P. Skands
QCD Lecture III

Monte Carlo Generators

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A Monte Carlo technique: is any technique making use of random numbers to solve a problem

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Monte Carlo Generators

Convergence:

Calculus: {A} converges to B
if an n exists for which
$|A_{i>n} - B| < \varepsilon$, for any $\varepsilon > 0$

Monte Carlo: {A} converges to B
if n exists for which
the probability for
$|A_{i>n} - B| < \varepsilon$, for any $\varepsilon > 0$,
is > P, for any P[0<P<1]

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QCD Lecture III

A Monte Carlo technique: is any technique making use of random numbers to solve a problem

Convergence:

Monte Carlo Generators

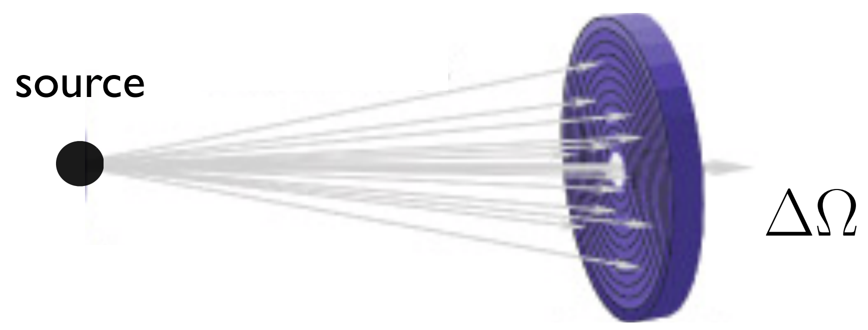
Calculus: {A} converges to B if an n exists for which $|A_{i>n} - B| < \varepsilon$, for any $\varepsilon > 0$

Monte Carlo: {A} converges to B if n exists for which the probability for $|A_{i>n} - B| < \varepsilon$, for any $\varepsilon > 0$, is > P, for any P[0<P<1]

"This risk, that convergence is only given with a certain probability, is inherent in Monte Carlo calculations and is the reason why this technique was named after the world's most famous gambling casino. Indeed, the name is doubly appropriate because the style of gambling in the Monte Carlo casino, not to be confused with the noisy and tasteless gambling houses of Las Vegas and Reno, is serious and sophisticated."

F. James, "Monte Carlo theory and practice", Rept. Prog. Phys. 43 (1980) 1145

Scattering Experiments



LHC detector
Cosmic-Ray detector
Neutrino detector
X-ray telescope
...

→ Integrate differential cross sections over specific phase-space regions

Predicted number of counts
= integral over solid angle

$$N_{\rm count}(\Delta\Omega) \propto \int_{\Delta\Omega} \mathrm{d}\Omega \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}$$

QCD
Lecture III

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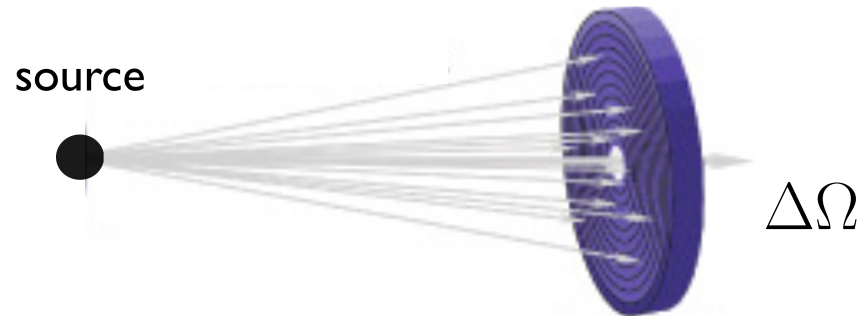

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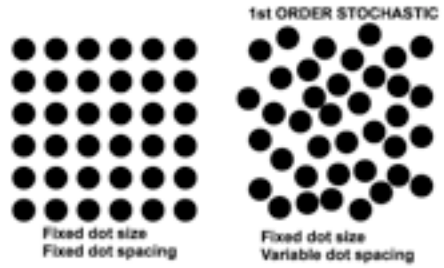
In particle physics:
Integrate over all quantum histories

Convergence

MC convergence is Stochastic!

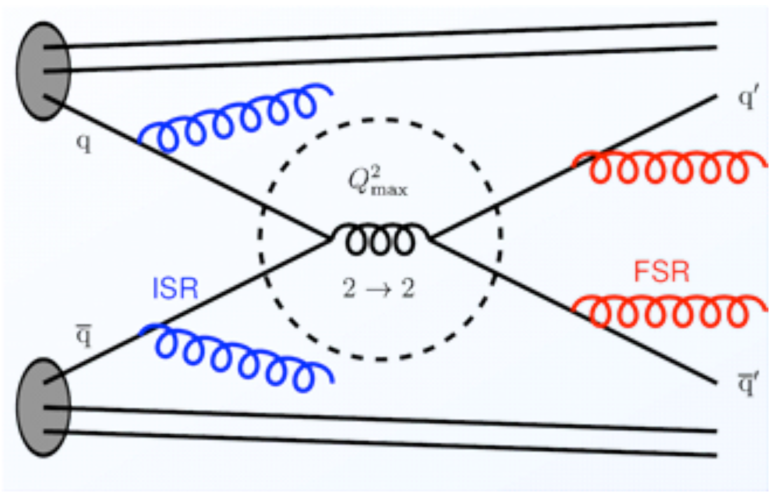
$\frac{1}{\sqrt{n}}$ in any dimension



Uncertainty (after n function evaluations)	n_{eval} / bin	Approx Conv. Rate (in 1D)	Approx Conv. Rate (in D dim)
Trapezoidal Rule (2-point)	2^D	$1/n^2$	$1/n^{2/D}$
Simpson's Rule (3-point)	3^D	$1/n^4$	$1/n^{4/D}$
… m-point (Gauss rule)	m^D	$1/n^{2m-1}$	$1/n^{(2m-1)/D}$
Monte Carlo	1	$1/n^{1/2}$	$1/n^{1/2}$

+ many ways to optimize: stratification, adaptation, ...
+ gives "events" → iterative solutions,
+ interfaces to detector simulation & propagation codes

Monte Carlo Generators



Calculate Everything ≈ solve QCD → requires compromise!

Improve lowest-order perturbation theory,
by including the 'most significant' corrections
→ complete events (can evaluate any observable you want)

Existing Approaches

PYTHIA : Successor to JETSET (begun in 1978). Originated in hadronization studies: Lund String.
HERWIG : Successor to EARWIG (begun in 1984). Originated in coherence studies: angular ordering.
SHERPA : Begun in 2000. Originated in "matching" of matrix elements to showers: CKKW.
+ MORE SPECIALIZED: ALPGEN, MADGRAPH, ARIADNE, VINCIA, WHIZARD, MC@NLO, POWHEG, …

PYTHIA anno 1978

(then called JETSET)

LU TP 78-18
November, 1978

A Monte Carlo Program for Quark Jet Generation

T. Sjöstrand, B. Söderberg

A Monte Carlo computer program is presented, that simulates the fragmentation of a fast parton into a jet of mesons. It uses an iterative scaling scheme and is compatible with the jet model of Field and Feynman.

Note: Field-Feynman was an early fragmentation model
Now superseded by the String (in PYTHIA) and Cluster (in HERWIG & SHERPA) models.

(PYTHIA)

