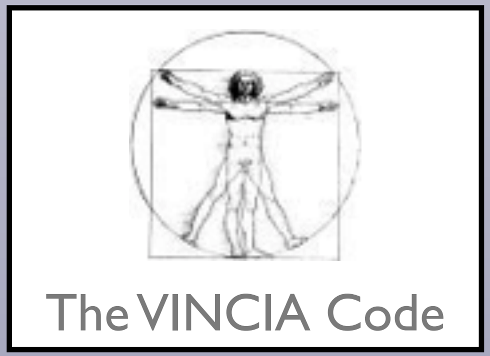
VINCIA: Markovian pQCD*

*)pQCD : perturbative QCD

Start at Born level

$$|M_F|^2$$





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PYTHIA: Sjöstrand, Mrenna, Skands, JHEP 0605 (2006) 026 & CPC 178 (2008) 852

Note: other teams working on alternative strategies
Perturbation theory is solvable → expect improvements

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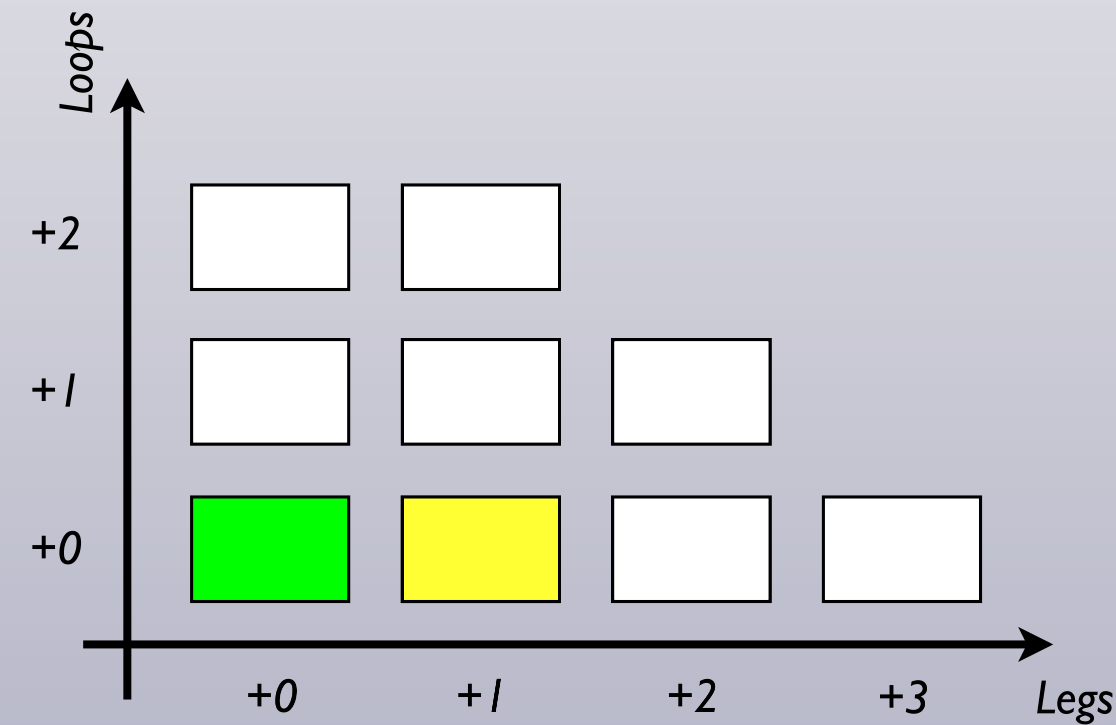

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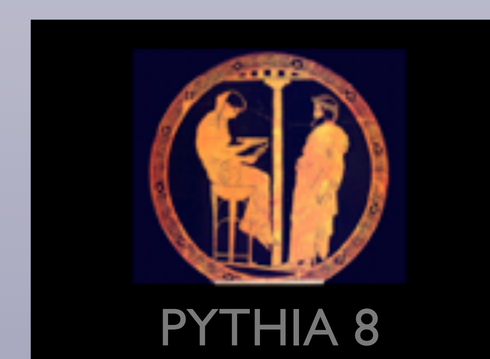

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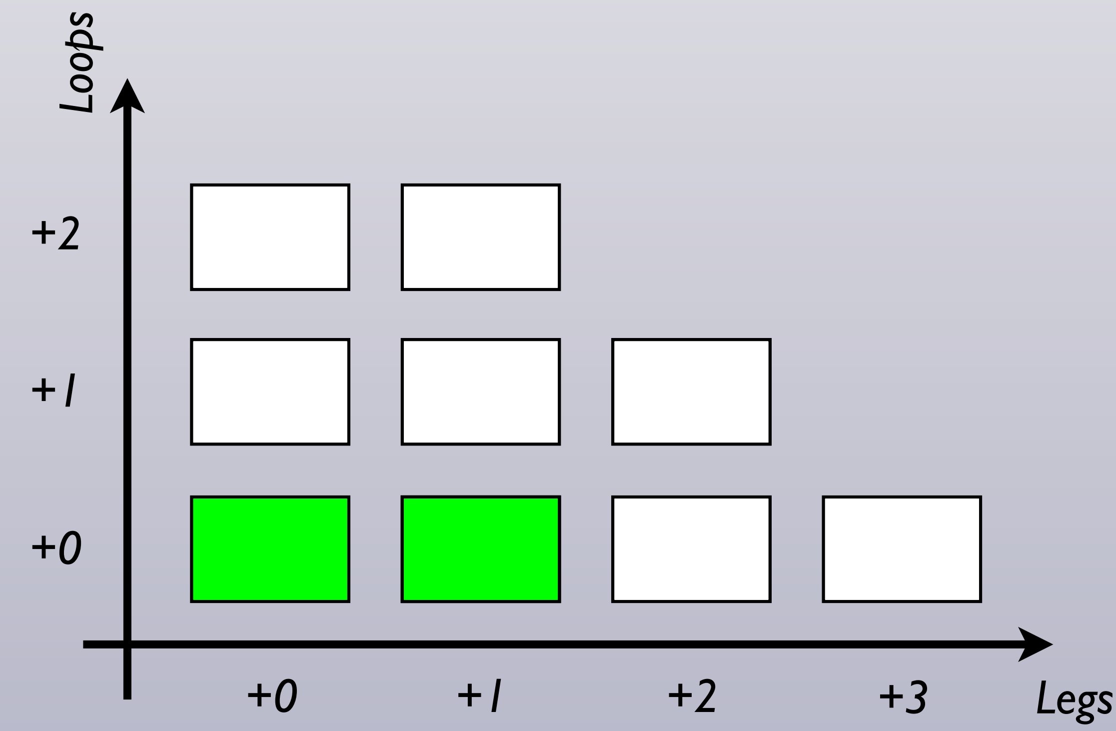

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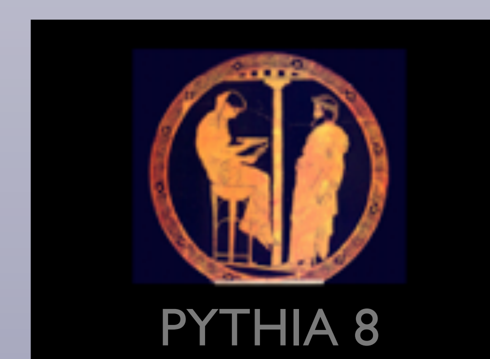

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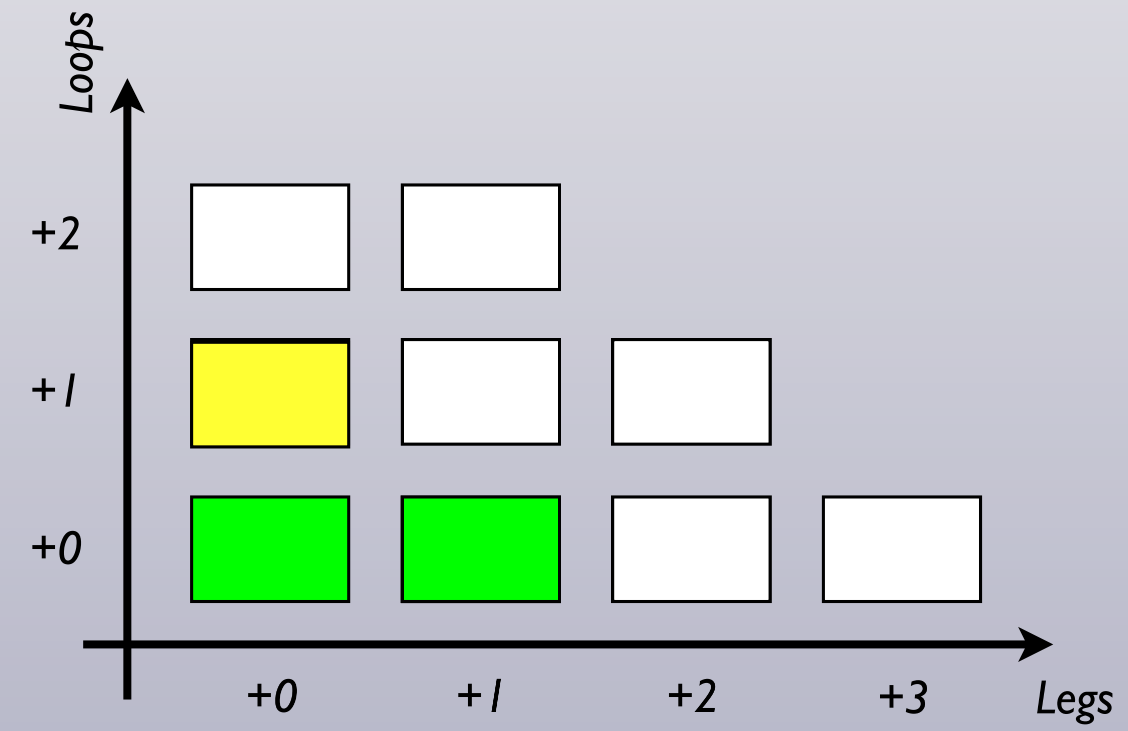
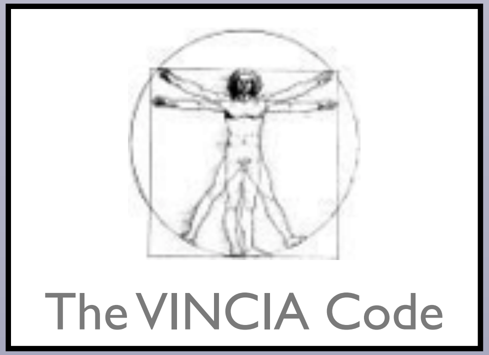
Unitarity of Shower

$$\mathrm{Virtual} = -\int \mathrm{Real}$$





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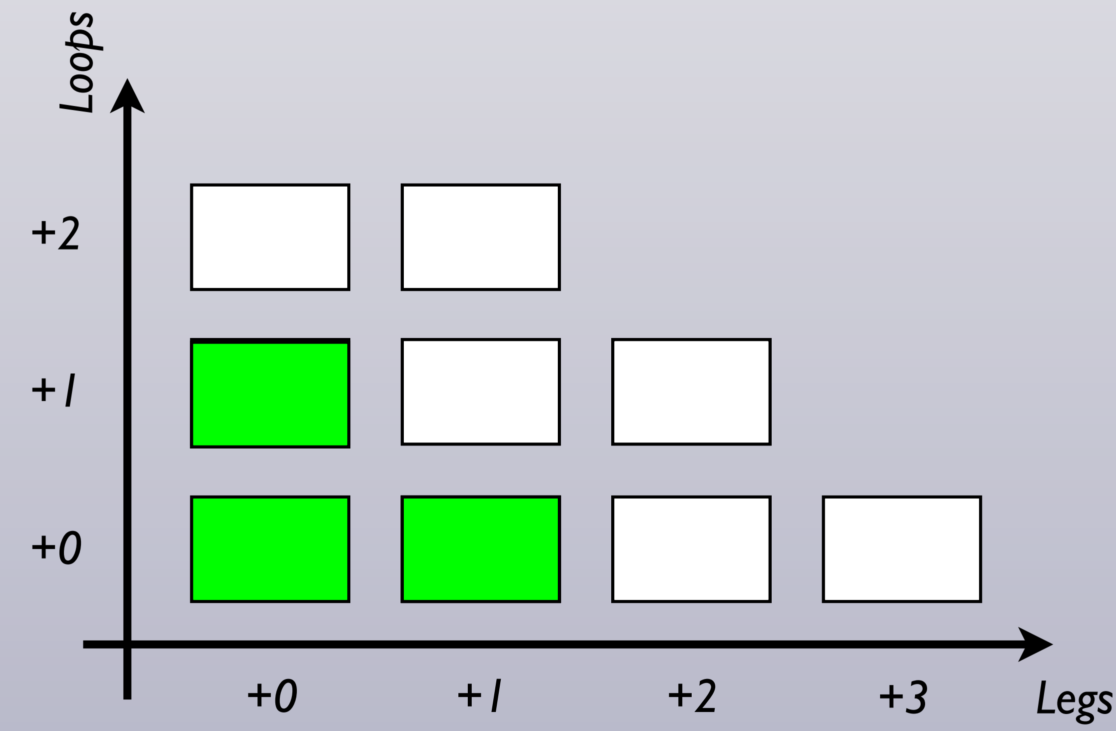

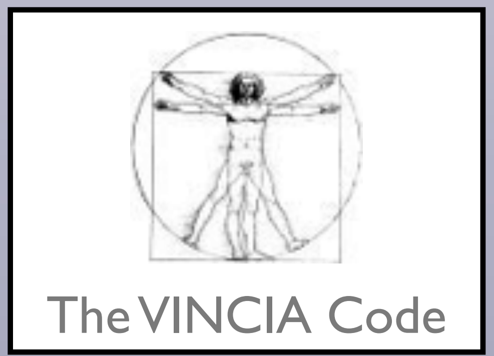

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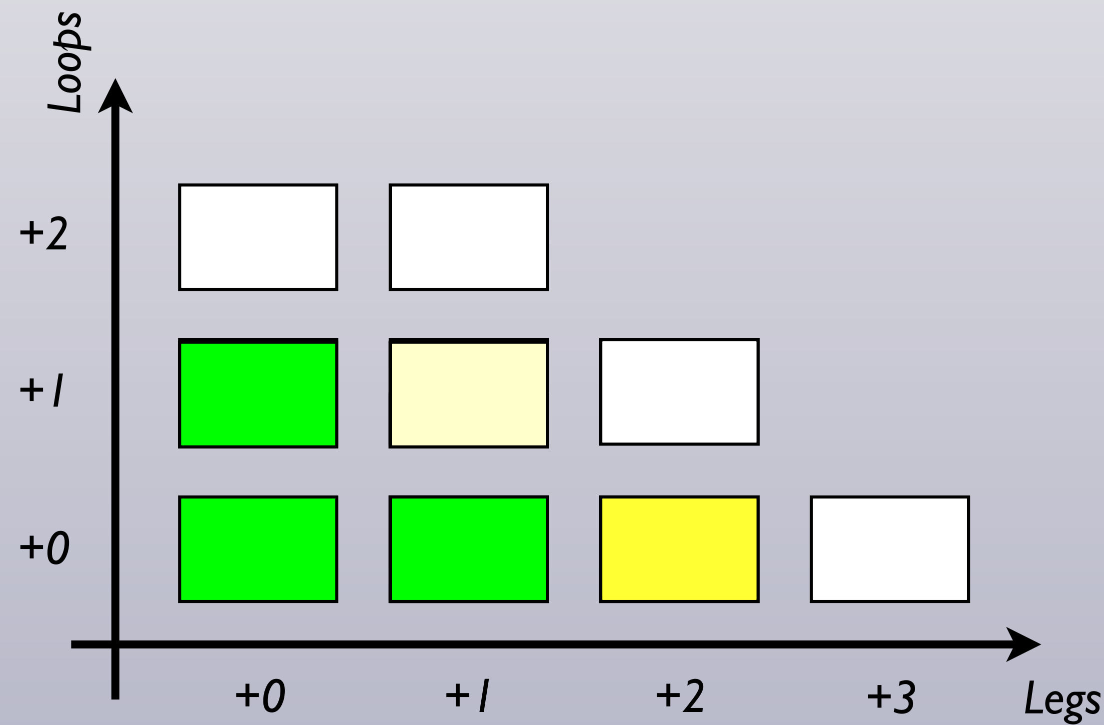

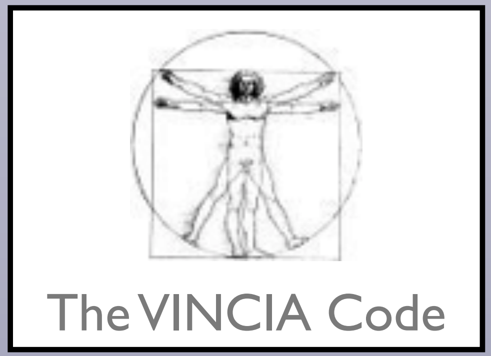

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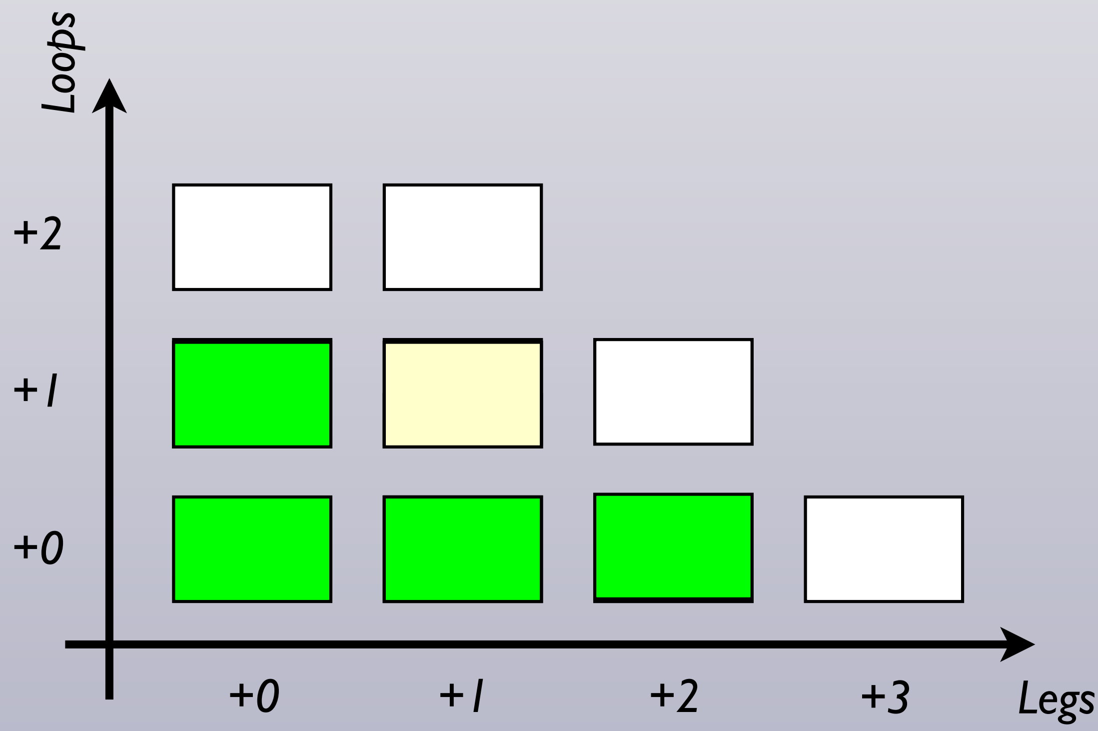