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```
      SUBROUTINE JETGEN(N)
      COMMON /JET/ K(100,2), P(100,5)
      COMMON /PAR/ PUD, PS1, SIGMA, CX2, EBEG, WFIN, IFLBEG
      COMMON /DATA1/ MESO(9,2), CMIX(6,2), PMAS(19)
      IFLSGN=(10-IFLBEG)/5
      W=2.*EBEG
      I=0
      IPD=0
C 1 FLAVOUR AND PT FOR FIRST QUARK
      IFL1=IABS(IFLBEG)
      PT1=SIGMA*SQRT(-ALOG(RANF(0)))
      PHI1=6.2832*RANF(0)
      PX1=PT1*COS(PHI1)
      PY1=PT1*SIN(PHI1)
  100 I=I+1
C 2 FLAVOUR AND PT FOR NEXT ANTIQUARK
      IFL2=1+INT(RANF(0)/PUD)
      PT2=SIGMA*SQRT(-ALOG(RANF(0)))
      PHI2=6.2832*RANF(0)
      PX2=PT2*COS(PHI2)
      PY2=PT2*SIN(PHI2)
C 3 MESON FORMED, SPIN ADDED AND FLAVOUR MIXED
      K(I,1)=MESO(3*(IFL1-1)+IFL2,IFLSGN)
      ISPIN=INT(PS1+RANF(0))
      K(I,2)=1+9*ISPIN+K(I,1)
      IF(K(I,1).LE.6) GOTO 110
      TMIX=RANF(0)
      KM=K(I,1)-6+3*ISPIN
      K(I,2)=8+9*ISPIN+INT(TMIX+CMIX(KM,1))+INT(TMIX+CMIX(KM,2))
C 4 MESON MASS FROM TABLE, PT FROM CONSTITUENTS
  110 P(I,5)=PMAS(K(I,2))
      P(I,1)=PX1+PX2
      P(I,2)=PY1+PY2
      PMTS=P(I,1)**2+P(I,2)**2+P(I,5)**2
C 5 RANDOM CHOICE OF X=(E+PZ)MESON/(E+PZ)AVAILABLE GIVES E AND PZ
      X=RANF(0)
      IF(RANF(0).LT.CX2) X=1.-X**(1./3.)
      P(I,3)=(X*W-PMTS/(X*W))/2.
      P(I,4)=(X*W+PMTS/(X*W))/2.
C 6 IF UNSTABLE, DECAY CHAIN INTO STABLE PARTICLES
  120 IPD=IPD+1
      IF(K(IPD,2).GE.8) CALL DECAY(IPD,I)
      IF(IPD.LT.I.AND.I.LE.96) GOTO 120
C 7 FLAVOUR AND PT OF QUARK FORMED IN PAIR WITH ANTIQUARK ABOVE
      IFL1=IFL2
      PX1=-PX2
      PY1=-PY2
C 8 IF ENOUGH E+PZ LEFT, GO TO 2
      W=(1.-X)*W
      IF(W.GT.WFIN.AND.I.LE.95) GOTO 100
      N=I
      RETURN
      END
```

Note: Field-Feynman was an early fragmentation model
Now superseded by the String (in PYTHIA) and Cluster (in HERWIG & SHERPA) models.

(PYTHIA)

PYTHIA anno 2012

(now called PYTHIA 8)

LU TP 07-28 (CPC 178 (2008) 852)
October, 2007

A Brief Introduction to PYTHIA 8.1

T. Sjöstrand, S. Mrenna, P. Skands

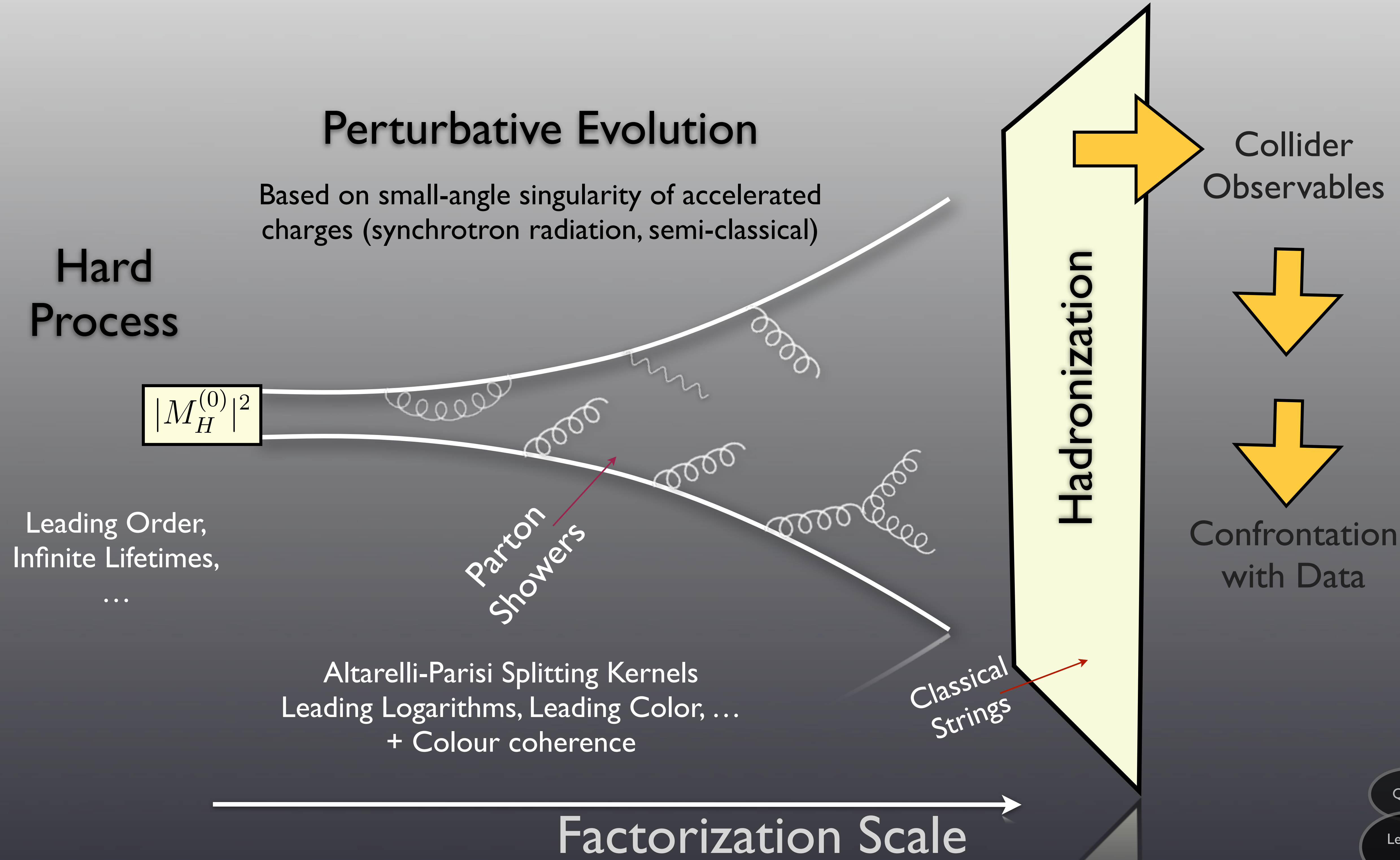
The Pythia program is a standard tool for the generation of high-energy collisions, comprising a coherent set of physics models for the evolution from a few-body hard process to a complex multihadronic final state. It contains a library of hard processes and models for initial- and final-state parton showers, multiple parton-parton interactions, beam remnants, string fragmentation and particle decays. It also has a set of utilities and interfaces to external programs. […]