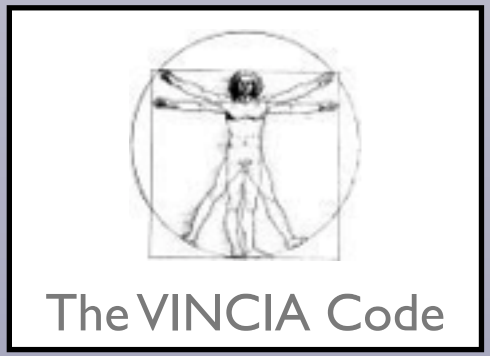

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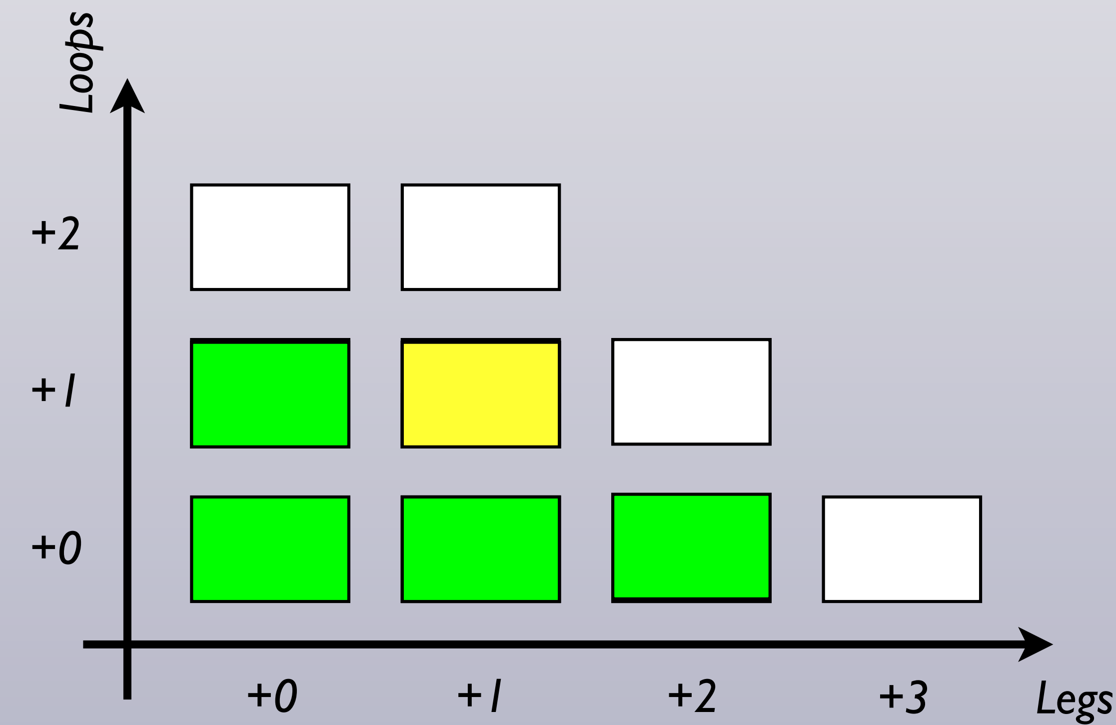

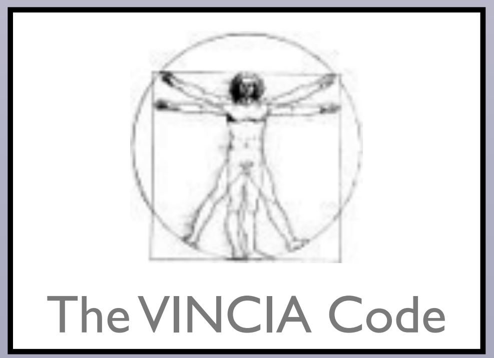

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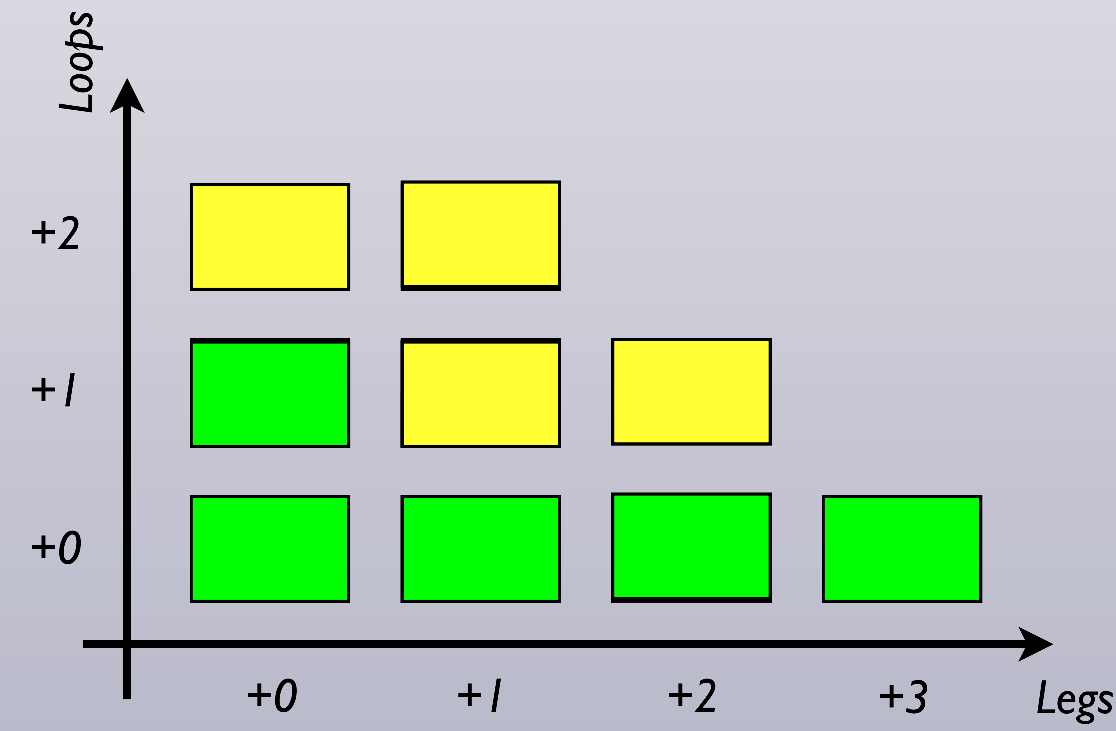

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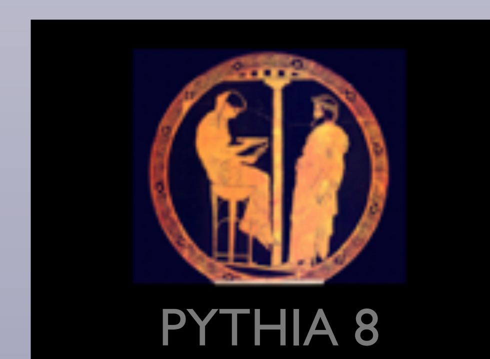

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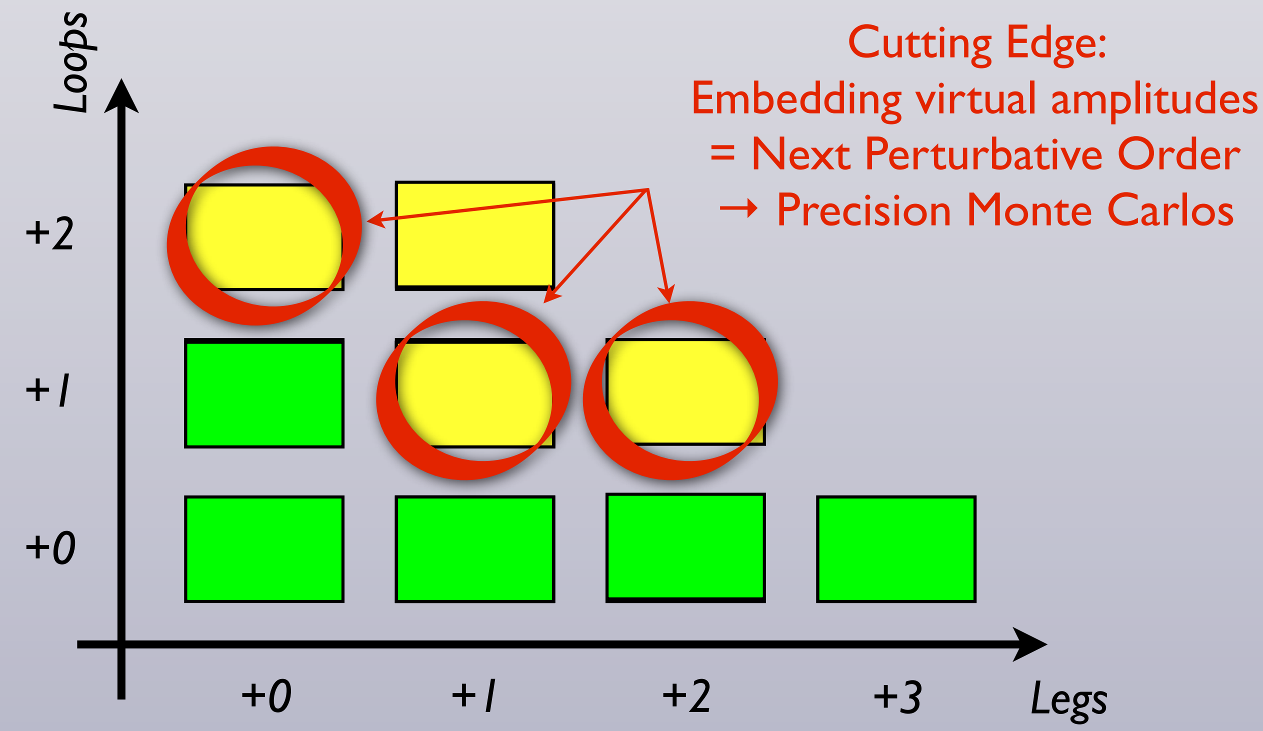
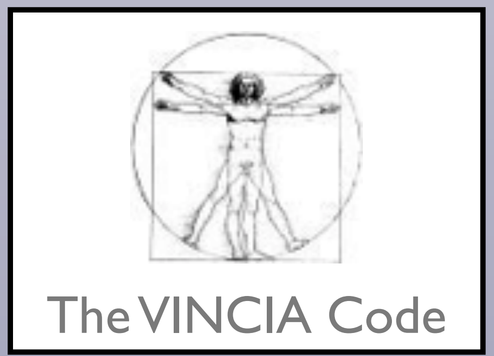
Repeat





+

VINCIA: Giele, Kosower, Skands, PRD78(2008)014026 & PRD84(2011)054003
+ ongoing work with M. Ritzmann, E. Laenen, L. Hartgring, A. Larkoski, J. Lopez-Villarejo
PYTHIA: Sjöstrand, Mrenna, Skands, JHEP 0605 (2006) 026 & CPC 178 (2008) 852

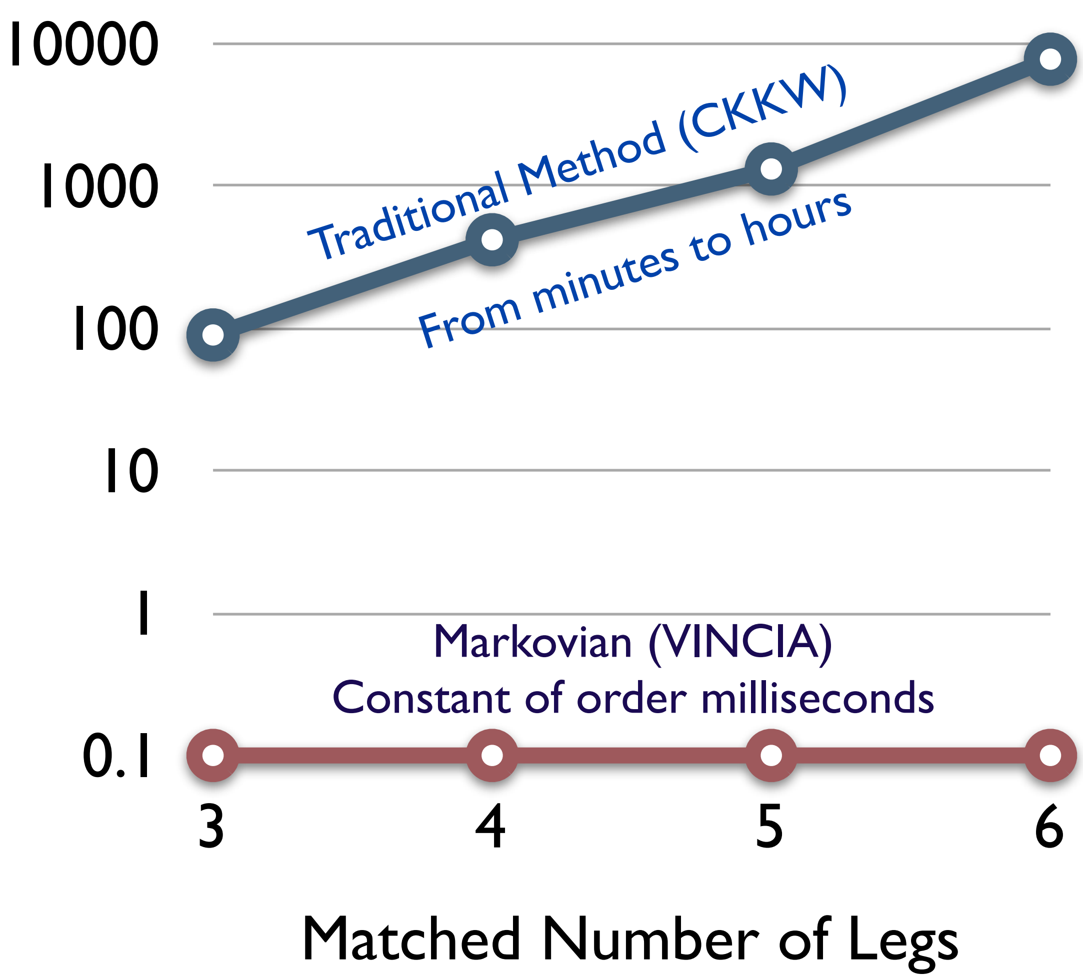
Note: other teams working on alternative strategies
Perturbation theory is solvable → expect improvements

Markov+Unitarity: SPEED

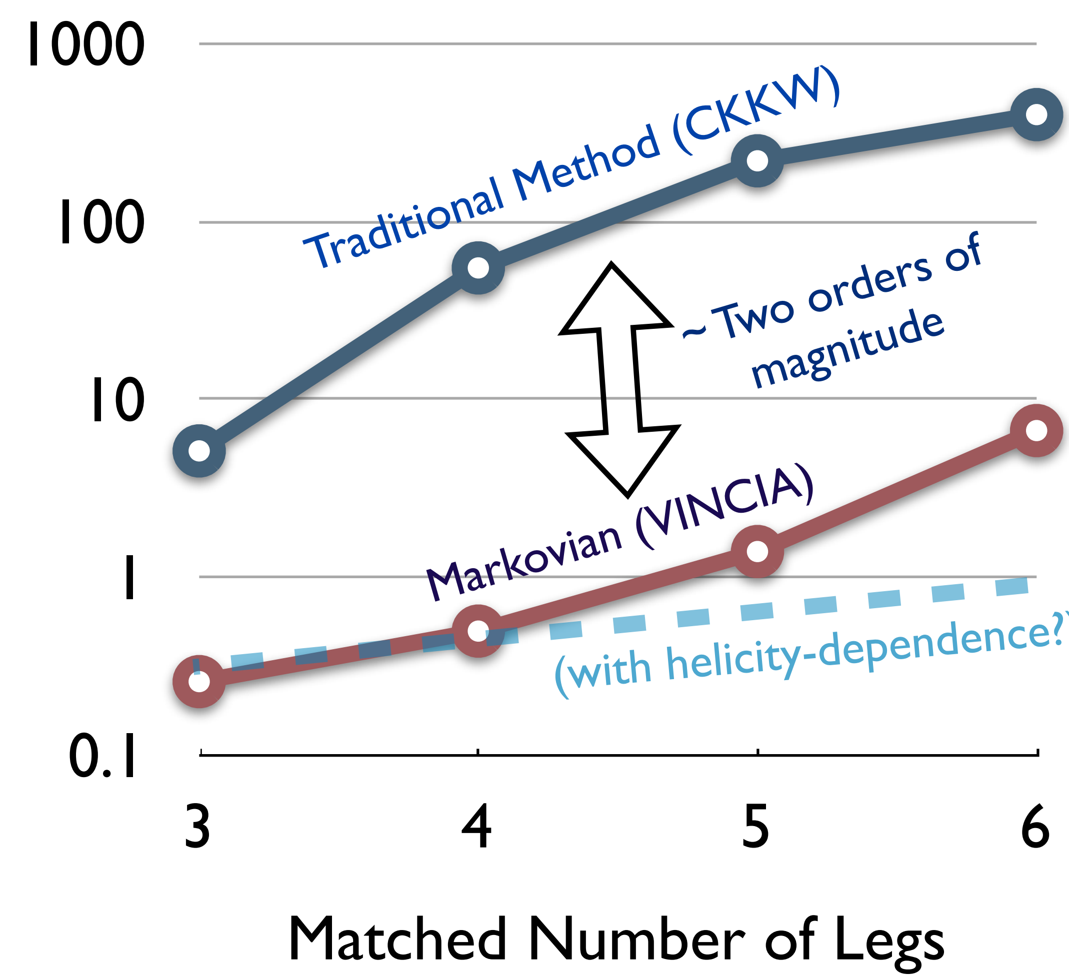
(Why I believe Markov + unitarity is the method of choice for complex problems)

Efficient Matching with Sector Showers
J. Lopez-Villarejo & PS : JHEP 1111 (2011) 150

Initialization Time
(seconds)



Time to Generate 1000 $Z \to qq$ showers
(seconds)



$Z \to qq$ *(q=udscb)* + shower. Matched and unweighted. Hadronization off
gfortran/g++ with gcc v.4.4 -O2 on single 3.06 GHz processor with 4GB memory
Generator Versions: Pythia 6.425 *(Perugia 2011 tune)*, Pythia 8.150, Sherpa 1.3.0, Vincia 1.026 *(without uncertainty bands, NLL/NLC=OFF)*

Approaches on the Market

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Hw/Py standalone

1[st] order matching for many processes, especially resonance decays

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MLM-slicing + HW or PY showers

NOTE: If you just write "AlpGen" on a plot, we assume AlpGen standalone! (no showering or matching!) - very different from Alp+Py/Hw

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QCD

Lecture IV

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NLO with subtraction, ~10% w<0

+ Herwig showers

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+ "truncated" showers + HW or PY

VINCIA + Py (Still only for Final State)

NLO + multileg with unitarity

+ dipole-antenna showers

QCD Lecture IV

Matching: Summary

- Shower off X already contains LL part of all $X+n$

$$d\sigma_{X+1} \sim 2g^2 d\sigma_X \frac{ds_{a1}}{s_{a1}} \frac{ds_{1b}}{s_{1b}}$$

- Adding back full ME for $X+n$ would be overkill

Solution 1: "Slicing" (most widespread)

HERWIG: Seymour, CPC 90 (1995) 95
ALPGEN, MADGRAPH: MLM
SHERPA: CKKW, JHEP 0111 (2001) 063
ARIADNE: Lönnblad, JHEP 0205 (2002) 046

Good for generating Born + several hard jets + shower

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Solution 1: "Slicing" (most widespread)

Add event samples. Use ME above p_{tmatch} and PS below it

$$w_X = |M_X|^2 \quad + \text{Shower} \times \text{Veto above } p_{T\mathrm{match}}$$

$$w_{X+m<n} = |M_{X+1}|^2 \times \Delta_{X+1} \quad + \text{Shower} \times \text{Veto above } p_{T\mathrm{match}}$$

$$w_{X+n} = |M_{X+n}|^2 \times \Delta_{X+n} \quad + \text{Shower}$$

MULTILEG: Only CKKW and MLM

HERWIG: for X+1 @ LO (Used to populate dead zone of angular-ordered shower)

CKKW & MLM : for all X+n @ LO (with n up to 3-4)
SHERPA (CKKW), ALPGEN (MLM + HW/PY), MADGRAPH (MLM + HW/PY), PYTHIA8 (CKKW-L from LHE files), …

Good for generating Born + several hard jets + shower

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$$d\sigma_{X+1} \sim 2g^2 d\sigma_X \frac{ds_{a1}}{s_{a1}} \frac{ds_{1b}}{s_{1b}}$$

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Solution 2: “Subtraction” (for NLO)

Frixione-Webber (MC@NLO), JHEP 0206 (2002) 029
+ many more recent ...