~ 80,000 lines of C++

What a modern MC generator has inside:

- Hard Processes (internal, semi-internal, or via Les Houches events)
- BSM (internal or via interfaces)
- PDFs (internal or via interfaces)
- Showers (internal or inherited)
- Multiple parton interactions
- Beam Remnants
- String Fragmentation
- Decays (internal or via interfaces)
- Examples and Tutorial
- Online HTML / PHP Manual
- Utilities and interfaces to external programs

(Traditional) Monte Carlo Generators



From Fixed to Infinite Order

Fixed Order : All resolved scales >> Λ_{QCD} **AND** no large hierarchies

Trivially untrue for QCD

We're colliding, and observing, hadrons → small scales

We want to consider high-scale processes → large scale differences

→ A Priori, no perturbatively calculable observables in hadron-hadron collisions

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$$\frac{\mathrm{d}\sigma}{\mathrm{d}X} = \sum_{a,b} \sum_{f} \int_{\hat{X}_f} f_a(x_a, Q_i^2) f_b(x_b, Q_i^2) \frac{\mathrm{d}\hat{\sigma}_{ab\to f}(x_a, x_b, f, Q_i^2, Q_f^2)}{\mathrm{d}\hat{X}_f} D(\hat{X}_f \to X, Q_i^2, Q_f^2)$$

PDFs: needed to compute inclusive cross sections

FFs: needed to compute (semi-)exclusive cross sections

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PDFs: needed to compute inclusive cross sections

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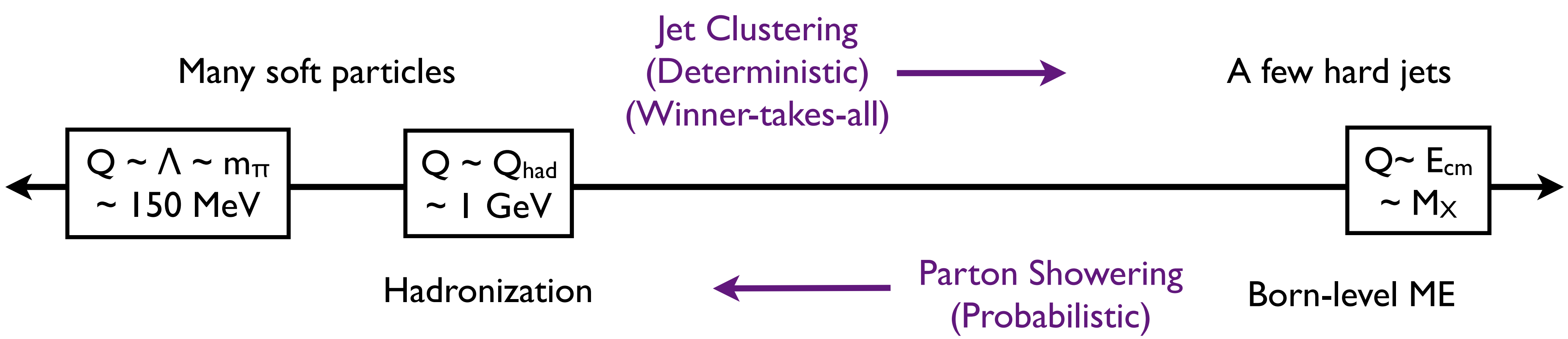
Resummed: All resolved scales >> Λ_{QCD} **AND** X Infrared Safe

Jets and Showers

Jet clustering algorithms

Map event from low resolution scale (i.e., with many partons/hadrons, most of which are soft) to a higher resolution scale (with fewer, hard, jets)



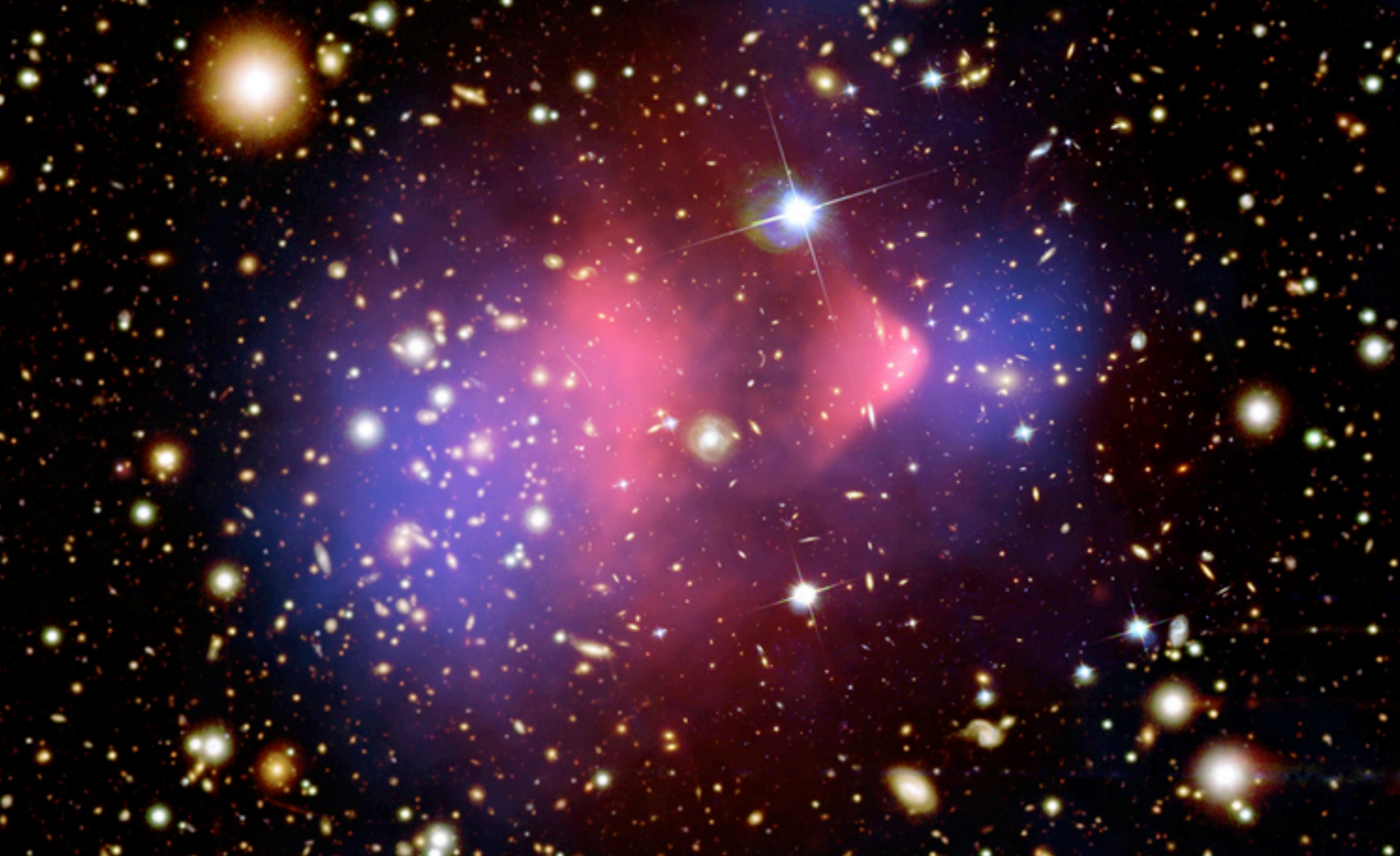
Parton shower algorithms

Map a few hard partons to many softer ones

Probabilistic → closer to nature, but normally not uniquely invertible by any jet algorithm

(see Lopez-Villarejo & PS [JHEP 1111 (2011) 150] for a shower that *is* invertible)

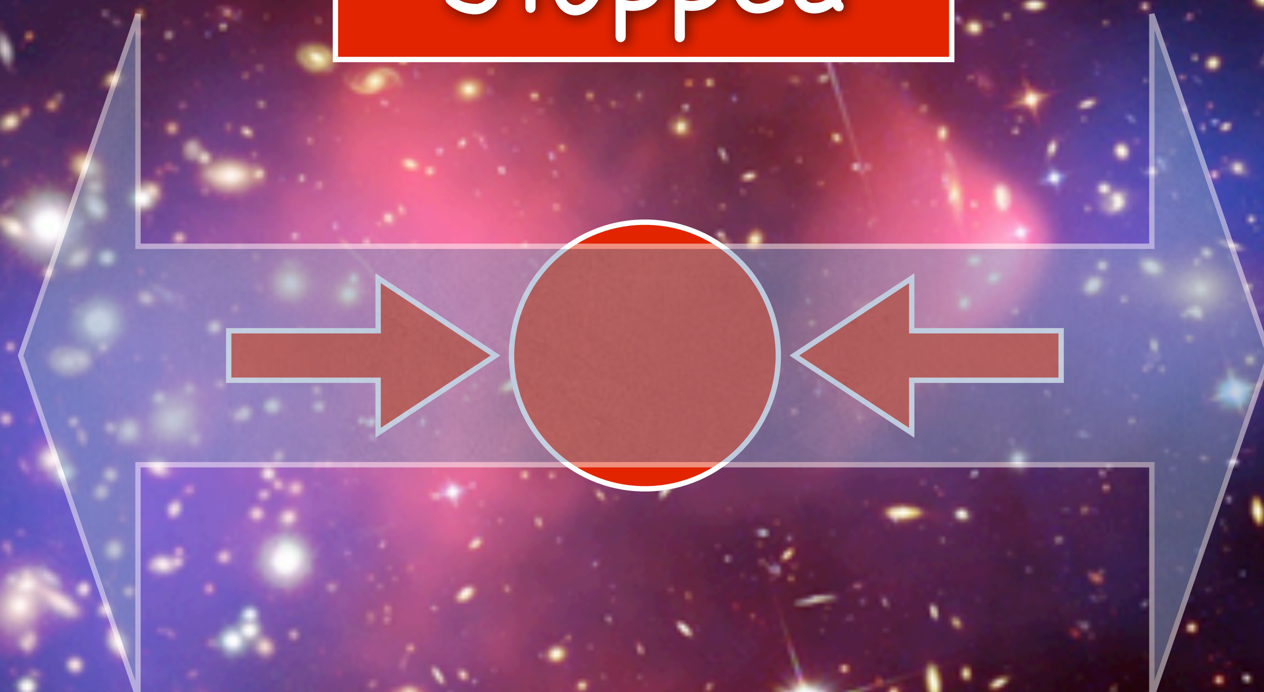
Perturbative Evolution: Bremsstrahlung



Perturbative Evolution: Bremsstrahlung

Perturbative Evolution: Bremsstrahlung