Good for generating NLO Born + shower

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- Adding back full ME for X+n would be overkill

Solution 2: "Subtraction" (for NLO)

Frixione-Webber (MC@NLO), JHEP 0206 (2002) 029
+ many more recent ...

Add event samples, with modified weights

$w_X = |M_X|^2 \,(1 + (NLO - Shower\{w_X\}))$ + *Shower*

$w_{X+1} = |M_{X+1}|^2 - Shower\{w_X\}$ + *Shower*

MC@NLO: for X+1 @ LO and X @ NLO (note: correction can be negative)

aMC@NLO: for X+1 @ LO and X @ NLO (note: correction can be negative)

Good for generating NLO Born + shower

QCD

Lecture IV

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- Shower off X already contains LL part of all X+n

$$d\sigma_{X+1} \sim 2g^2 d\sigma_X \frac{ds_{a1}}{s_{a1}} \frac{ds_{1b}}{s_{1b}}$$

- Adding back full ME for X+n would be overkill

Solution 3: "Unitarity"

Bengtsson-Sjöstrand (Pythia), PLB 185 (1987) 435 + more
Bauer-Tackmann-Thaler (GenEva), JHEP 0812 (2008) 011
Giele-Kosower-Skands (Vincia), PRD84 (2011) 054003

Matching: Summary

- Shower off X already contains LL part of all $X+n$

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One event sample

$w_X = |M_X|^2$ + *Shower*

Make a "course correction" to the shower at each order

$R_{X+1} = |M_{X+1}|^2/\text{Shower}\{w_X\}$ + *Shower*

$R_{X+n} = |M_{X+n}|^2/\text{Shower}\{w_{X+n-1}\}$ + *Shower* **Only VINCIA**

PYTHIA: for X+1 @ LO (for color-singlet production and ~ all SM and BSM decay processes)

POWHEG: for X+1 @ LO and X @ NLO (note: positive weights) → POWHEG Box, HERWIG++, …

VINCIA: for all X+n @ LO and X @ NLO (only worked out for decay processes so far)

LHC@home 2.0

Test4Theory - A Virtual Atom Smasher

http://lhcathome2.cern.ch/

Helping to crunch numbers for the mcplots.cern.ch web site
Next large calculation attempt: NNLO top pair production

Additional Slides

Vetoed Parton Showers

(used in Phase Space Slicing, a.k.a. CKKW or MLM matching)

Common (at ME level):

1. Generate one ME sample for each of $\sigma_n(p_{Tcut})$ (using large, fixed α_{s0})
2. Use a jet algorithm (e.g., k_T) to determine an approximate shower history for each ME event
3. Construct the would-be shower α_s factor and reweight

$$w_n = \text{Prod}[\alpha_s(k_{Ti})]/\alpha_{s0}^n$$

→ "Renormalization-improved" ME weights

CKKW and CKKW-L

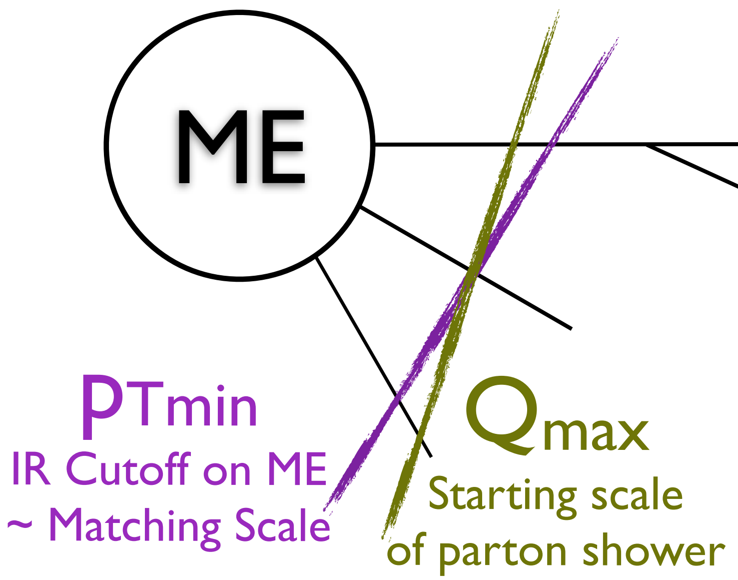
1. Apply Sudakov $\Delta(t_{start},t_{end})$ for each reconstructed internal line (NLL for CCKW, trial-shower for CKKW-L)
2. Accept/Reject: w_n ×= Prod[Δ_i]
3. Do parton shower, vetoing any emissions above cutoff

MLM

1. Do normal parton showers
2. Cluster showered event (cone)
3. Match ME partons to jets
4. If (all partons matched && $n_{partons}$ == n_{jets}) Accept : Reject;

Scales: the devil in the details 1

Clean Slicing: Shower **Starts** at ME **cutoff** scale (=matching scale)



Be safe: start at s and veto shower emissions above p_{Tmin}

But ME cut not necessarily = shower evolution variable (even if shower ordered in p_T)

Scales: the devil in the details 1

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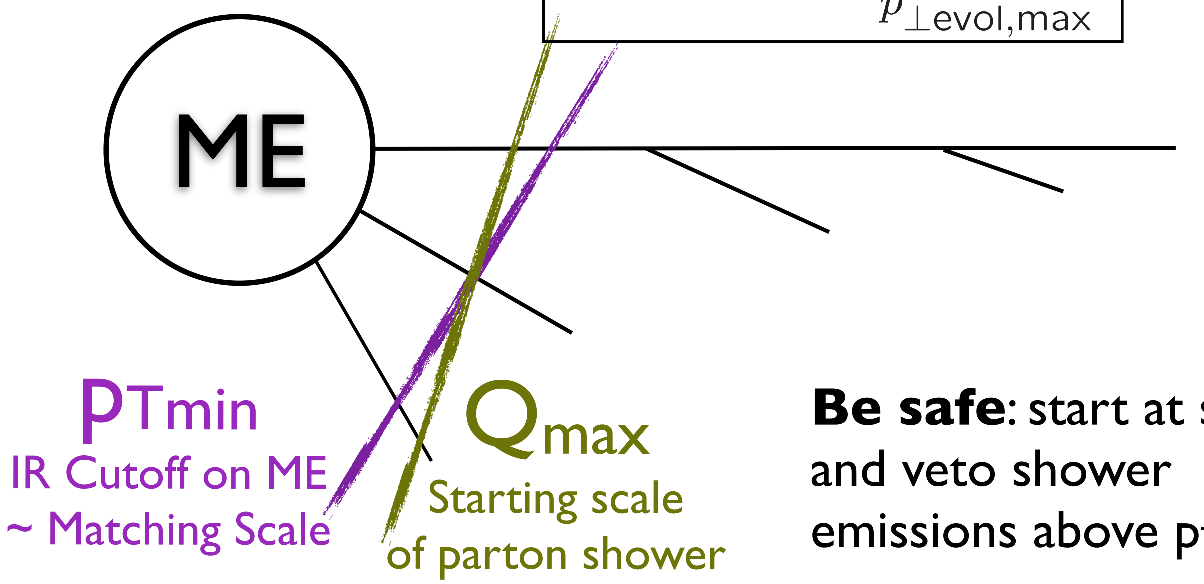
Example: PYTHIA uses p_{Tevol} ~ lightcone p_T

$$p_\perp^2 = p_{\perp\mathrm{evol}}^2 - \frac{p_{\perp\mathrm{evol}}^4}{p_{\perp\mathrm{evol,max}}^2}$$



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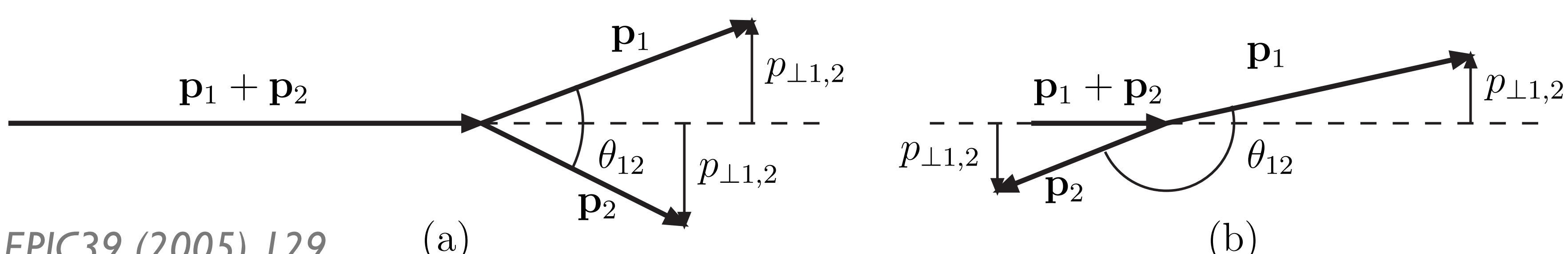

T. Sjöstrand & PS, EPJC39 (2005) 129

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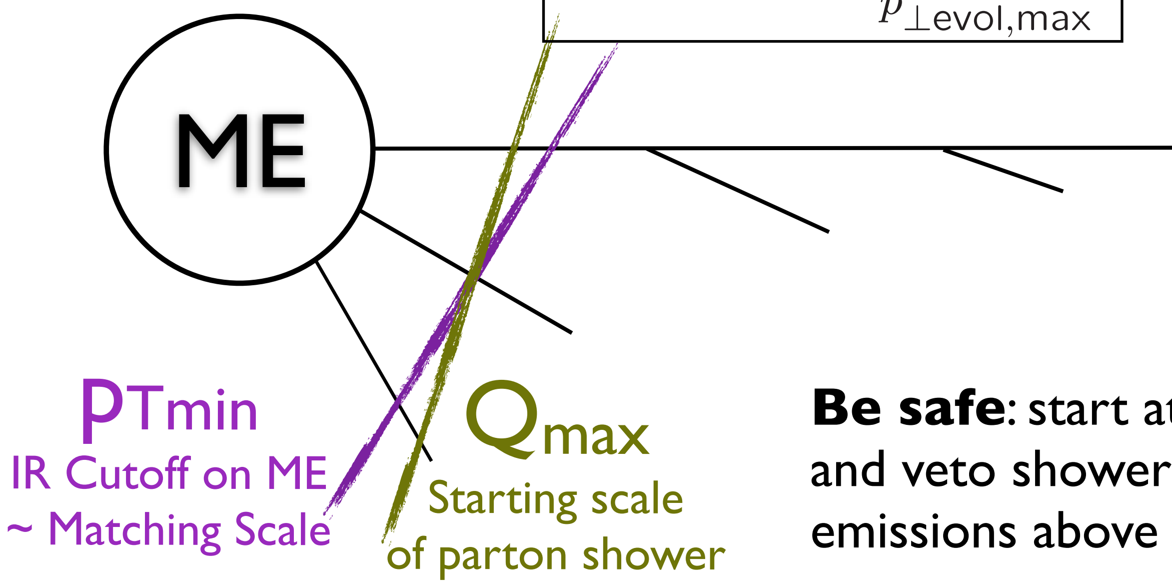

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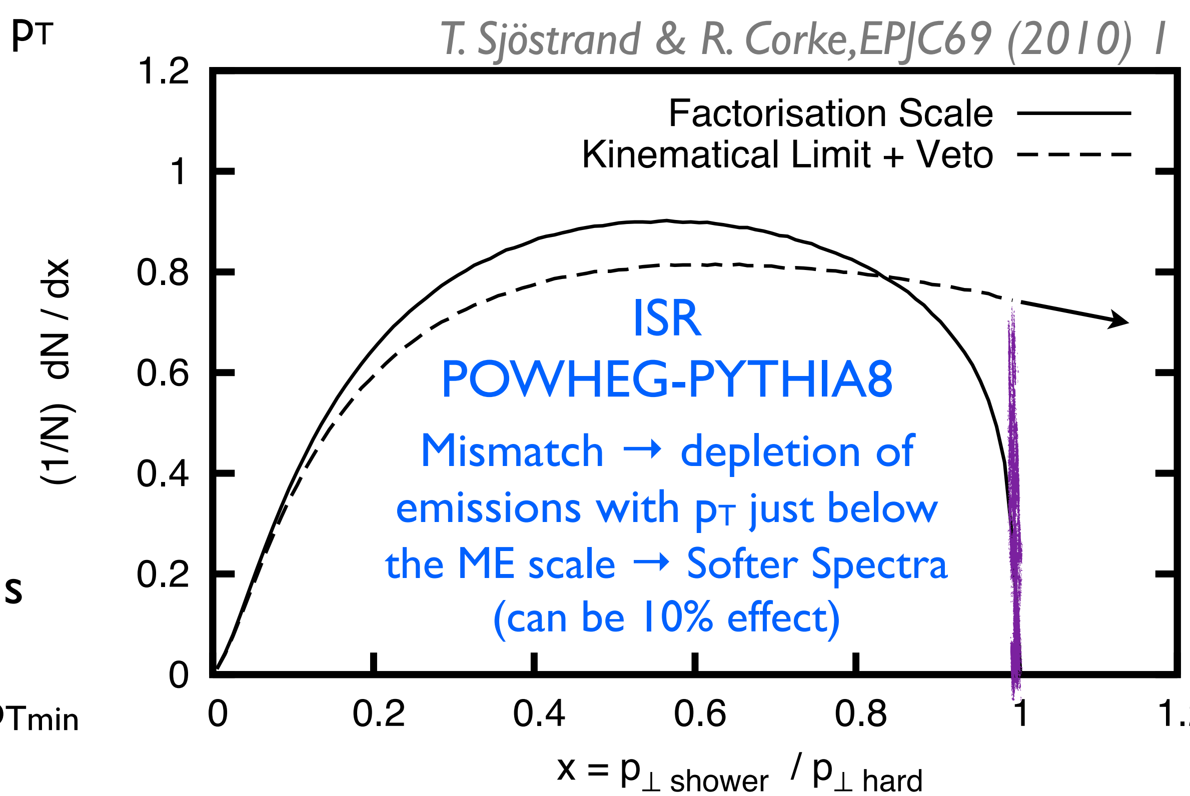
T. Sjöstrand & R. Corke, EPJC69 (2010) 1



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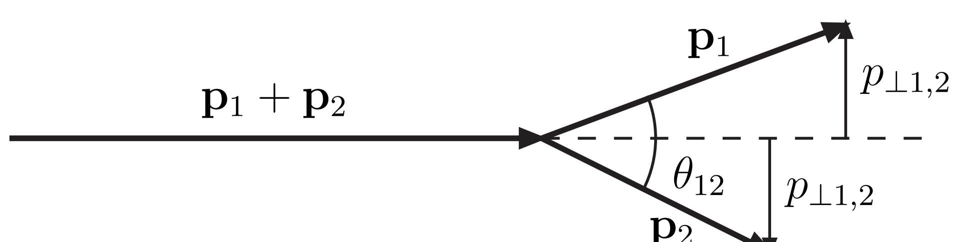
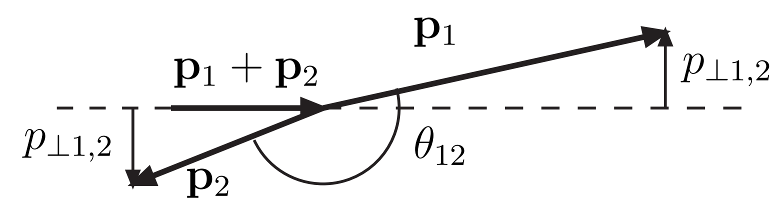


(a)



(b)

T. Sjöstrand & PS, EPJC39 (2005) 129

1st Order: PYTHIA and POWHEG

PYTHIA

FSR: Sjöstrand & Bengtsson, PLB185(1987)435, NPB289(1987)810
Drell-Yan: Miu & Sjöstrand, PLB449(1999)313

Real Radiation:

$$\left.\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\hat{t}}\right|_{\mathrm{PS}} = \left.\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\hat{t}}\right|_{\mathrm{PS1}} + \left.\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\hat{t}}\right|_{\mathrm{PS2}} = \frac{\sigma_0}{\hat{s}}\frac{\alpha_{\mathrm{s}}}{2\pi}\frac{4}{3}\frac{\hat{s}^2+m_{\mathrm{W}}^4}{\hat{t}\hat{u}} .$$

(for qg→q'W)

Use PS as overestimate. Correct to R/B via veto:

$$R_{\mathrm{qg}\to\mathrm{q'W}}(\hat{s},\hat{t}) = \frac{(\mathrm{d}\hat{\sigma}/\mathrm{d}\hat{t})_{\mathrm{ME}}}{(\mathrm{d}\hat{\sigma}/\mathrm{d}\hat{t})_{\mathrm{PS}}} = \boxed{\frac{\hat{s}^2+\hat{u}^2+2m_{\mathrm{W}}^2\hat{t}}{\hat{s}^2+2m_{\mathrm{W}}^2(\hat{t}+\hat{u})}}$$

(+analogous for qq→gW)

Unitarity → Modified Sudakov Factor:

$$\exp\left(-\int_t^{t_{\max}}\mathrm{d}t'\,\frac{\alpha_{\mathrm{s}}(t')}{2\pi}\sum_a\int_x^1\mathrm{d}z\,\frac{x' f_a(x',t')}{x f_b(x,t')}P_{a\to bc}(z)\right)$$

Inclusive Cross Section (at fixed underlying Born variables):

Unitarity + no normalization correction → remains σ_0

$$\to B = \sigma_0 = |M_{\mathrm{Born}}|^2$$

Cancels when normalizing to $1/\sigma$ and integrating over Born

Note: → tuning of standalone PYTHIA done with this matching scheme
Should be OK for POWHEG, but could give worries for MLM
B. Cooper et al, arXiv:1109.5295

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POWHEG

Nason, JHEP 11(2004)040
Drell-Yan: Alioli et al., JHEP 07(2008)060

Real Radiation:

$$R_{g\bar{q},q} + R_{qg,\bar{q}} = \left.\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\hat{t}}\right|_{\mathrm{ME}} = \frac{\sigma_0}{\hat{s}}\frac{\alpha_\mathrm{s}}{2\pi}\frac{4}{3}\frac{\hat{t}^2+\hat{u}^2+2m_\mathrm{W}^2\hat{s}}{\hat{t}\hat{u}}$$

(for qg→q'W) (using Sjöstrand's notation)

Use R/B as splitting kernels (via overestimate + veto)

(+analogous for qq→gW)

Unitarity → Sudakov Factor:

(explicit formula only for final-state in org paper → no PDF factors here)

$$\Delta_R^{(\mathrm{NLO})}(p_\mathrm{T}) = e^{-\int d\Phi_r \frac{R(v,r)}{B(v)}\theta(k_\mathrm{T}(v,r)-p_\mathrm{T})}$$

= LL'

(not needed if shower ordered in p_T, though watch out, see next)

Inclusive Cross Section (at fixed underlying Born variables):

Include correction to NLO inclusive level → becomes σ_{NLO}

$$\to \bar{B}(v) = B(v) + V(v) + \int (R(v,r) - C(v,r))\, d\Phi_r$$

Cancels when normalizing to $1/\sigma$ and integrating over Born

μ_R in a matched setting (MLM)

B. Cooper et al., arXiv:1109.5295

If using one code for MEs and another for showering

Tree-level corrections use α_s from Matrix-element Generator

Virtual corrections use α_s from Shower Generator (Sudakov)