Charges Stopped



Associated field (fluctuations) continues

Perturbative Evolution: Bremsstrahlung

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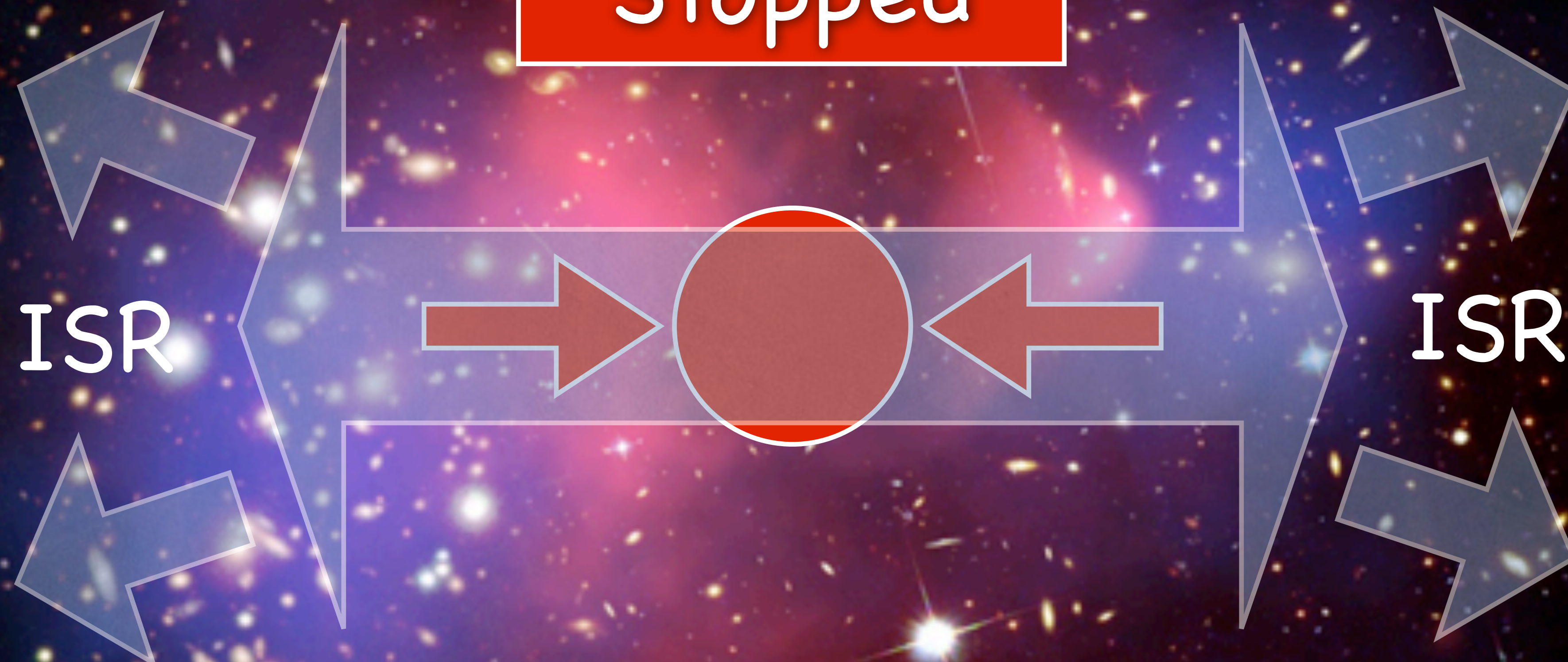

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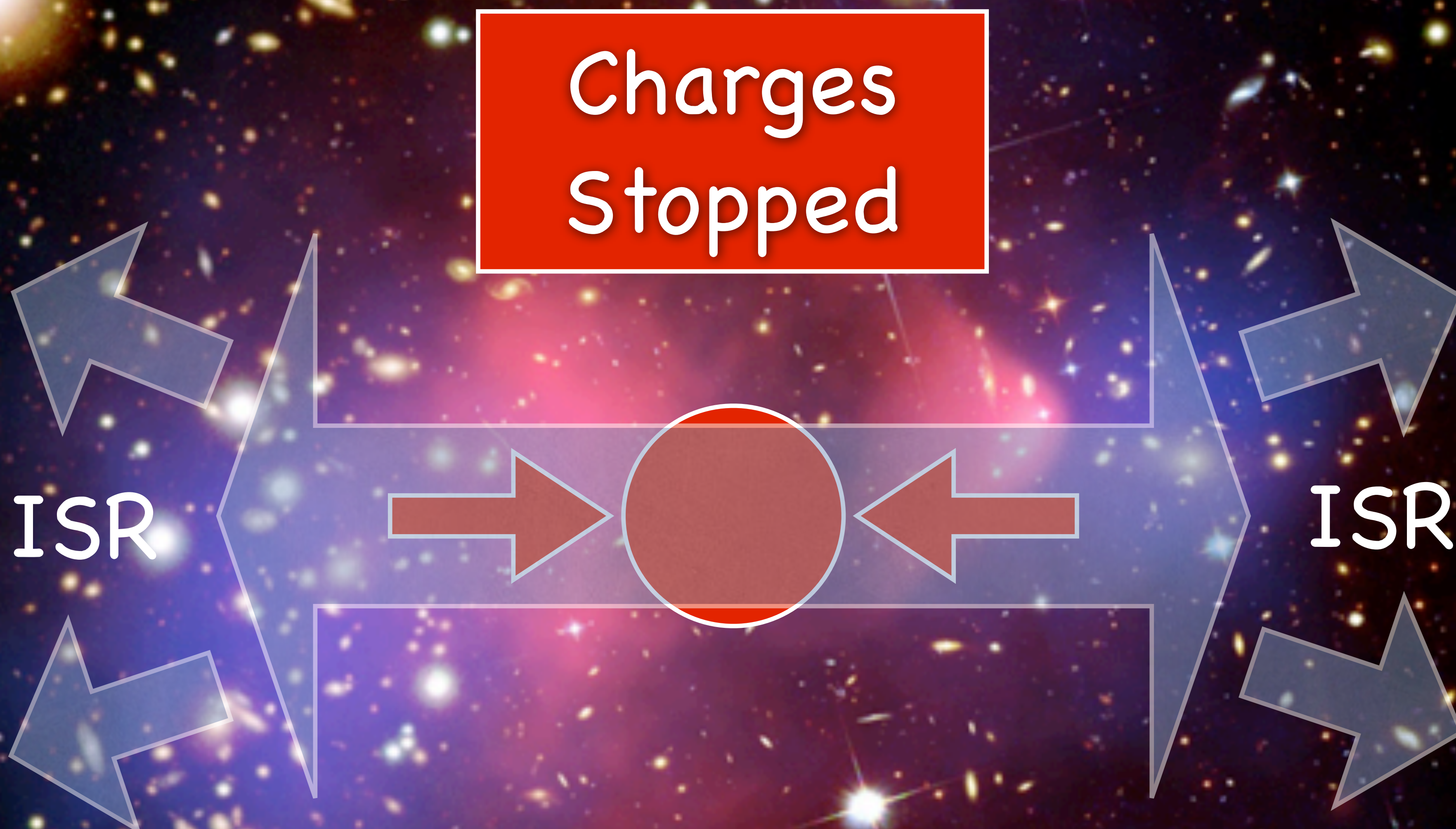
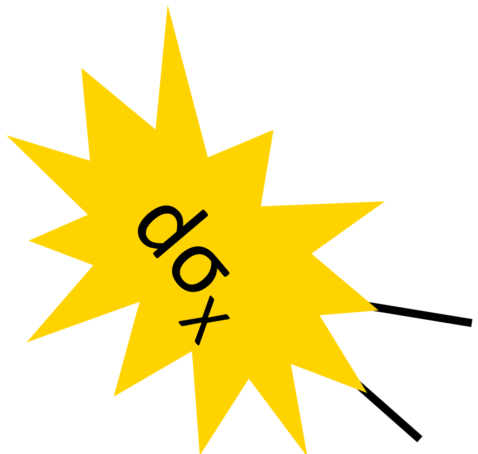
Perturbative Evolution: Bremsstrahlung



The harder they stop, the harder the fluctuations that continue to become strahlung

Bremsstrahlung



For any basic process $d\sigma_X$ = ✓ (calculated process by process)

Recall: Factorization in Soft and Collinear Limits

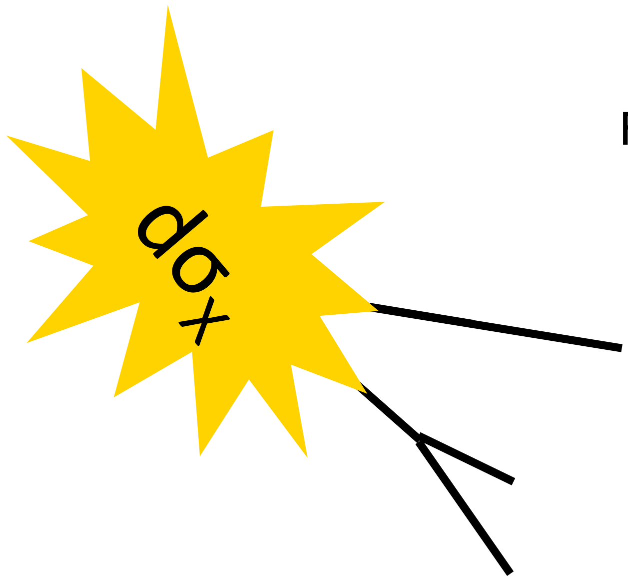
P(z) : "Altarelli-Parisi Splitting Functions" (more later)

$$|M(\ldots,p_i,p_j\ldots)|^2 \overset{i||j}{\to} g_s^2 \mathcal{C}\,\frac{P(z)}{s_{ij}}\,|M(\ldots,p_i+p_j,\ldots)|^2$$

"Soft Eikonal" : generalizes to Dipole/Antenna Functions (more later)

$$|M(\ldots,p_i,p_j,p_k\ldots)|^2 \overset{j_g\to 0}{\to} g_s^2 \mathcal{C}\,\frac{2s_{ik}}{s_{ij}s_{jk}}\,|M(\ldots,p_i,p_k,\ldots)|^2$$

Bremsstrahlung



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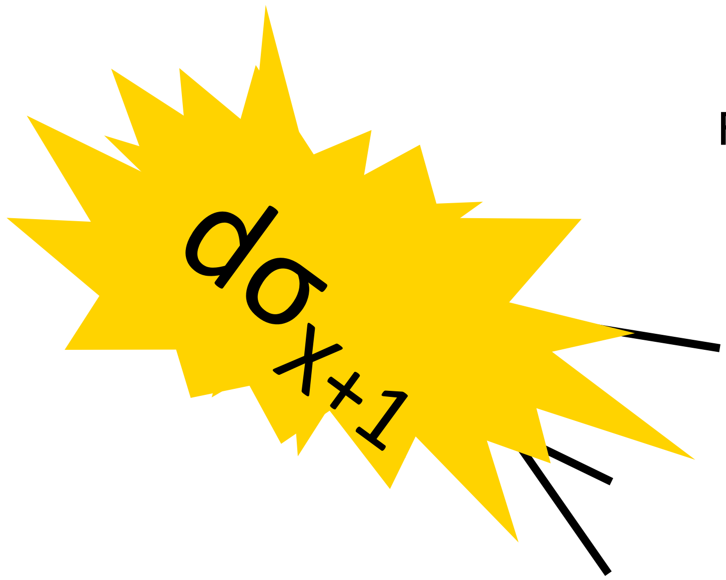

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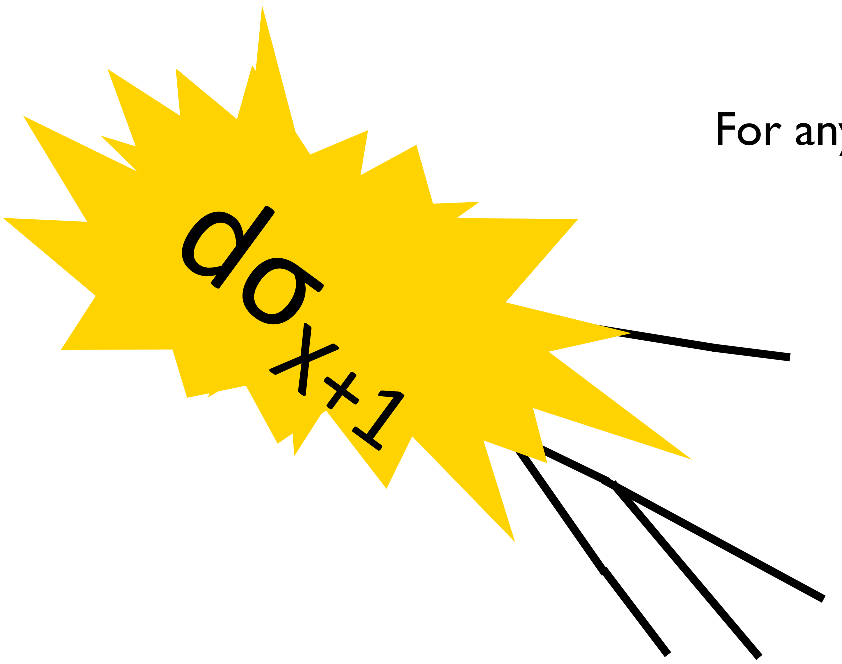
Bremsstrahlung



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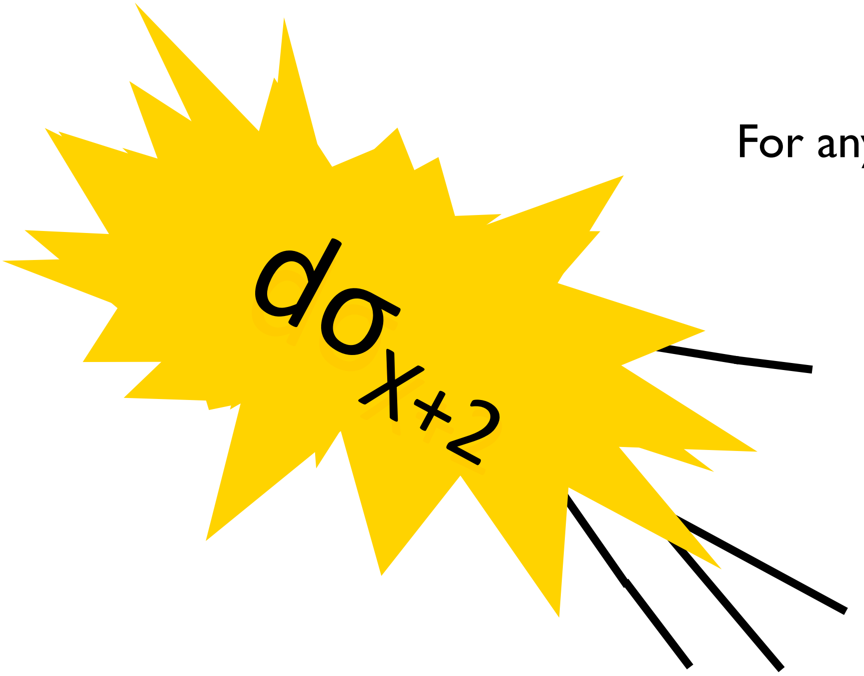

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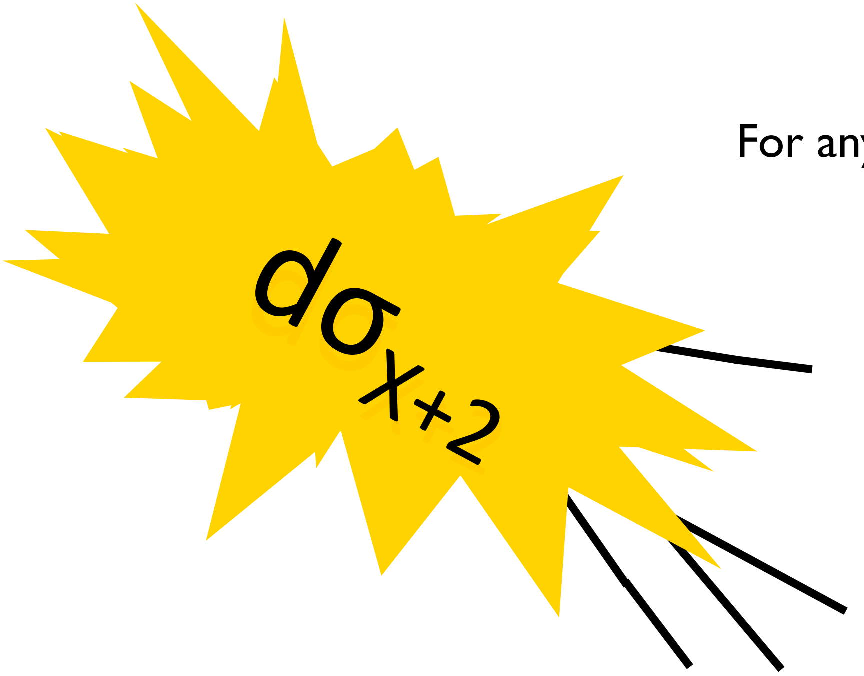

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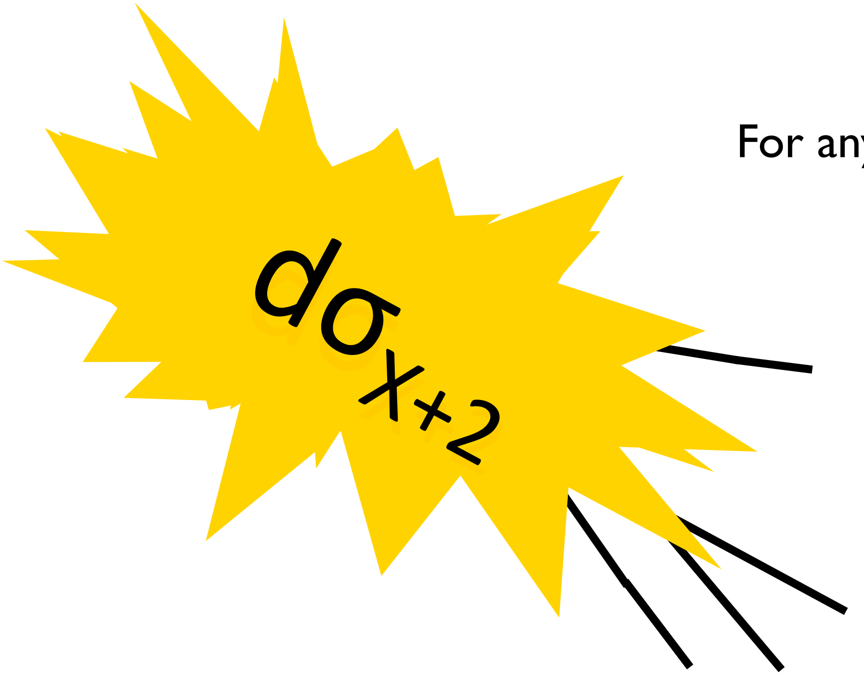
Bremsstrahlung



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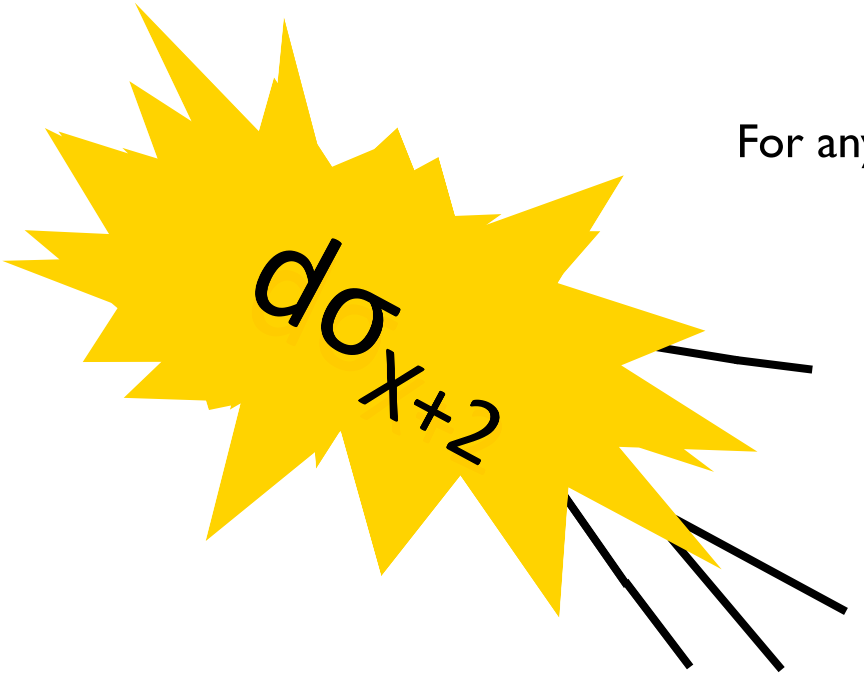
Recall: Singularities mandated by gauge theory
Non-singular terms: up to you

Bremsstrahlung



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Non-singular terms: up to you

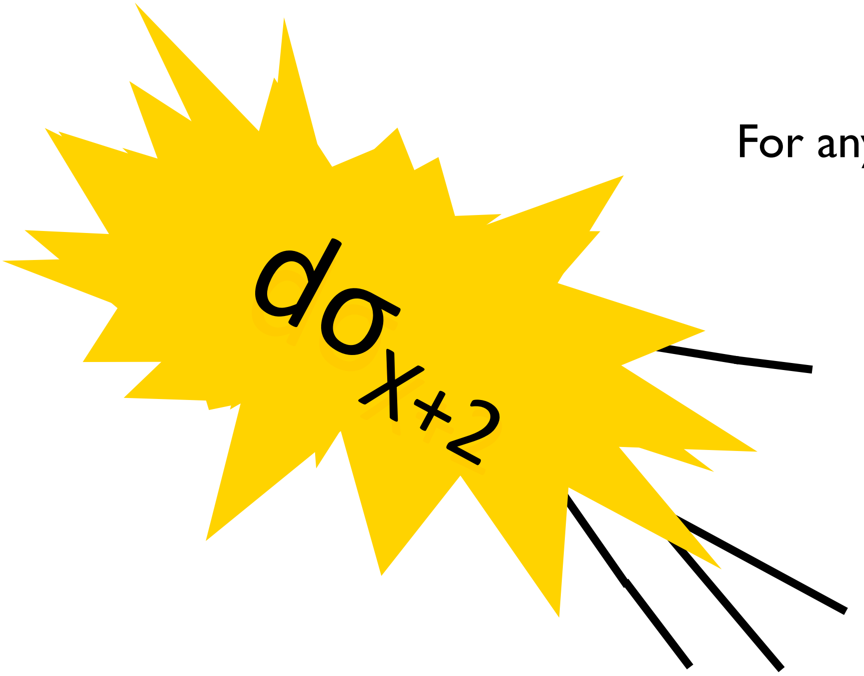
SOFT COLLINEAR

$$\frac{|\mathcal{M}(Z^0 \to q_i g_j \bar{q}_k)|^2}{|\mathcal{M}(Z^0 \to q_I \bar{q}_K)|^2} = g_s^2\, 2C_F \left[\frac{2s_{ik}}{s_{ij}s_{jk}} + \frac{1}{s_{IK}} \left(\frac{s_{ij}}{s_{jk}} + \frac{s_{jk}}{s_{ij}} \right) \right]$$

$$\frac{|\mathcal{M}(H^0 \to q_i g_j \bar{q}_k)|^2}{|\mathcal{M}(H^0 \to q_I \bar{q}_K)|^2} = g_s^2\, 2C_F \left[\frac{2s_{ik}}{s_{ij}s_{jk}} + \frac{1}{s_{IK}} \left(\frac{s_{ij}}{s_{jk}} + \frac{s_{jk}}{s_{ij}} + 2 \right) \right]$$

SOFT COLLINEAR +F

Bremsstrahlung



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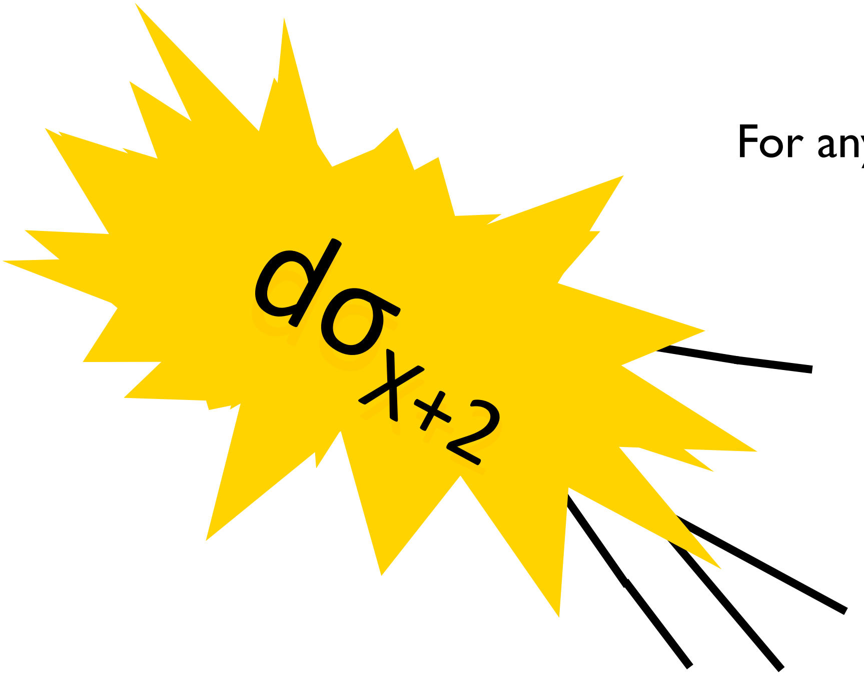
Iterated factorization

Gives us an approximation to ∞-order tree-level cross sections.
Exact in singular (strongly ordered) limit.
Finite terms → Uncertainty on non-singular (hard) radiation

Bremsstrahlung



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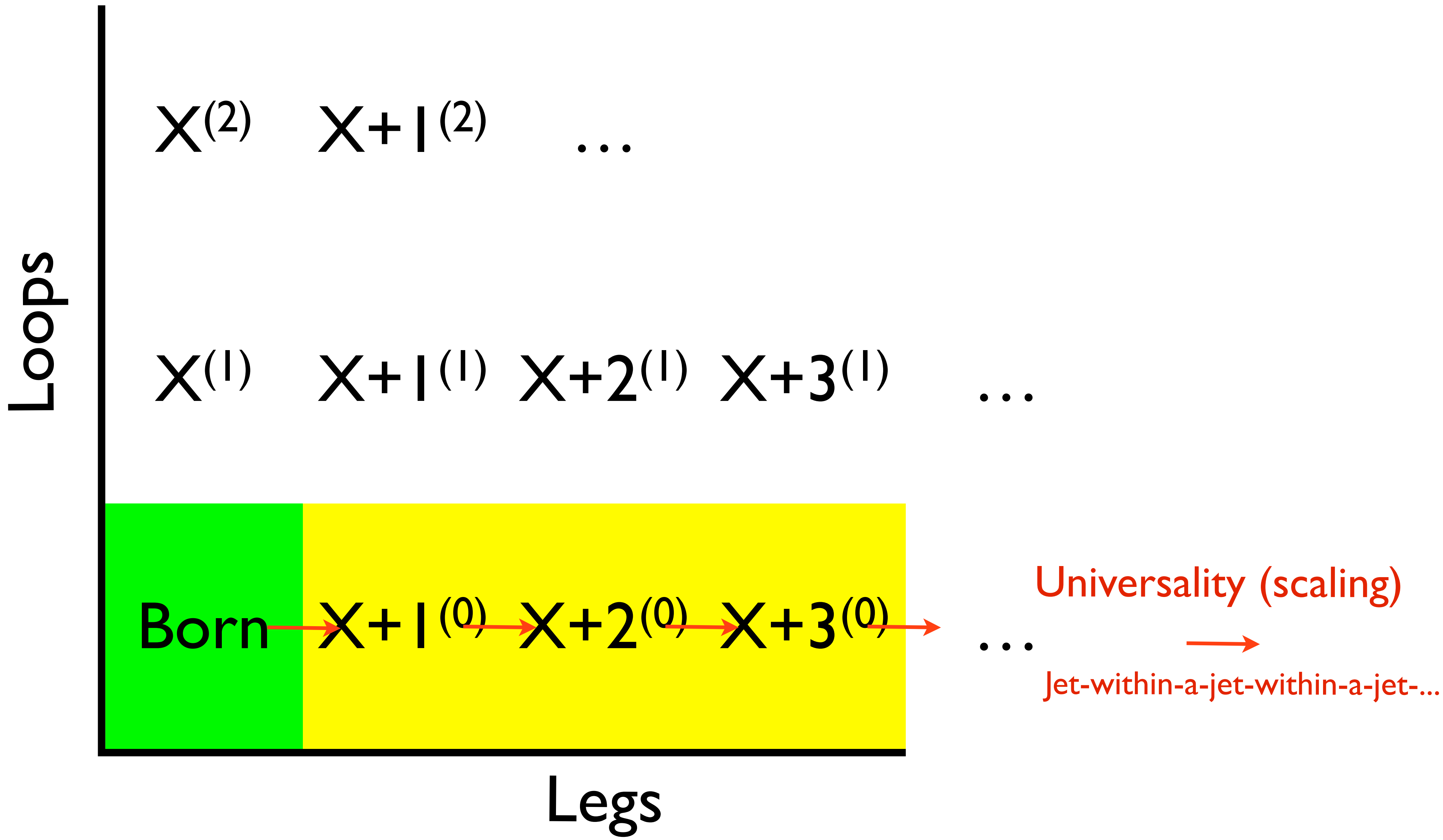
Gives us an approximation to ∞-order tree-level cross sections.
Exact in singular (strongly ordered) limit.
Finite terms → Uncertainty on non-singular (hard) radiation

But something is not right … Total σ would be infinite …

Loops and Legs

Coefficients of the Perturbative Series



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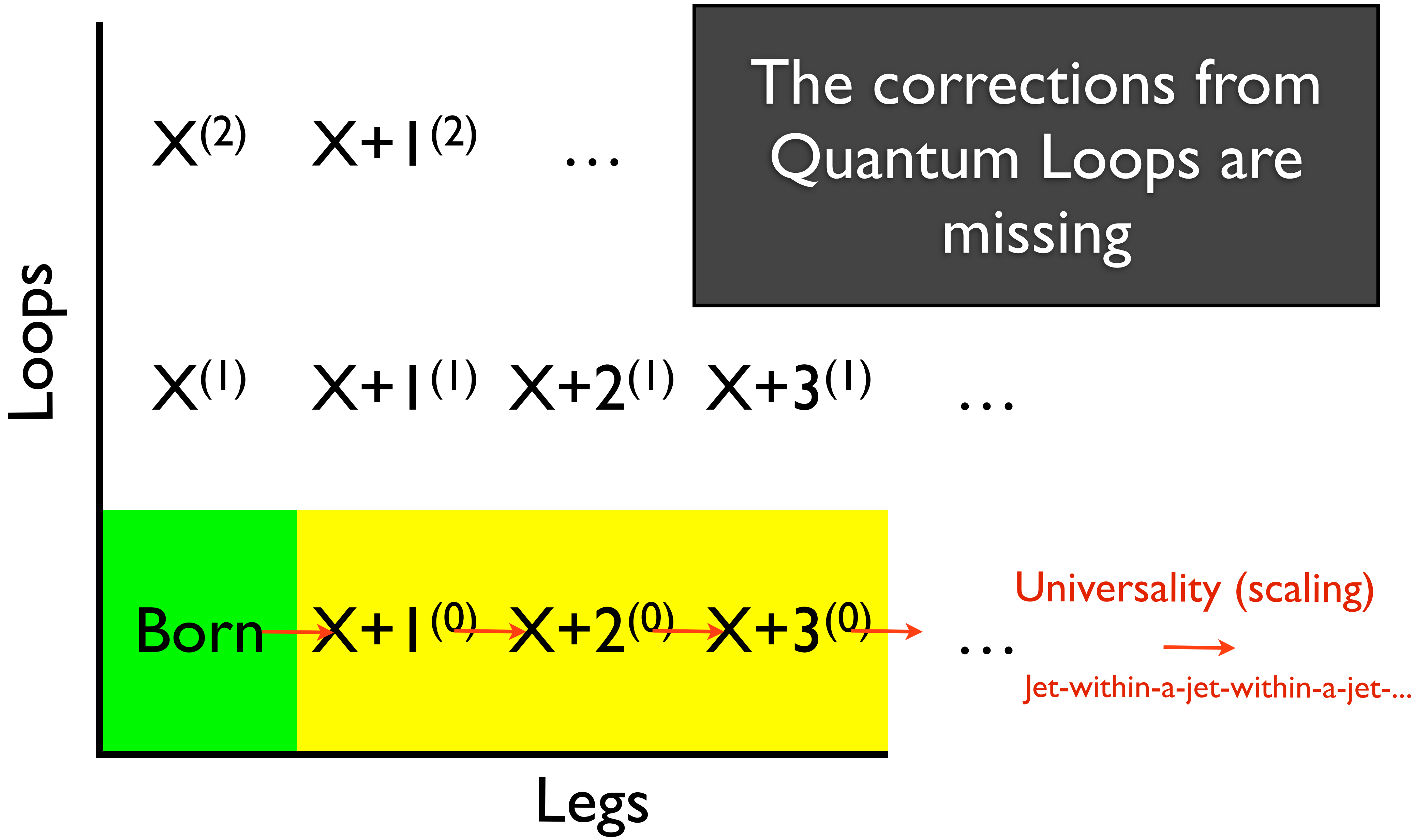
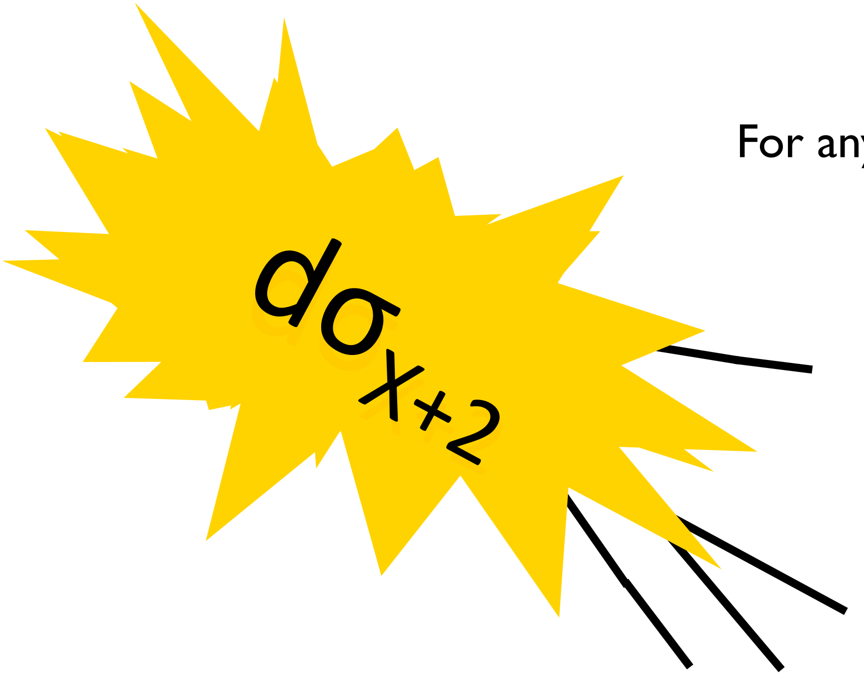


The Resummation Idea



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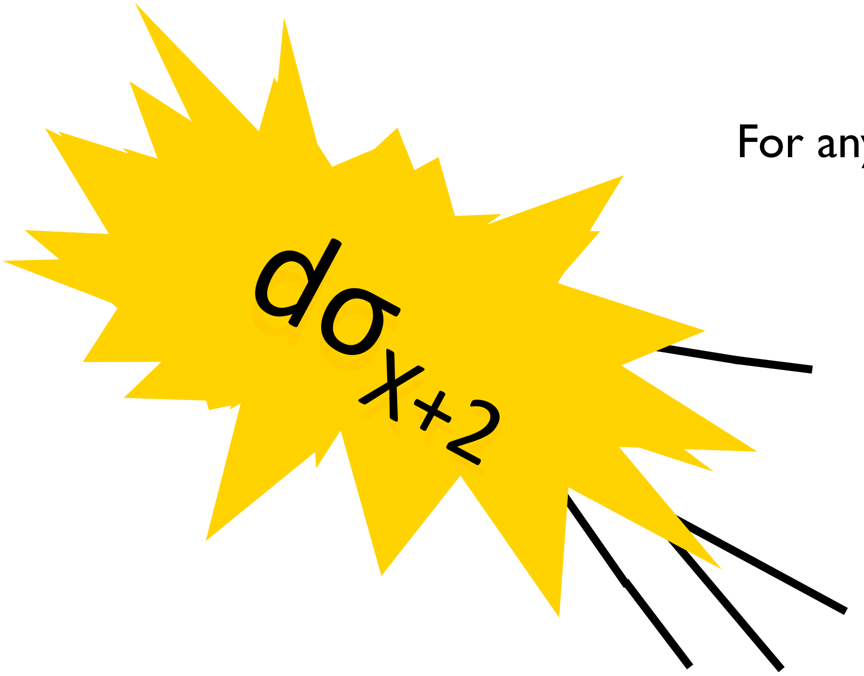
- Interpretation: the structure evolves! (example: X = 2-jets)
  - Take a jet algorithm, with resolution measure “Q”, apply it to your events
  - At a very crude resolution, you find that everything is 2-jets

QCD

Lecture III

The Resummation Idea



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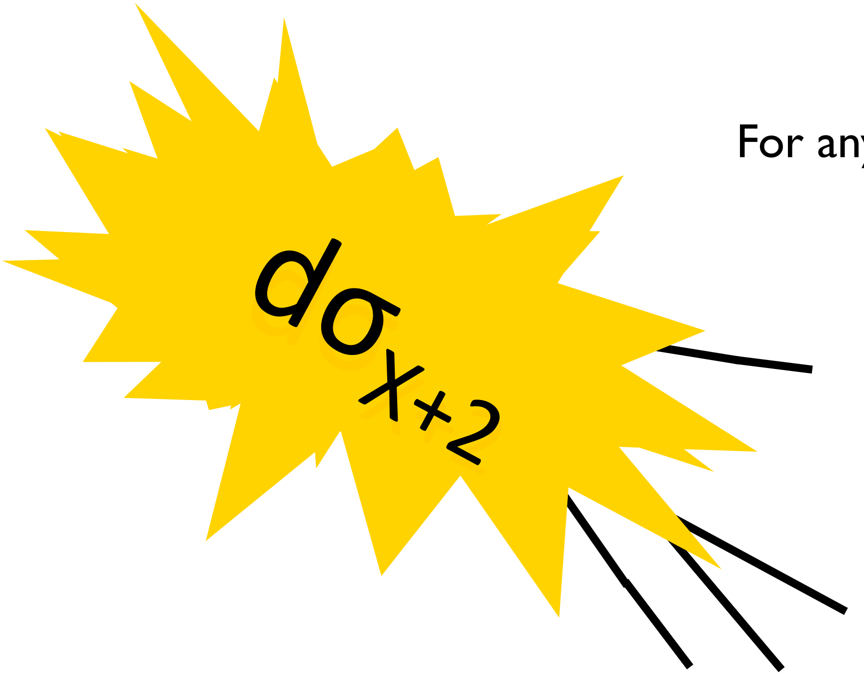
QCD

Lecture III

The Resummation Idea



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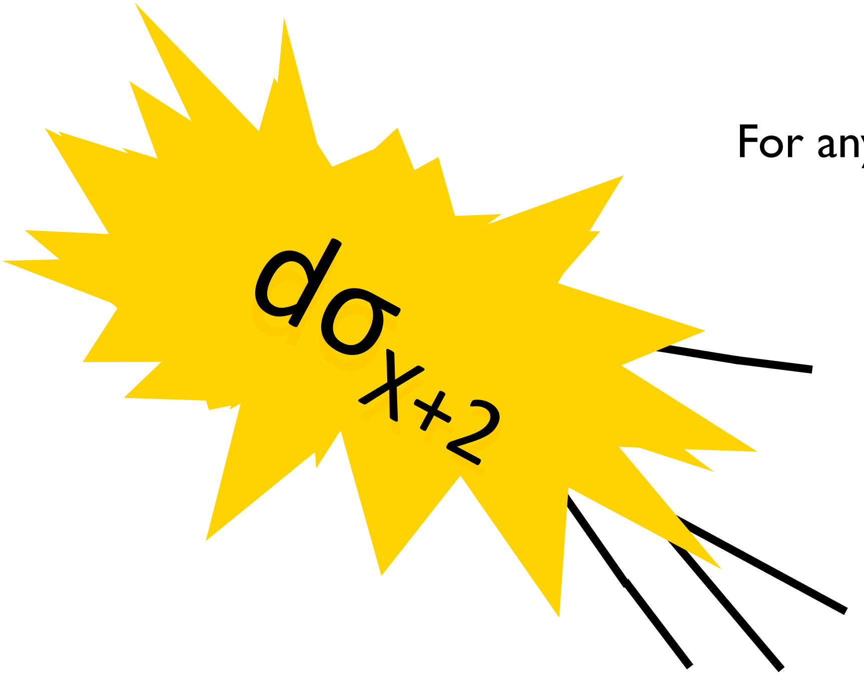

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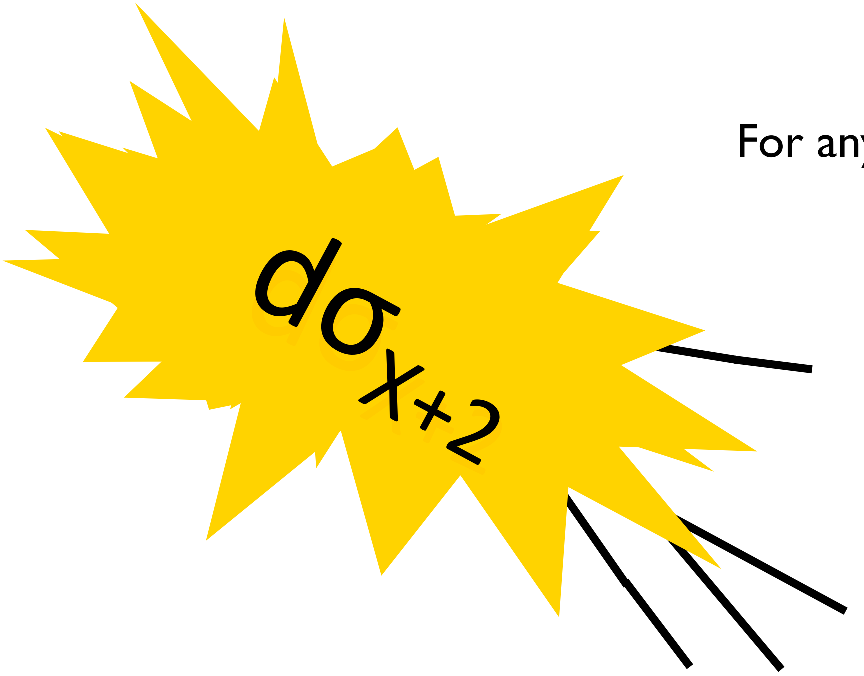

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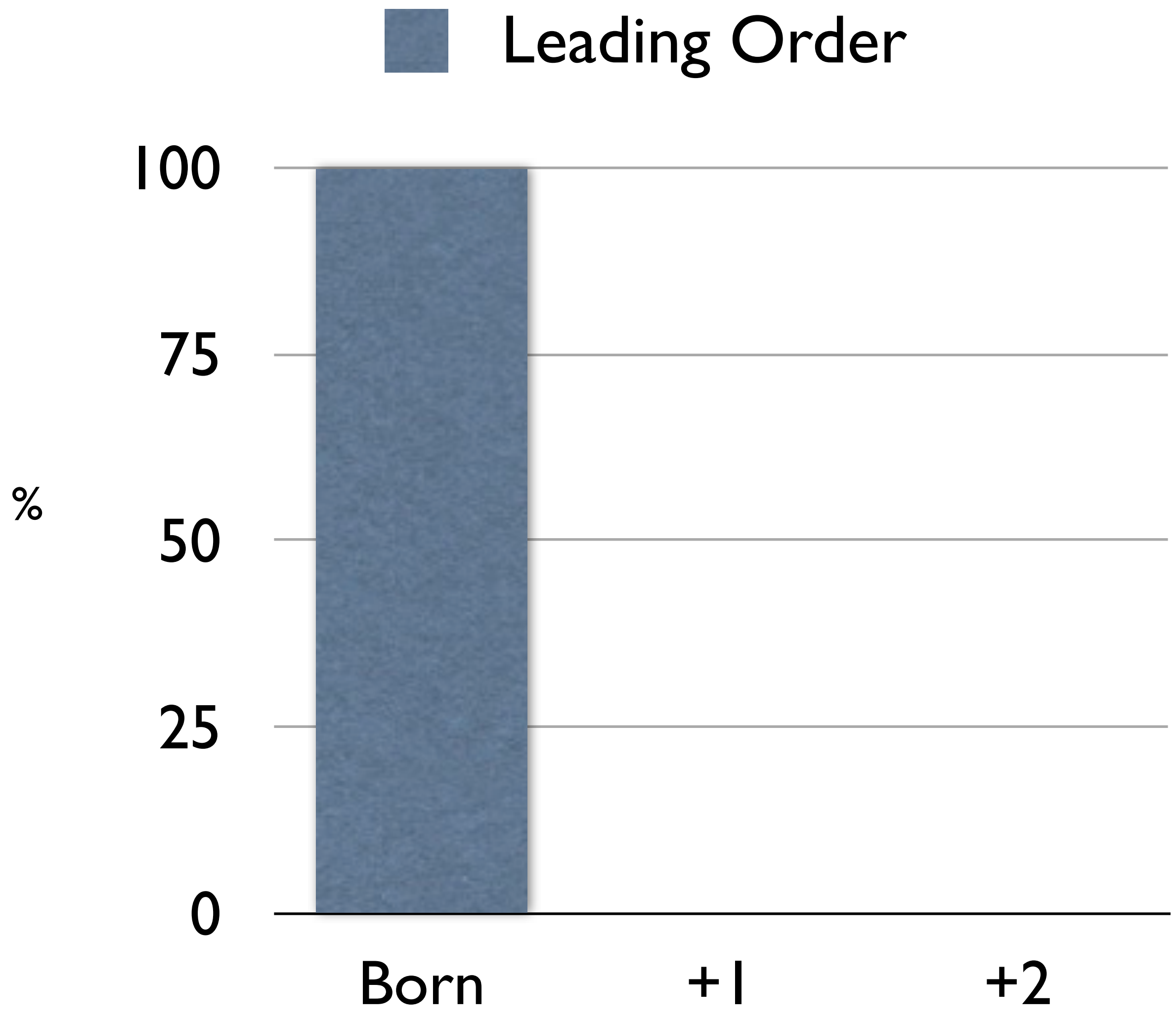
  - This evolution takes place between two scales, Q_{in} ~ **s** and Q_{end} ~ $\mathbf{Q_{had}}$
- $\sigma_{X;\mathrm{tot}} = \mathrm{Sum}\,(\sigma_{X+0,1,2,3,\ldots;\mathrm{excl}}) = \mathrm{int}(d\sigma_X)$

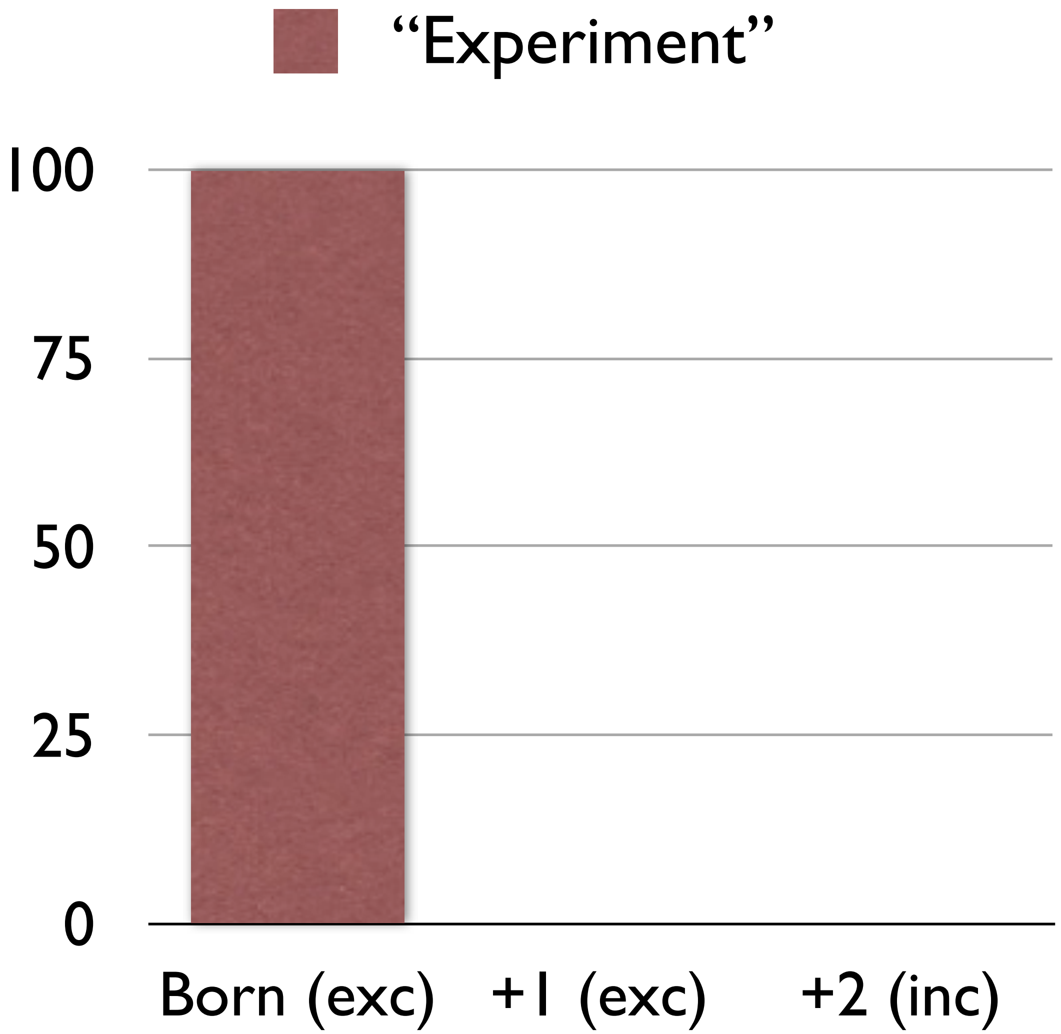
QCD

Lecture III

Evolution

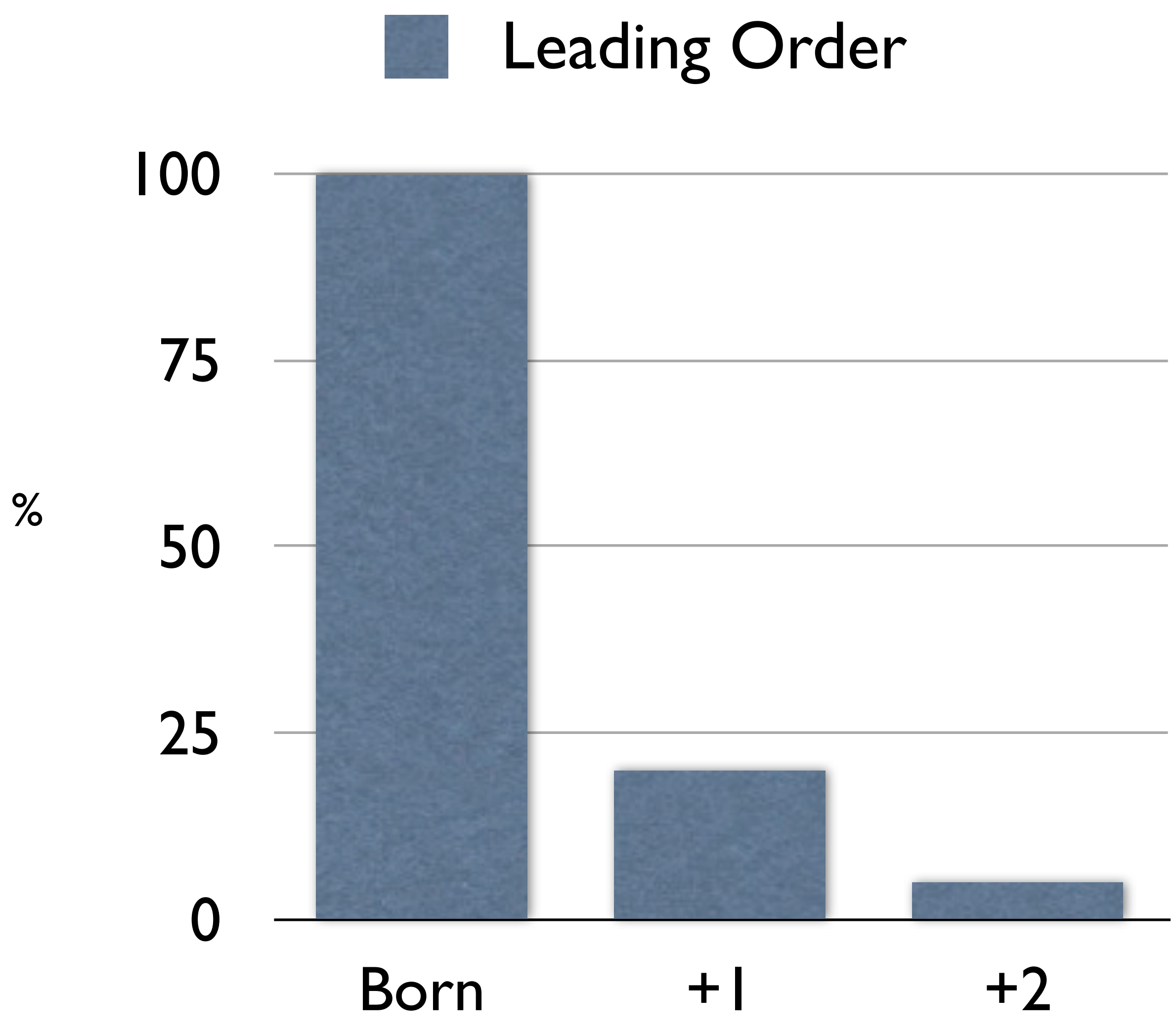
$$Q \sim Q_X$$

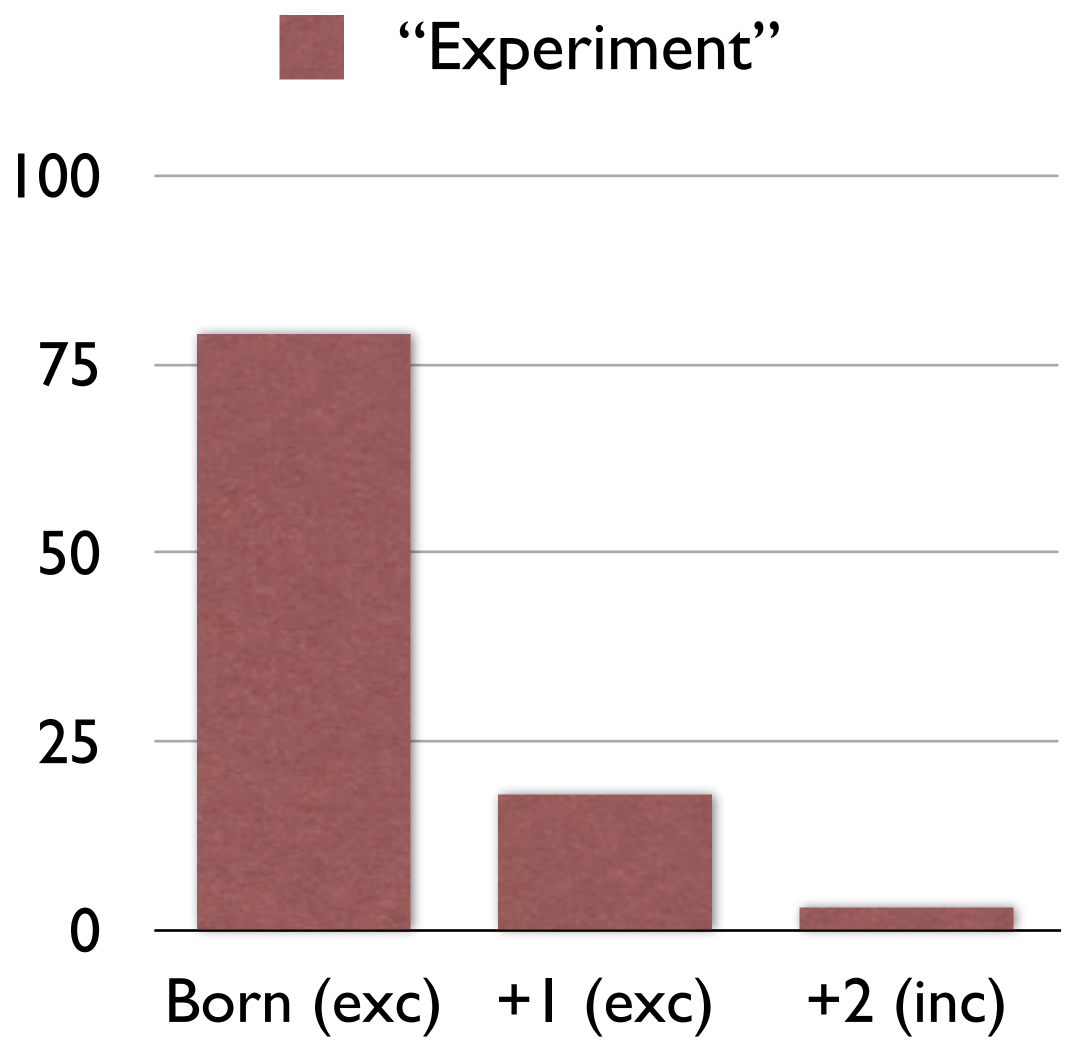




Evolution

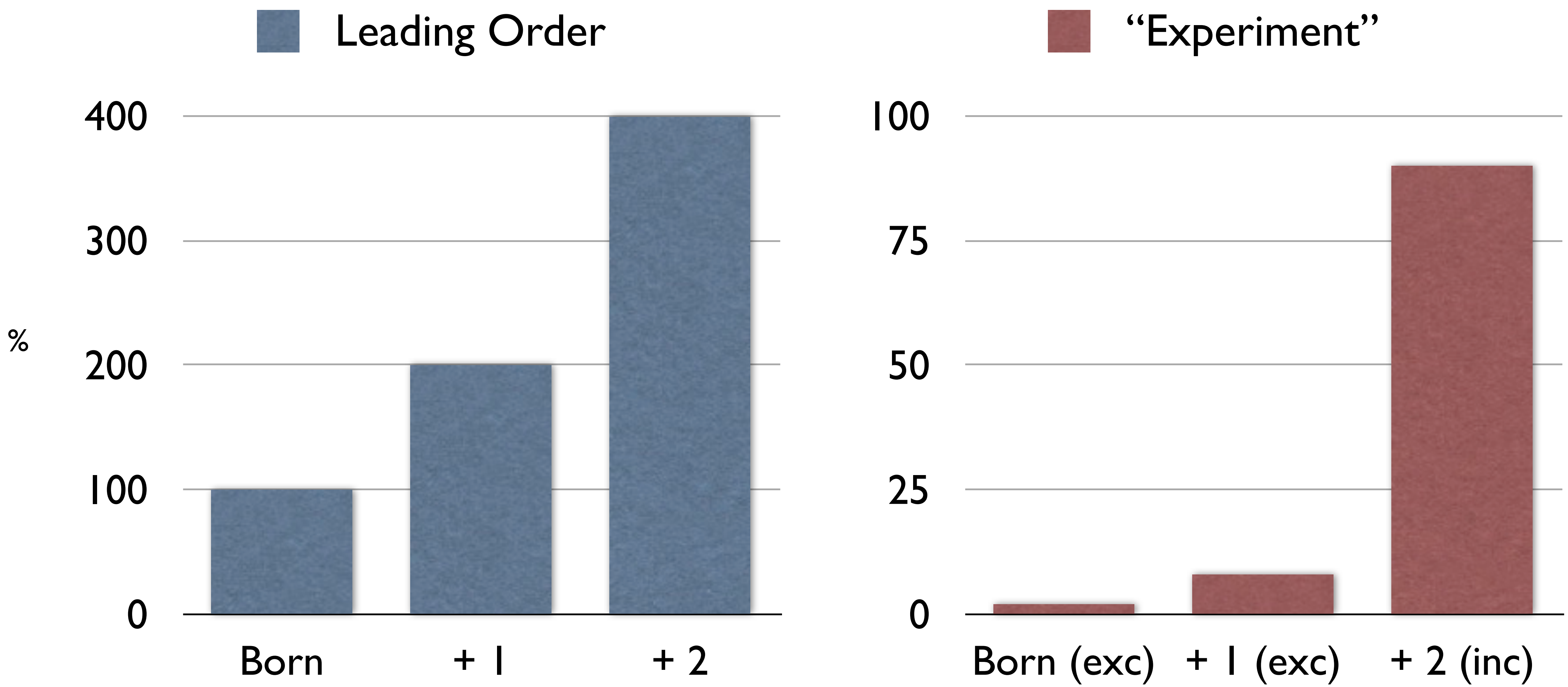
$$Q \sim \frac{Q_X}{\text{"A few"}}$$





Evolution

$$Q \ll Q_X$$



Cross Section Diverges

Cross Section Remains = Born (IR safe)
Number of Partons Diverges (IR unsafe)

QCD
Lecture III

Evolution Equations