μ_R in a matched setting (MLM)

B. Cooper et al., arXiv:1109.5295

If using one code for MEs and another for showering

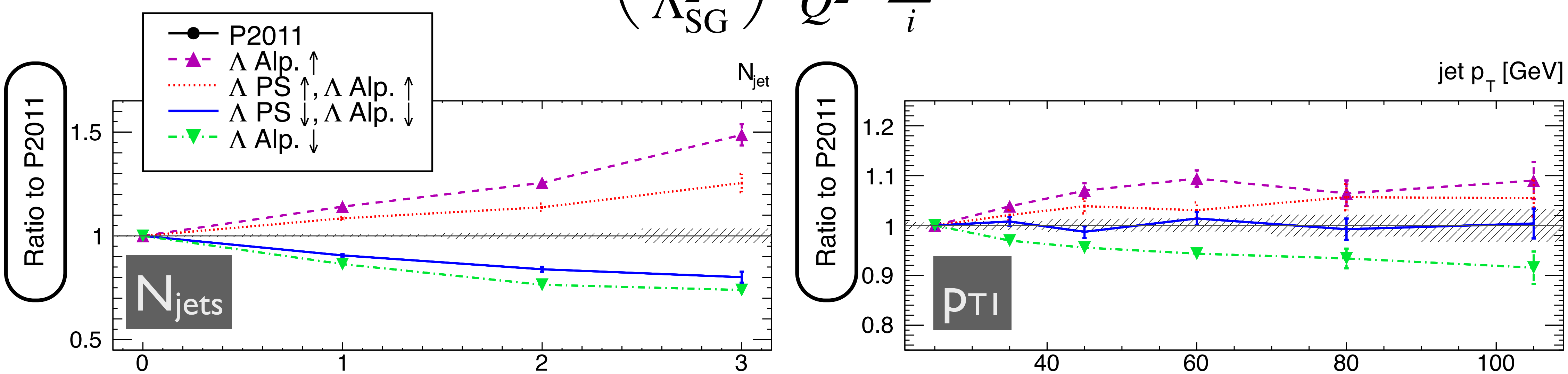
Tree-level corrections use α_s from Matrix-element Generator

Virtual corrections use α_s from Shower Generator (Sudakov)

Mismatch if the two do not use same Λ_{QCD} or $\alpha_s(m_Z)$

$$\alpha_s^2 \, b_0 \ln\left(\frac{\Lambda_{MG}^2}{\Lambda_{SG}^2}\right) \frac{dQ^2}{Q^2} \sum_i P_i(z) \, |M_F|^2 \; .$$

note: running **order** also has a (subleading) effect



AlpGen: can set xlclu = Λ_{QCD} since v.2.14 (default remains to inherit from PDF)
Pythia 6: set common PARP(61)=PARP(72)=PARP(81) = Λ_{QCD} in Perugia 2011 tunes
Pythia 8: use TimeShower:alphaSvalue and SpaceShower:alphaSvalue

QCD
Lecture IV

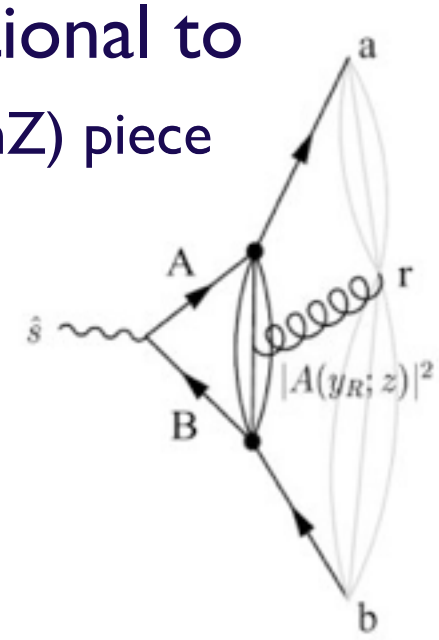
Choice of Renormalization Scale

in Parton Showers

One-loop radiation functions contain pieces proportional to the β function (E.g.,: e+e-→3 jets, for arbitrary choice of μR (e.g., μR= mZ) piece from integrating quark loops over all of phase space

$$n_f \ A_3^0 \left(\ln\left(\frac{s_{23}}{\mu_R^2}\right) + \ln\left(\frac{s_{13}}{\mu_R^2}\right) \right)$$

+ gluon loops



Proportional to the β function (b_0).

Can be absorbed by using $\mu_R^4 = s_{13}\, s_{23} = p_T^2\, s$.

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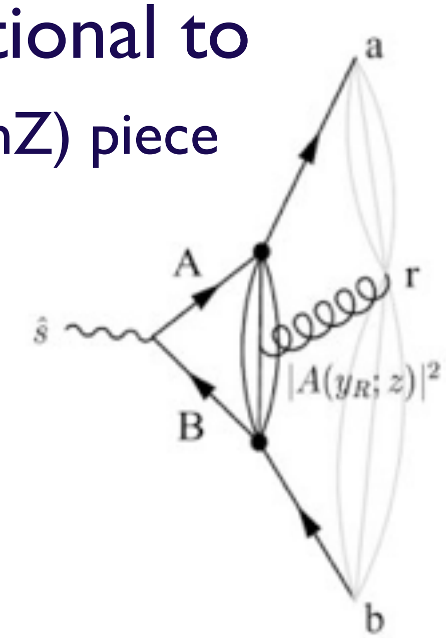


$$n_f \; A_3^0 \left(\ln\left(\frac{s_{23}}{\mu_R^2}\right) + \ln\left(\frac{s_{13}}{\mu_R^2}\right) \right)$$ + gluon loops

Proportional to the β function (b_0).

Can be absorbed by using $\mu_R^4 = s_{13}\, s_{23} = p_T^2\, s$.

In an ordered shower, quark (and gluon) loop integrals are restricted by strong-ordering condition → modified to

$\mu_R = p_T$ (but depends on ordering variable? Anyway, we're using pT here)

Additional logs induced by gluon loops can be absorbed by replacing Λ^{MS} by $\Lambda^{MC} \sim 1.5\, \Lambda^{MS}$ (with mild dependence on number of flavors)

Catani, Marchesini, Webber, NPB349 (1991) 635

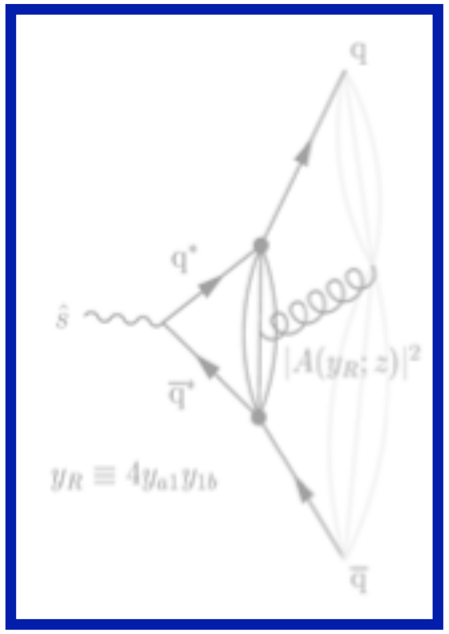
Remaining ambiguity → tuning

Note: CMW not automatic in PYTHIA, has to be done by hand, by choosing effective Λ or $\alpha_s(M_Z)$ values instead of $\overline{MS}$ ones
Note 2: There are obviously still order 2 uncertainties on μ_R, but this is the background for the central choice made in showers

NLO Matching in 1 Slide

- First Order Shower expansion

PS $\int \mathrm{d}\Phi_2$ Born $\int_{Q^2_{\mathrm{had}}}^{s} \frac{\mathrm{d}\Phi_3}{\mathrm{d}\Phi_2}$ LL $\delta\left(\mathcal{O} - \mathcal{O}(\{p\}_3)\right)$

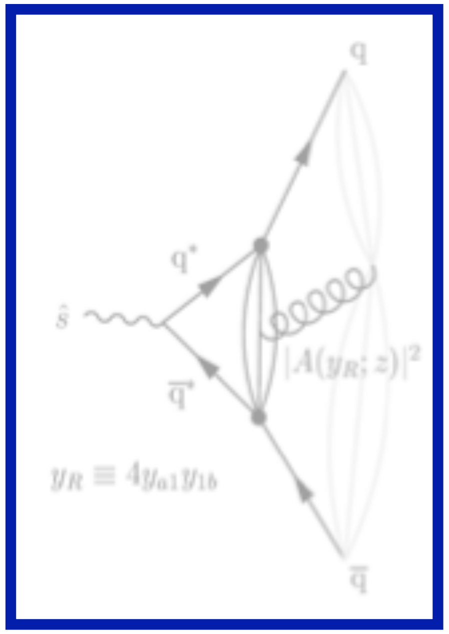


NLO Matching in 1 Slide

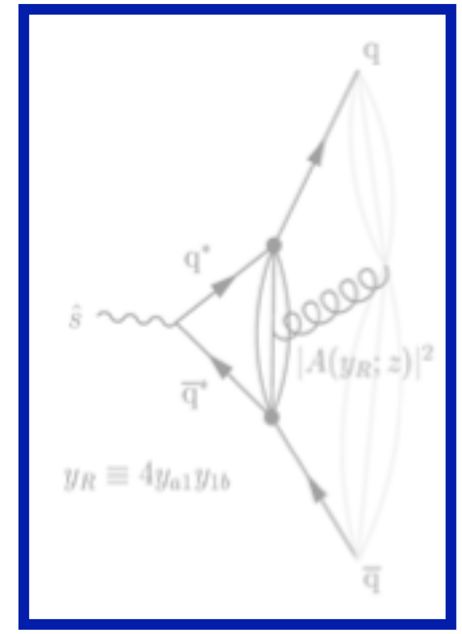
- First Order Shower expansion

PS $$\int \mathrm{d}\Phi_2 \; \boxed{\text{Born}} \int_{Q^2_{\mathrm{had}}}^{s} \frac{\mathrm{d}\Phi_3}{\mathrm{d}\Phi_2} \; \boxed{\text{LL}} \; \delta\left(\mathcal{O} - \mathcal{O}(\{p\}_3)\right)$$