What we need is a differential equation

- Boundary condition: a few partons defined at a high scale (Q_F)
- Then evolves (or "runs") that parton system down to a low scale (the hadronization cutoff ~ 1 GeV) → It's an evolution equation in Q_F

Evolution Equations

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Close analogue: nuclear decay

Evolve an unstable nucleus. Check if it decays + follow chains of decays.

Decay constant

$$\frac{\mathrm{d}P(t)}{\mathrm{d}t} = c_N$$

Probability to remain undecayed in the time interval [t_1,t_2]

$$\Delta(t_1,t_2) = \exp\left(-\int_{t_1}^{t_2} c_N \, \mathrm{d}t\right) = \exp\left(-c_N \, \Delta t\right)$$

$$= 1 - c_N \Delta t + \mathcal{O}(c_N^2)$$

Decay probability per unit time

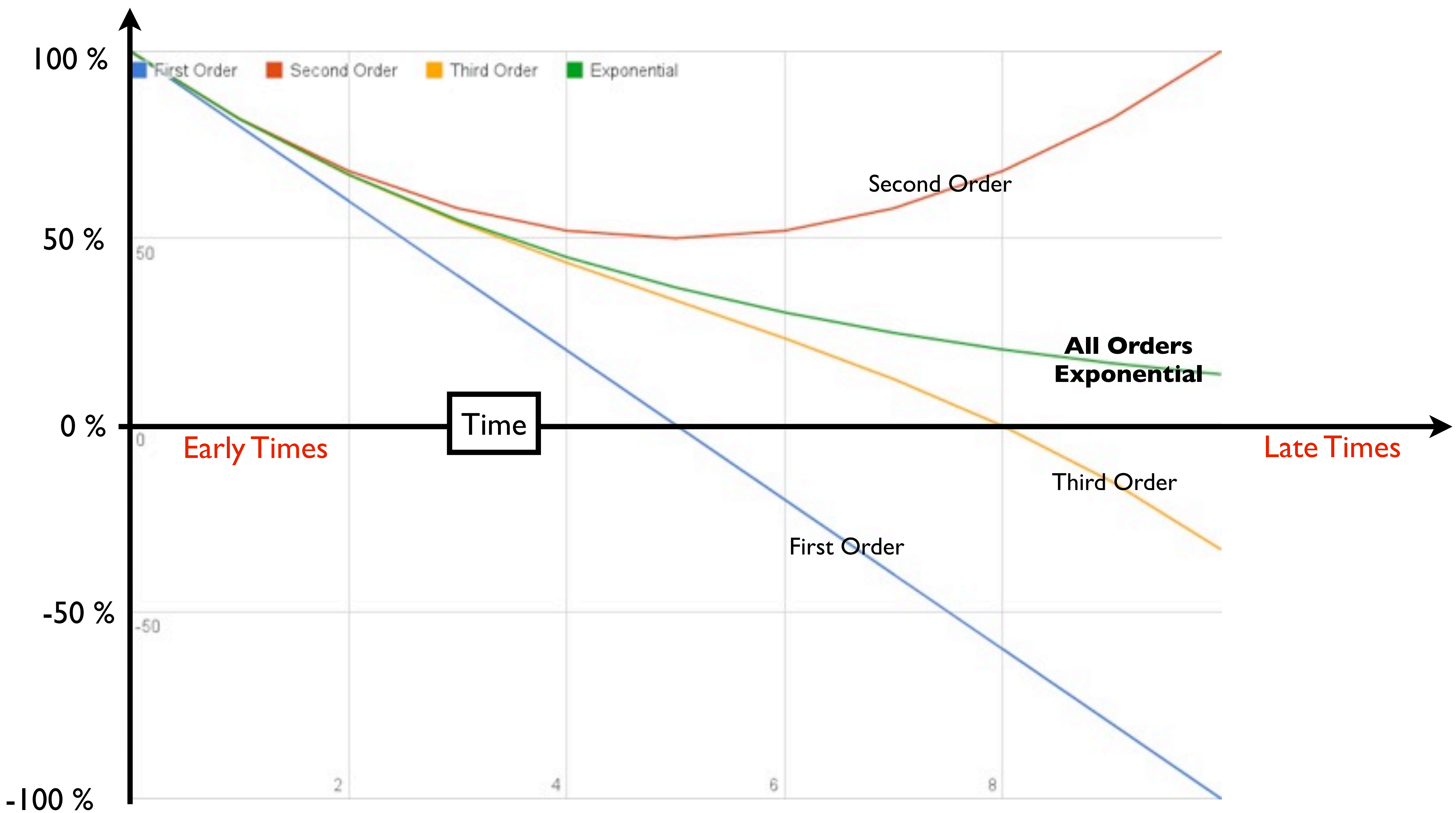
$$\frac{\mathrm{d}P_{\mathrm{res}}(t)}{\mathrm{d}t} = \frac{-\mathrm{d}\Delta}{\mathrm{d}t} = c_N \, \Delta(t_1,t)$$

(requires that the nucleus did not already decay)

$\Delta(t_1,t_2)$: "Sudakov Factor"

Nuclear Decay

Nuclei remaining undecayed after time t = $\Delta(t_1, t_2) = \exp\left(-\int_{t_1}^{t_2} \mathrm{d}t \, \frac{\mathrm{d}\mathcal{P}}{\mathrm{d}t}\right)$



The Sudakov Factor

In nuclear decay, the Sudakov factor counts:

How many nuclei remain undecayed after a time t

Probability to remain undecayed in the time interval [t_1,t_2]

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QCD

Lecture III

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In parton showers, we may also define a Sudakov factor for the parton system. It counts

The probability that the parton system doesn't evolve (emit) when I run the factorization scale (~1/time) from a high to a lower scale

Evolution probability per unit time

(replace c_N by proper shower evolution kernels)

$$\frac{\mathrm{d}P_{\mathrm{res}}(t)}{\mathrm{d}t} = \frac{-\mathrm{d}\Delta}{\mathrm{d}t} = c_N \, \Delta(t_1,t)$$

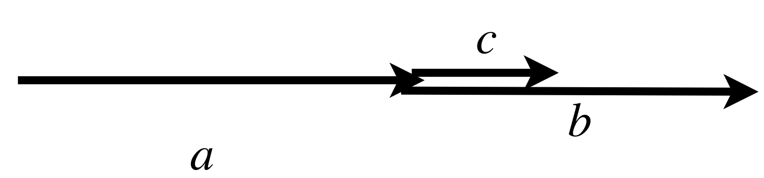
What's the evolution kernel?

Altarelli-Parisi splitting functions

Can be derived (*in the collinear limit*) from requiring invariance of the physical result with respect to Q_F → RGE

Altarelli-Parisi
(E.g., PYTHIA)

$$\mathrm{d}\mathcal{P}_a = \sum_{b,c} \frac{\alpha_{abc}}{2\pi} P_{a\to bc}(z)\,\mathrm{d}t\,\mathrm{d}z\ .$$



$p_b = z\, p_a$

$p_c = (1\text{-}z)\, p_a$

$$\mathrm{d}t = \frac{\mathrm{d}Q^2}{Q^2} = \mathrm{d}\ln Q^2$$

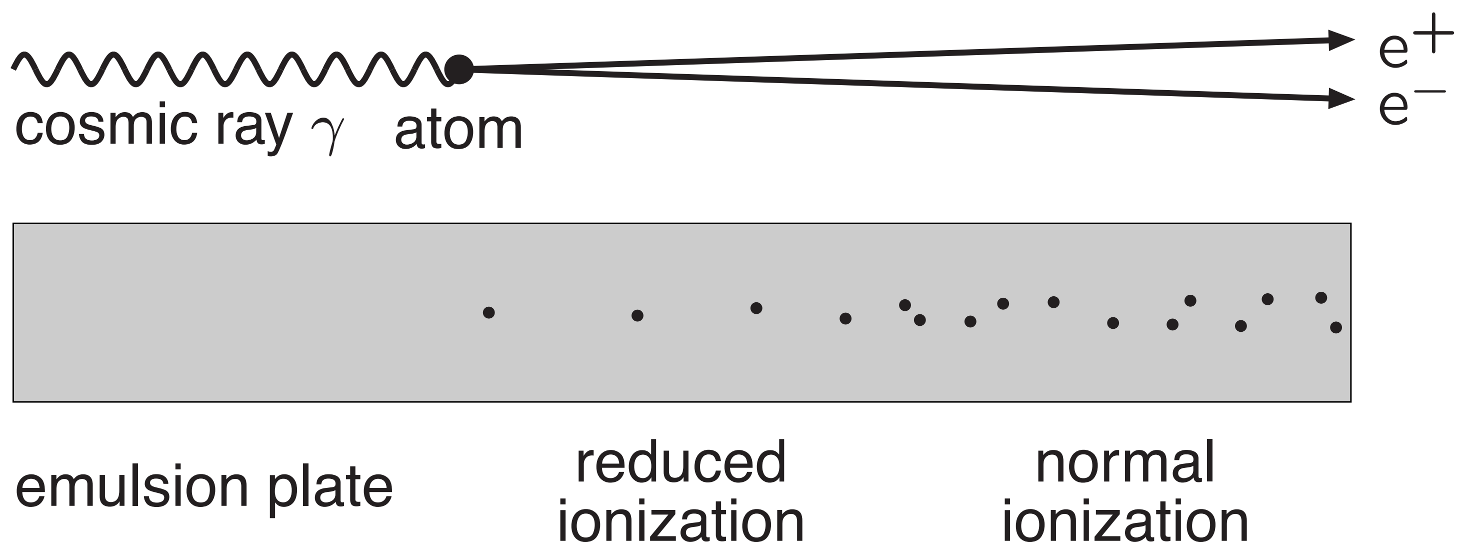
$$\begin{aligned}
P_{\mathrm{q}\to\mathrm{qg}}(z) &= C_F \frac{1+z^2}{1-z}\ , \\
P_{\mathrm{g}\to\mathrm{gg}}(z) &= N_C \frac{(1-z(1-z))^2}{z(1-z)}\ , \\
P_{\mathrm{g}\to\mathrm{q\bar{q}}}(z) &= T_R\,(z^2+(1-z)^2)\ , \\
P_{\mathrm{q}\to\mathrm{q}\gamma}(z) &= e_{\mathrm{q}}^2 \frac{1+z^2}{1-z}\ , \\
P_{\ell\to\ell\gamma}(z) &= e_\ell^2 \frac{1+z^2}{1-z}\ ,
\end{aligned}$$

… with Q^2 some measure of event/jet resolution measuring parton virtualities / formation time / …
Different models make different choices
But choice is not entirely free …

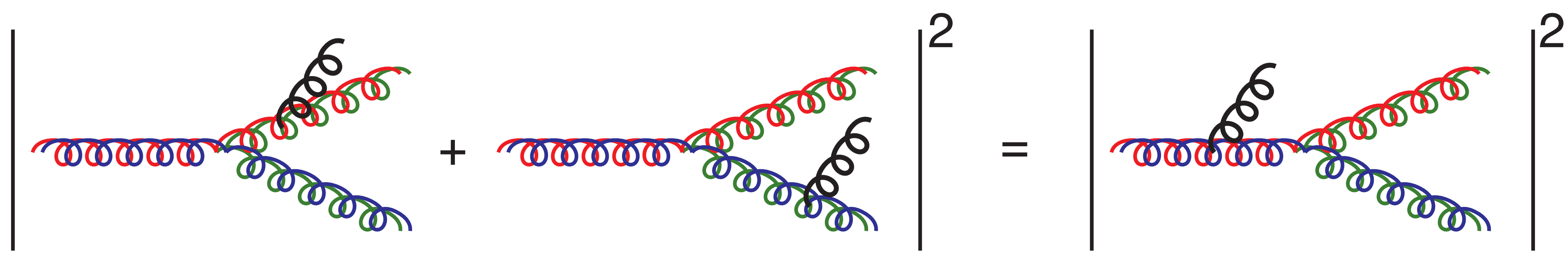
Coherence

Illustrations by T. Sjöstrand

QED: Chudakov effect (mid-fifties)



QCD: colour coherence for **soft** gluon emission



→ an example of an interference effect that can be treated probabilistically

More interference effects can be included by matching to full matrix elements → tomorrow

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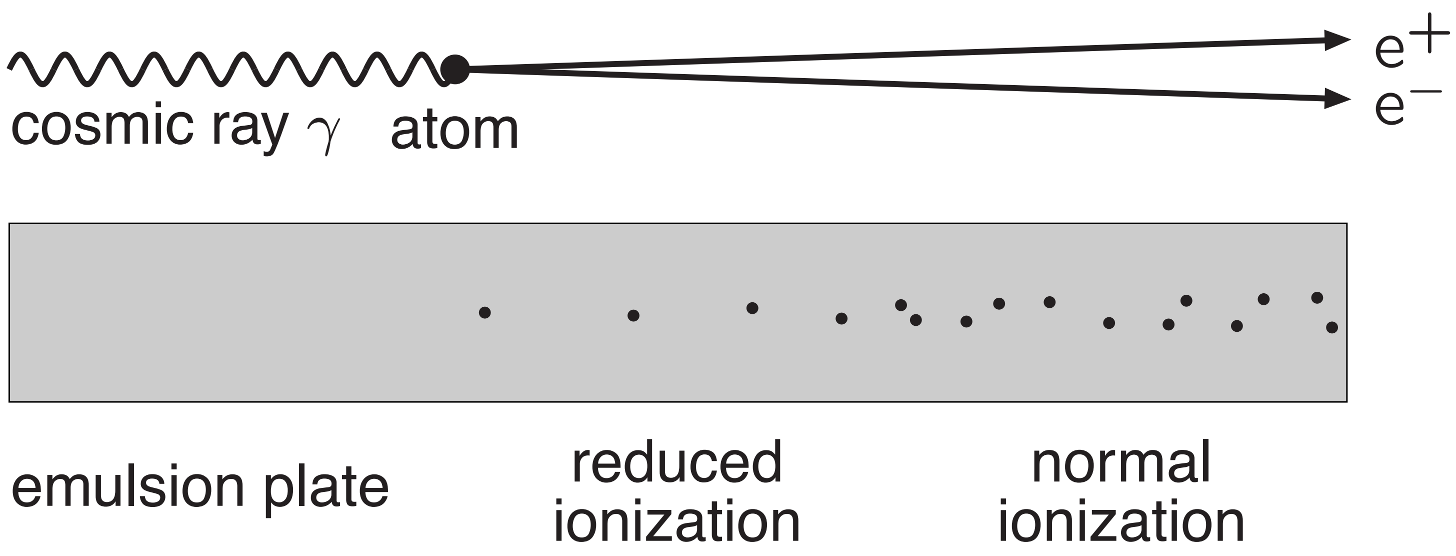


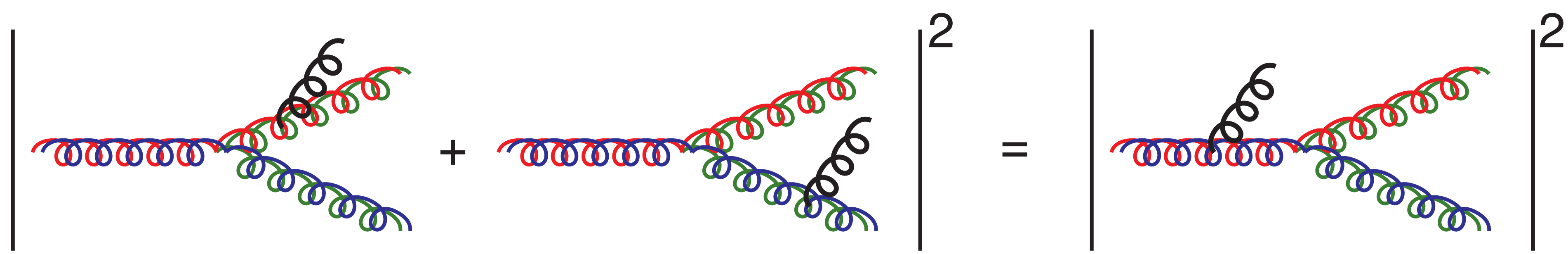
Approximations to Coherence:

Angular Ordering (HERWIG)

Angular Vetos (PYTHIA)

Coherent Dipoles/Antennae (ARIADNE, CS, VINCIA)

QCD: colour coherence for **soft** gluon emission



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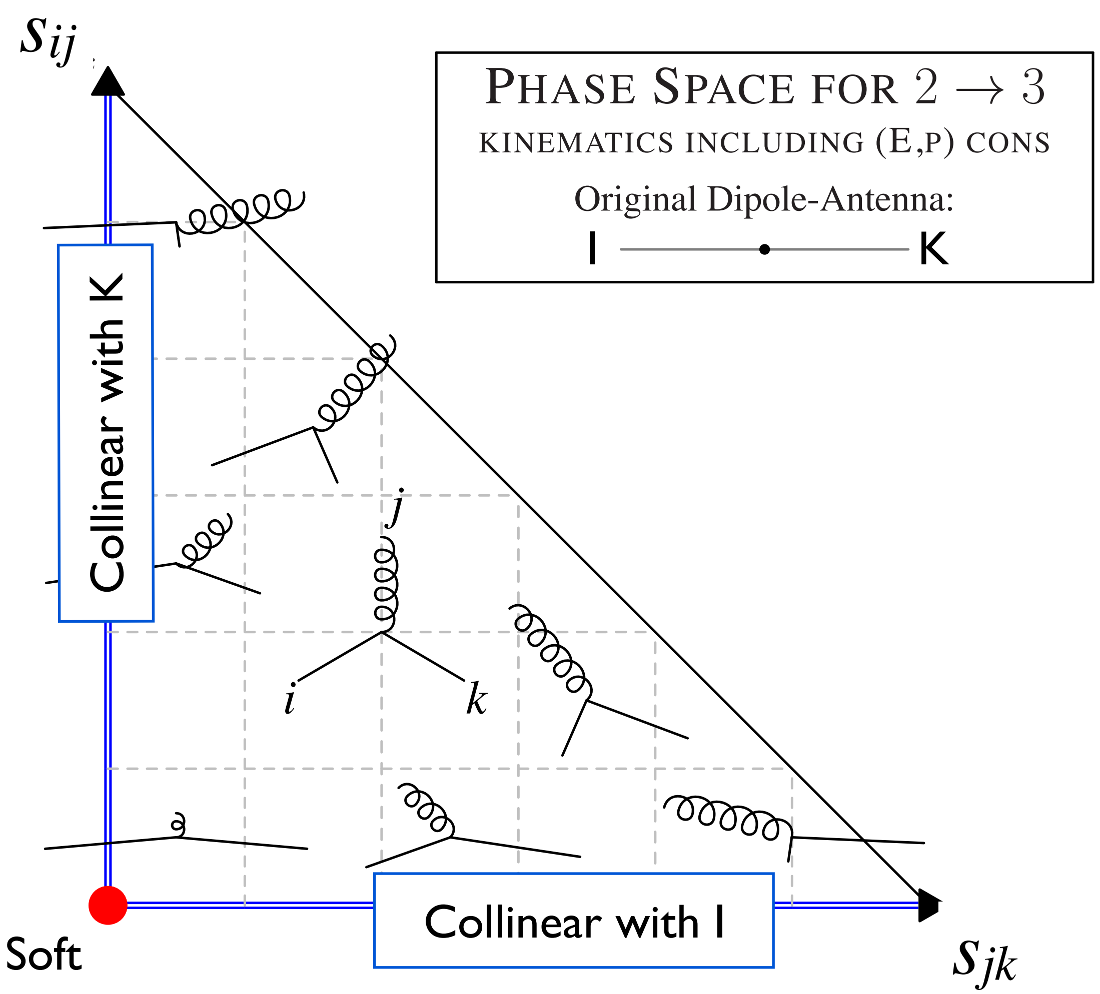
What is t ?

t : Shower Evolution Measure

~ Jet Resolution Measure

~ Sliding Factorization Scale

$$\mathrm{d}t = \frac{\mathrm{d}Q^2}{Q^2} = \mathrm{d}\ln Q^2$$



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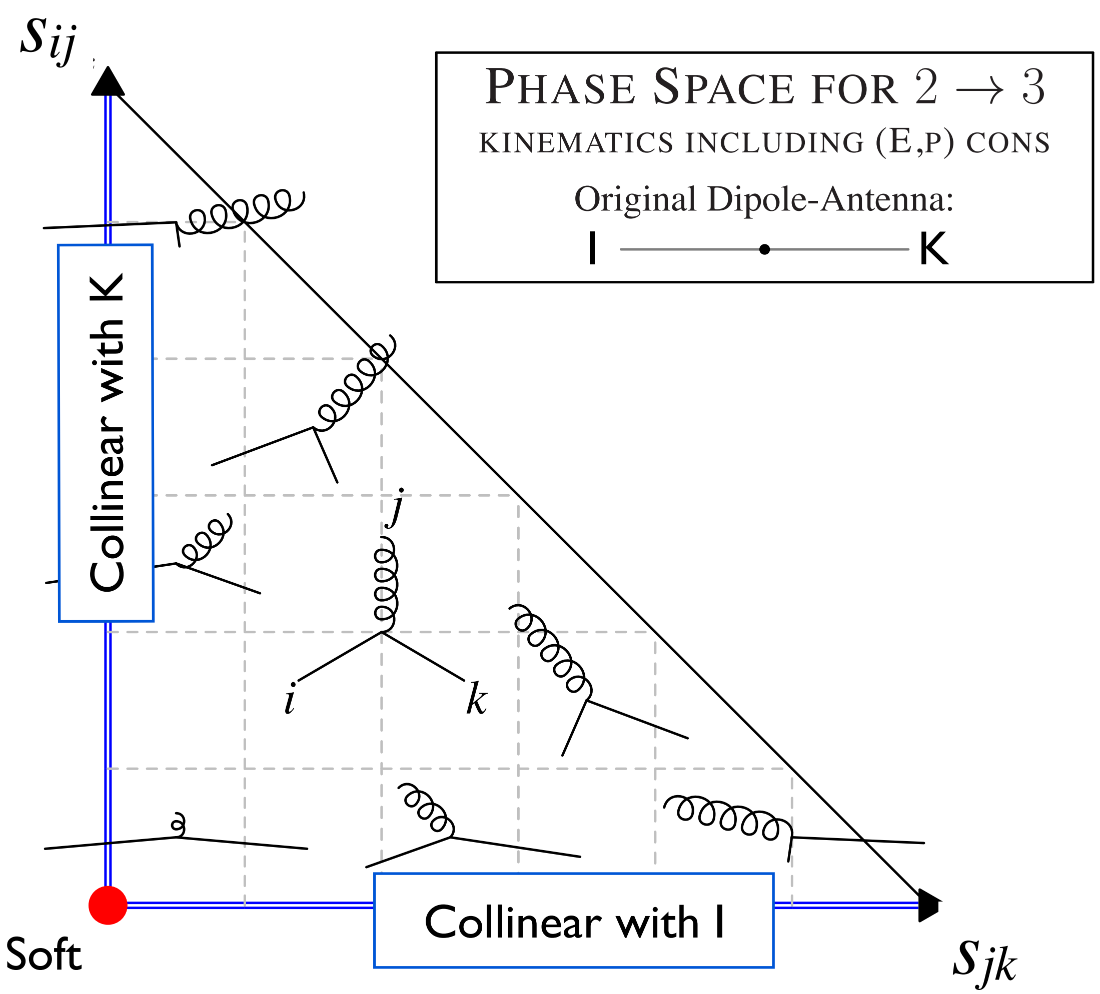
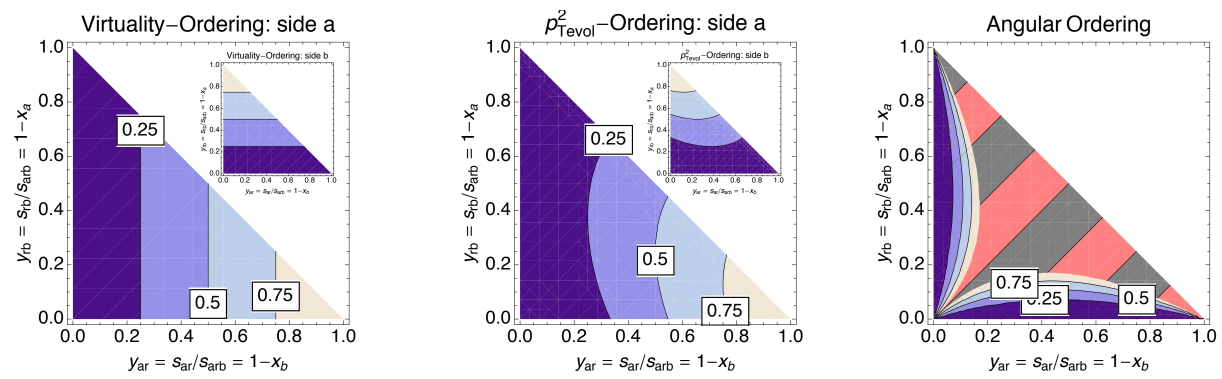


Parton Showers (PYTHIA & HERWIG)



PYTHIA: imposes angular vetos to obtain coherence

HERWIG:coherent (by angular ordering) but has dead zone

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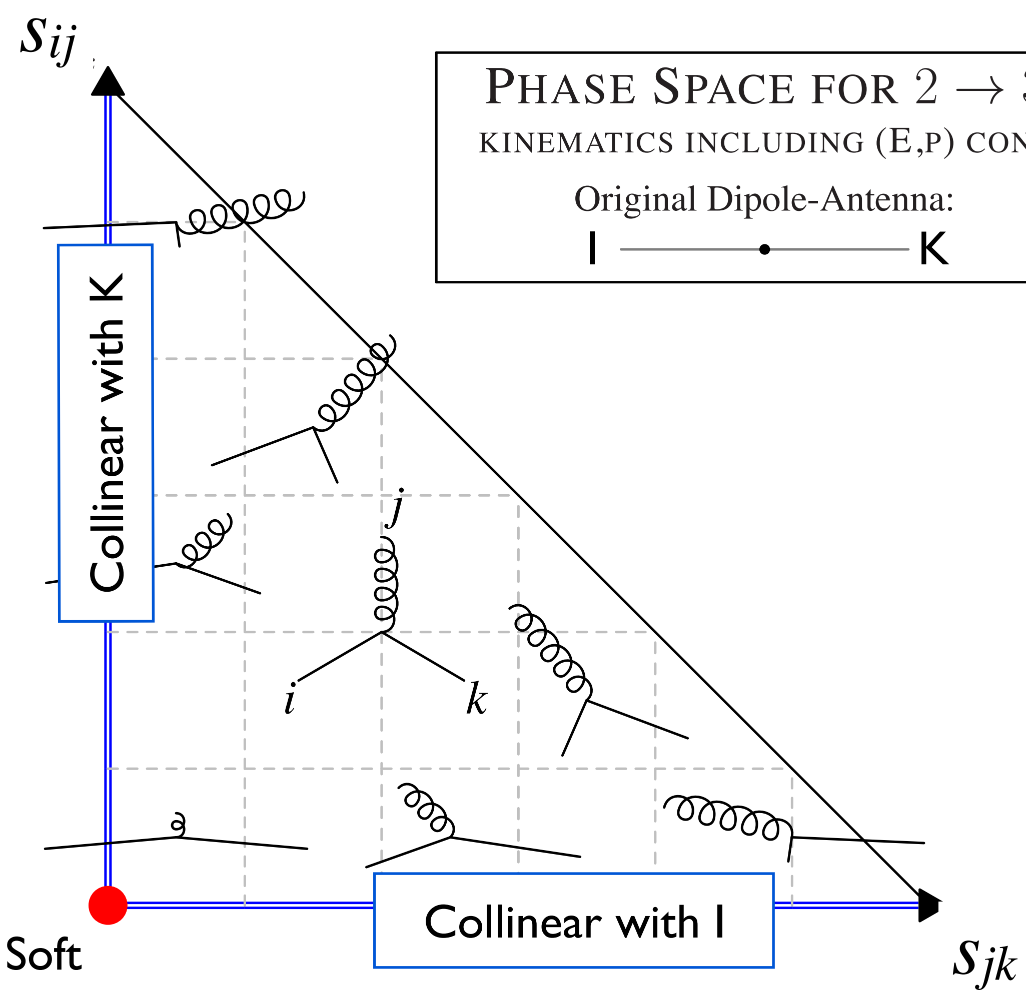

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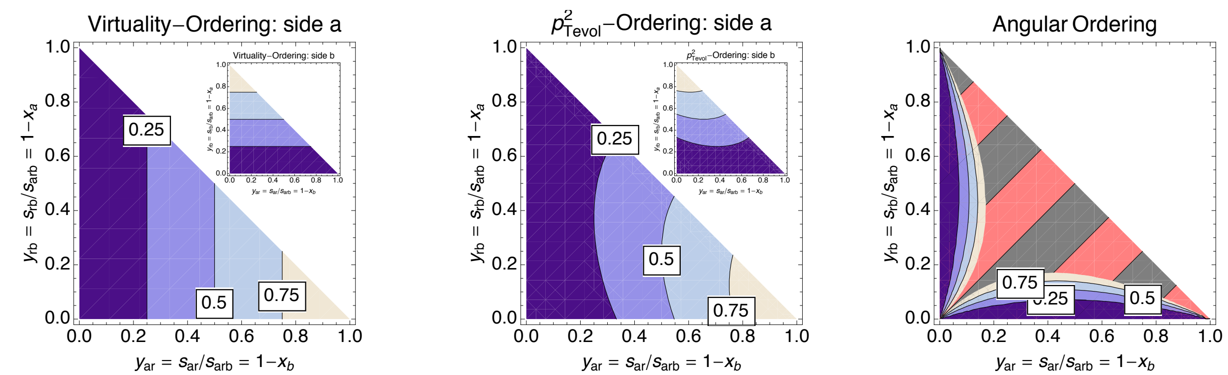

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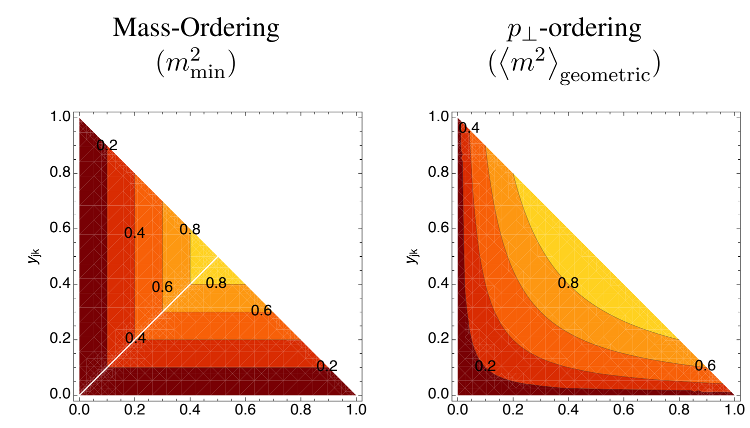
↓

Dipole/Antenna Showers (ARIADNE, SHERPA,VINCIA)



(a) $Q_E^2 = m_D^2 = 2\min(y_{ij}, y_{jk})s$ (b) $Q_E^2 = 2p_\perp\sqrt{s} = 2\sqrt{y_{ij}y_{jk}}s$

Intrinsically Coherent

QCD