Unitarity of shower ➔ 3-parton real = ÷ 2-parton “virtual”



NLO Matching in 1 Slide



- First Order Shower expansion

PS $\quad \int \mathrm{d}\Phi_2$ [Born] $\int_{Q^2_{\mathrm{had}}}^{s} \frac{\mathrm{d}\Phi_3}{\mathrm{d}\Phi_2}$ [LL] $\delta\left(\mathcal{O} - \mathcal{O}(\{p\}_3)\right)$

Unitarity of shower ➔ 3-parton real = − 2-parton "virtual"

- 3-parton real correction ($A_3 = |M_3|^2/|M_2|^2$ + finite terms; α, β)

[X+1$^{(0)}$] $=$ [X+1$^{(0)}$] $-\left(\frac{[\text{X+1}^{(0)}]}{[\text{Born}]} + \frac{4\pi\alpha_s\hat{C}_F}{s}\left(\alpha + \beta\frac{s_{ar}+s_{rb}}{s}\right)\right)$ [Born]

$$= -\frac{4\pi\alpha_s\hat{C}_F}{s}\left(\alpha + \beta\frac{s_{ar}+s_{rb}}{s}\right)|M_2^{(0)}|^2$$

NLO Matching in 1 Slide

- First Order Shower expansion

PS $\quad \int \mathrm{d}\Phi_2 \, [\text{Born}] \int_{Q^2_{\text{had}}}^{s} \frac{\mathrm{d}\Phi_3}{\mathrm{d}\Phi_2} \, [\text{LL}] \, \delta\left(\mathcal{O} - \mathcal{O}(\{p\}_3)\right)$

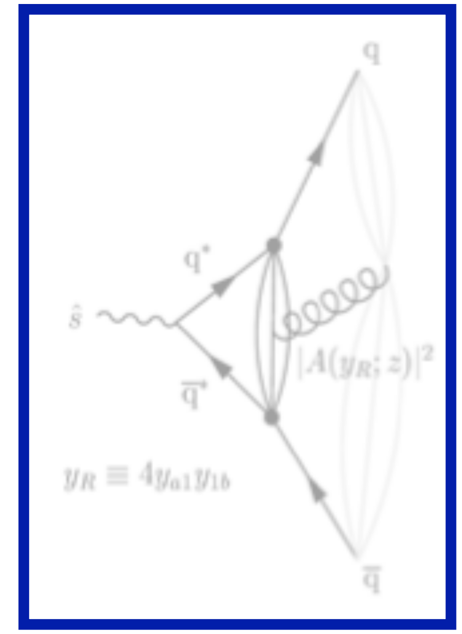
Unitarity of shower ➔ 3-parton real = ÷ 2-parton "virtual"

- 3-parton real correction ($A_3 = |M_3|^2/|M_2|^2$ + finite terms; α, β)

$$[\text{X+1}] = [\text{X+1}^{(0)}] - \left(\frac{[\text{X+1}^{(0)}]}{[\text{Born}]} + \frac{4\pi\alpha_s \hat{C}_F}{s}\left(\alpha + \beta\frac{s_{ar}+s_{rb}}{s}\right)\right) [\text{Born}]$$

$$= -\frac{4\pi\alpha_s \hat{C}_F}{s}\left(\alpha + \beta\frac{s_{ar}+s_{rb}}{s}\right)|M_2^{(0)}|^2 \quad \Rightarrow$$

Finite terms cancel in 3-parton O



NLO Matching in 1 Slide

- First Order Shower expansion

PS $$\int \mathrm{d}\Phi_2 \, [\text{Born}] \int_{Q^2_{\mathrm{had}}}^{s} \frac{\mathrm{d}\Phi_3}{\mathrm{d}\Phi_2} \, [\text{LL}] \, \delta\left(\mathcal{O} - \mathcal{O}(\{p\}_3)\right)$$

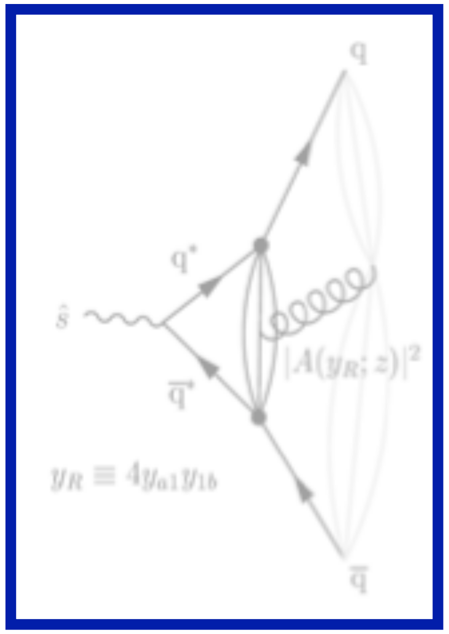
Unitarity of shower ➔ 3-parton real = ÷ 2-parton "virtual"

- 3-parton real correction ($A_3 = |M_3|^2/|M_2|^2$ + finite terms; α, β)

$$[\text{X+1}^{(0)}] = [\text{X+1}^{(0)}] - \left(\frac{[\text{X+1}^{(0)}]}{[\text{Born}]} + \frac{4\pi\alpha_s \hat{C}_F}{s}\left(\alpha + \beta\frac{s_{ar}+s_{rb}}{s}\right)\right)[\text{Born}]$$

$$= -\frac{4\pi\alpha_s \hat{C}_F}{s}\left(\alpha + \beta\frac{s_{ar}+s_{rb}}{s}\right)|M_2^{(0)}|^2 \quad \Rightarrow$$

Finite terms cancel in 3-parton O



- 2-parton virtual correction (same example)

$$[\text{X}^{(1)}] = [\text{X}^{(1)}] + [\text{Born}] \int_0^s \frac{\mathrm{d}\Phi_3}{\mathrm{d}\Phi_2} [\text{LL}] + \int_0^{Q^2_{\mathrm{had}}} \frac{\mathrm{d}\Phi_3}{\mathrm{d}\Phi_2} [\text{X+1}^{(0)}]$$

$$= \frac{\alpha_s \hat{C}_F}{2\pi}\left(2I^{(1)}_{q\bar{q}}(\epsilon, s) - 4 - 2I^{(1)}_{q\bar{q}}(\epsilon, s) + \frac{19 + \alpha + \frac{2}{3}\beta}{4}\right)[\text{Born}]$$

$$= \frac{\alpha_s}{\pi}\left(1 + \frac{1}{3}\left(\alpha + \frac{2}{3}\beta\right)\right)[\text{Born}] \quad \Rightarrow$$

Finite terms cancel in 2-parton O (normalization)

NLO Matching in 1 Slide

- First Order Shower expansion

PS $$\int \mathrm{d}\Phi_2 \, |M_2^{(0)}|^2 \int_{Q^2_{\rm had}}^{s} \frac{\mathrm{d}\Phi_3}{\mathrm{d}\Phi_2} A_{q\bar{q}}(\ldots) \delta\left(\mathcal{O} - \mathcal{O}(\{p\}_3)\right)$$

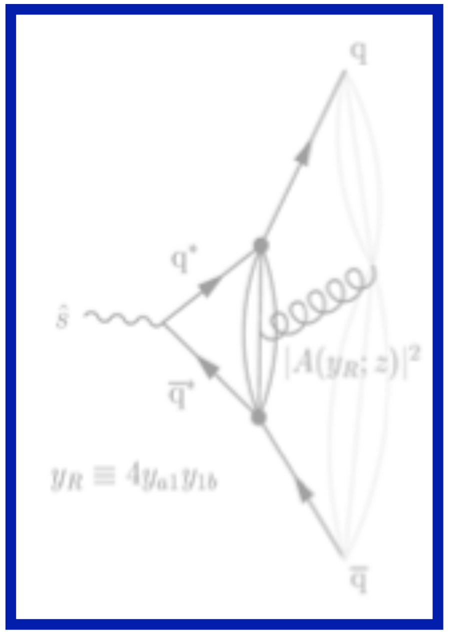
Unitarity of shower ➔ 3-parton real = ÷ 2-parton "virtual"

- 3-parton real correction ($A_3 = |M_3|^2/|M_2|^2$ + finite terms; α, β)

$$w_3^{(R)} = |M_3^{(0)}|^2 - \left(A_3^0(\ldots) + \frac{4\pi\alpha_s \hat{C}_F}{s} \left(\alpha + \beta \frac{s_{ar} + s_{rb}}{s} \right) \right) |M_2^{(0)}|^2$$

$$= -\frac{4\pi\alpha_s \hat{C}_F}{s} \left(\alpha + \beta \frac{s_{ar} + s_{rb}}{s} \right) |M_2^{(0)}|^2 \quad \Longrightarrow$$

Finite terms cancel in 3-parton O



- 2-parton virtual correction (same example)

$$w_2^{(V)} = 2\mathrm{Re}\left[M_2^{(1)} M_2^{(0)*}\right] + |M_2^{(0)}|^2 \int_0^s \frac{\mathrm{d}\Phi_3}{\mathrm{d}\Phi_2} A_{q\bar{q}}(\ldots) + \int_0^{Q^2_{\rm had}} \frac{\mathrm{d}\Phi_3}{\mathrm{d}\Phi_2} w_3^{(R)}$$

$$= \frac{\alpha_s \hat{C}_F}{2\pi} \left(2 I_{q\bar{q}}^{(1)}(\epsilon, s) - 4 - 2 I_{q\bar{q}}^{(1)}(\epsilon, s) + \frac{19 + \alpha + \frac{2}{3}\beta}{4} \right) |M_2^{(0)}|^2$$

$$= \frac{\alpha_s}{\pi} \left(1 + \frac{1}{3} \left(\alpha + \frac{2}{3}\beta \right) \right) |M_2^{(0)}|^2 \quad \Longrightarrow$$

Finite terms cancel in 2-parton O (normalization)

QCD Lecture IV