Lecture III

Antennae

Observation: the evolution kernel is responsible for generating real radiation.

→ Choose it to be the ratio of the real-emission matrix element to the Born-level matrix element

→ AP in coll limit, but also includes the Eikonal for soft radiation.

Dipole-Antennae
(E.g., ARIADNE, VINCIA)

$$\mathrm{d}\mathcal{P}_{IK\to ijk} = \frac{\mathrm{d}s_{ij}\mathrm{d}s_{jk}}{16\pi^2 s}\; a(s_{ij}, s_{jk})$$

2→3 instead of 1→2
(→ all partons on shell)

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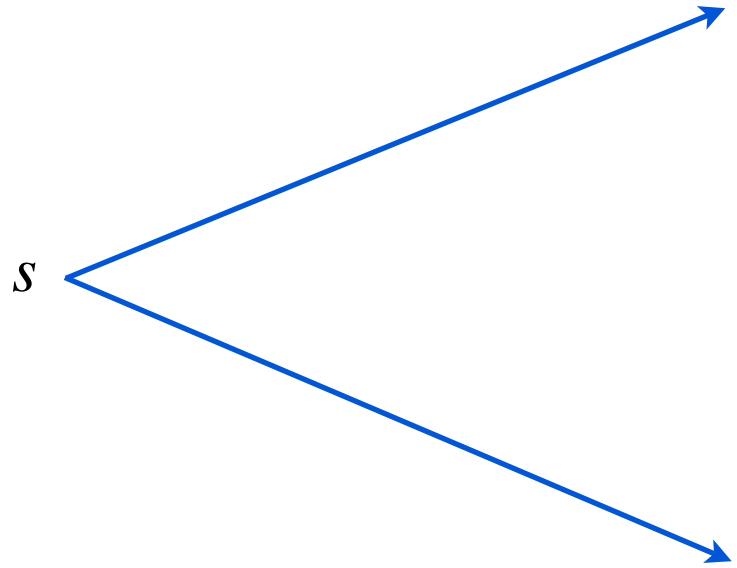

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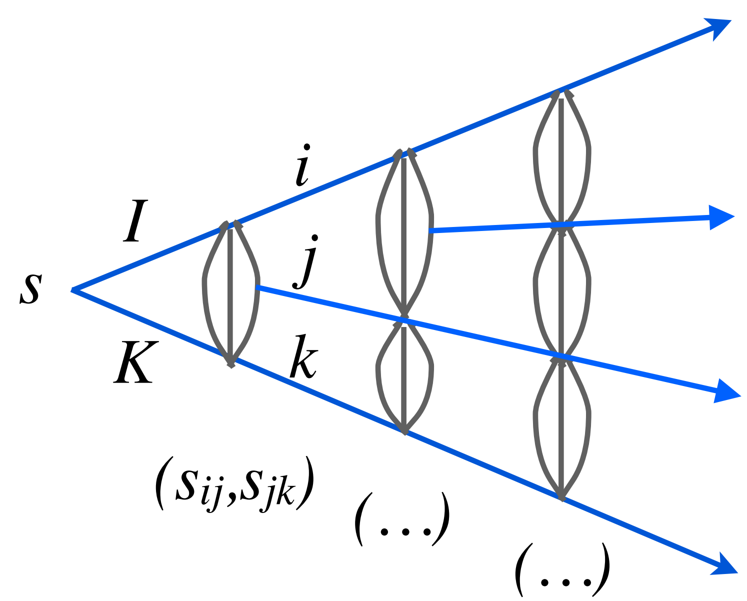

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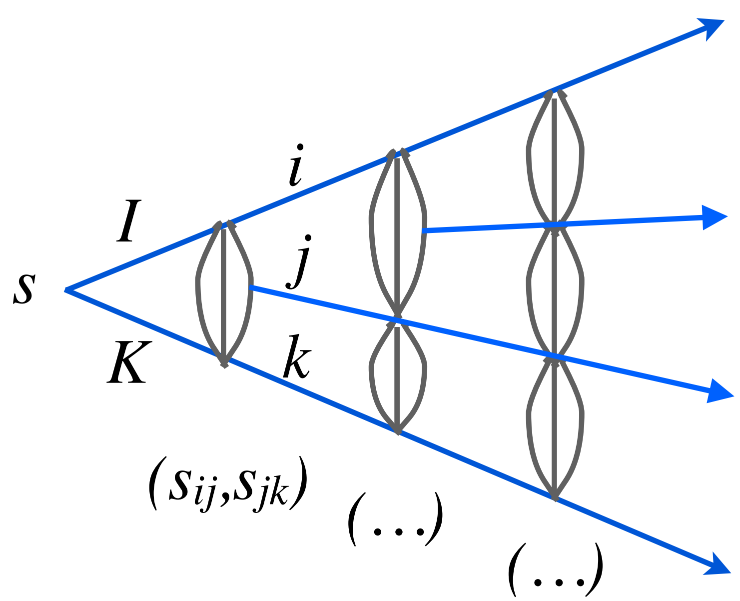


2→3 instead of 1→2
(→ all partons on shell)

$$a_{q\bar{q}\to qg\bar{q}} = \frac{2C_F}{s_{ij}s_{jk}}\left(2s_{ik}s + s_{ij}^2 + s_{jk}^2\right)$$

$$a_{qg\to qgg} = \frac{C_A}{s_{ij}s_{jk}}\left(2s_{ik}s + s_{ij}^2 + s_{jk}^2 - s_{ij}^3\right)$$

$$a_{gg\to ggg} = \frac{C_A}{s_{ij}s_{jk}}\left(2s_{ik}s + s_{ij}^2 + s_{jk}^2 - s_{ij}^3 - s_{jk}^3\right)$$

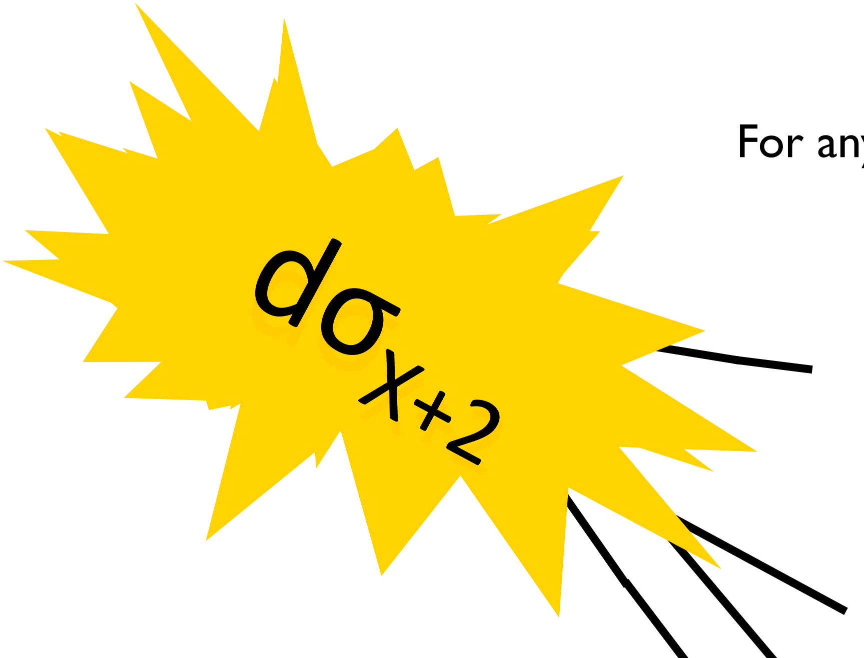
$$a_{qg\to q\bar{q}'q'} = \frac{T_R}{s_{jk}}\left(s - 2s_{ij} + 2s_{ij}^2\right)$$

$$a_{gg\to g\bar{q}'q'} = a_{qg\to q\bar{q}'q'}$$

… + non-singular terms

QCD
Lecture III

Evolution → Unitarity



For any basic process $d\sigma_X$ = ✓ (calculated process by process)

$$d\sigma_{X+1} \sim N_C 2 g_s^2 \frac{ds_{i1}}{s_{i1}} \frac{ds_{1j}}{s_{1j}} d\sigma_X \quad ✓$$

$$d\sigma_{X+2} \sim N_C 2 g_s^2 \frac{ds_{i2}}{s_{i2}} \frac{ds_{2j}}{s_{2j}} d\sigma_{X+1} \quad \ldots$$

Unitarity

Kinoshita-Lee-Nauenberg:

Loop = – Int(Tree) + F

Neglect F → Leading-Logarithmic (LL) Approximation

Imposed by Event *evolution*:

When (X) branches to (X+1):
Gain one (X+1). Loose one (X).

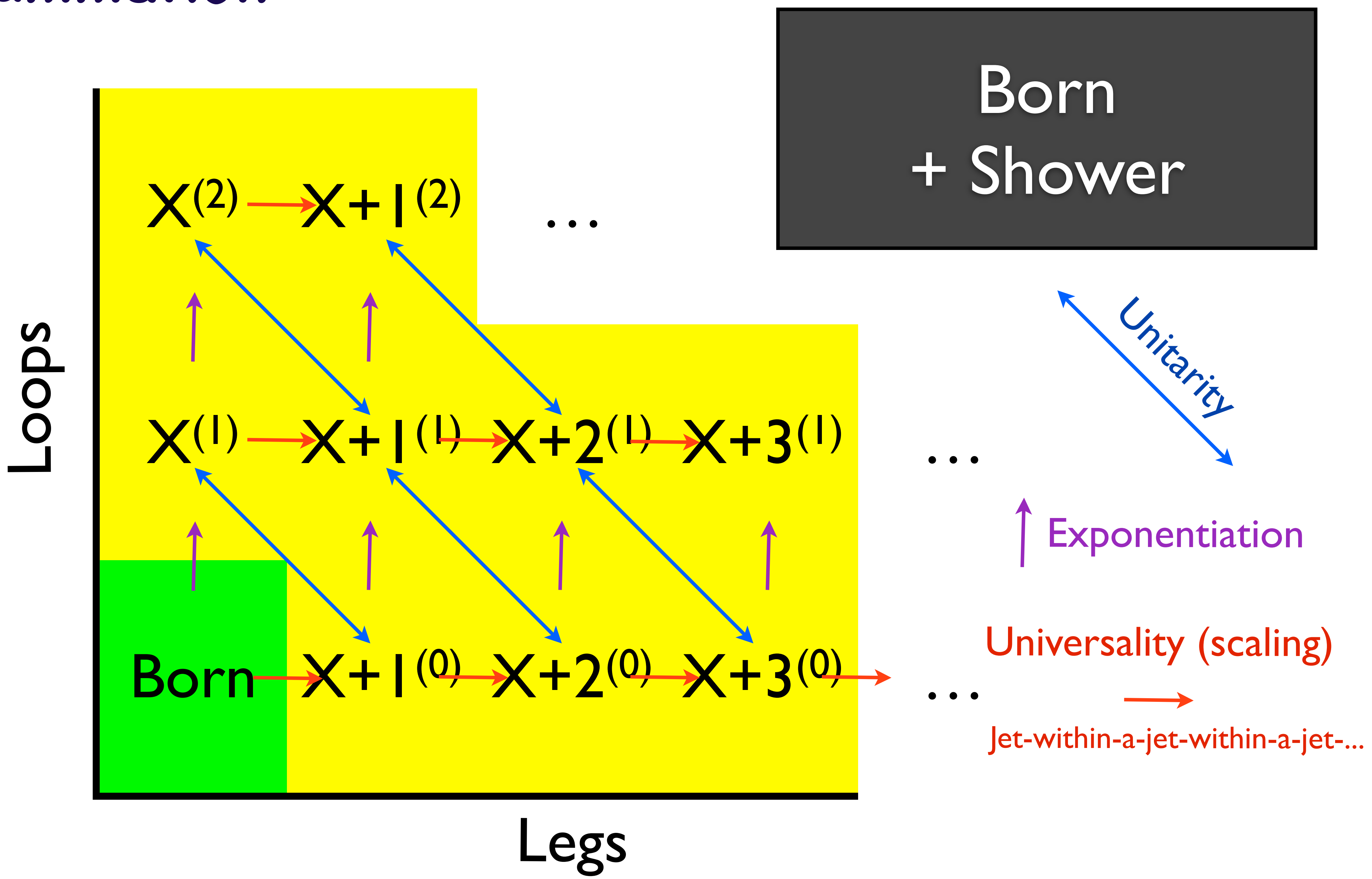
→ *evolution equation with kernel* $\frac{d\sigma_{X+1}}{d\sigma_X}$

Evolve in some measure of *resolution*
~ virtuality, energy, … ~ fractal scale

→ includes both real (tree) and virtual (loop) corrections

Bootstrapped Perturbation Theory

Resummation



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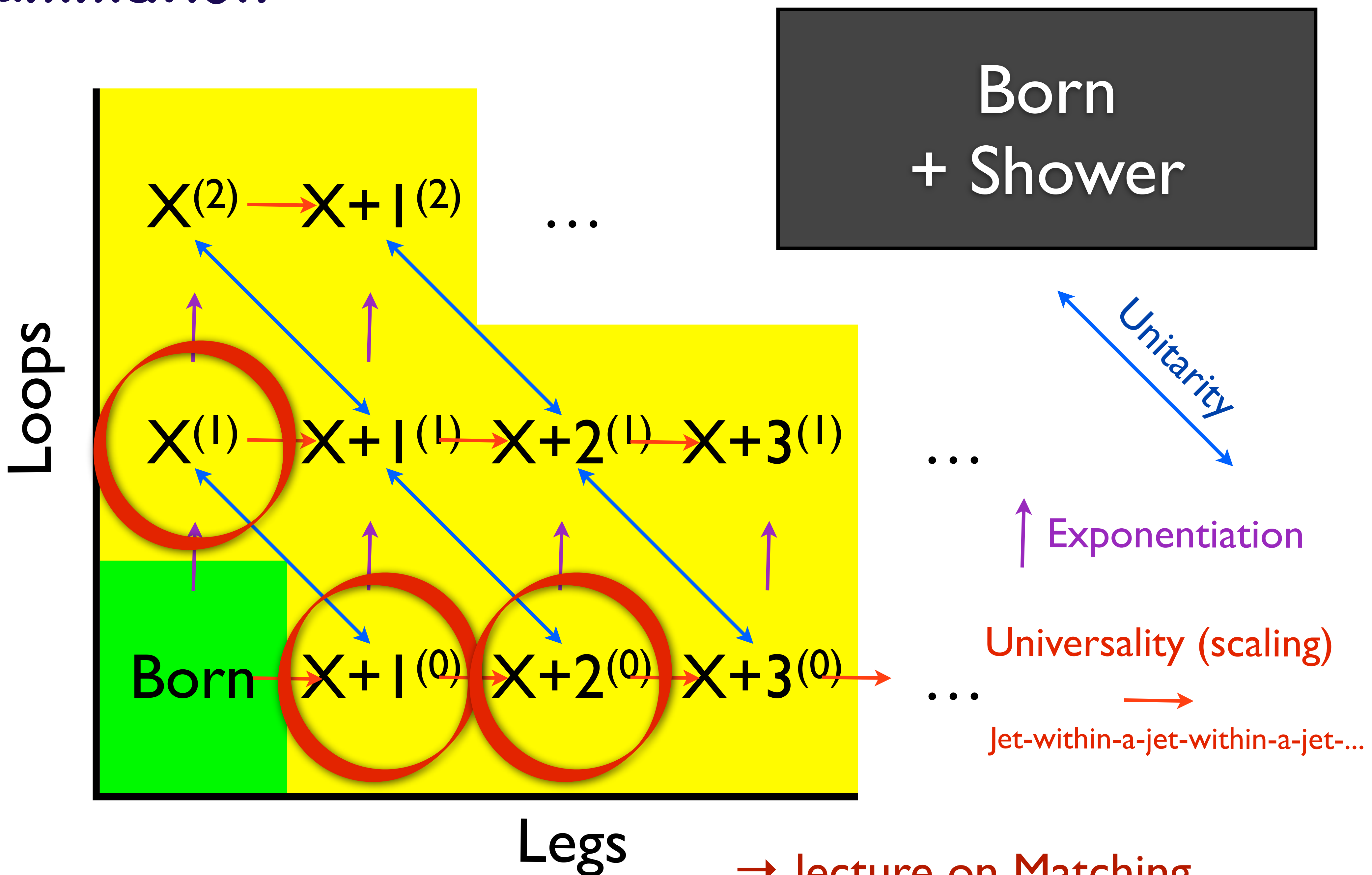

→ lecture on Matching

The Shower Operator

Born $\left.\frac{d\sigma_H}{d\mathcal{O}}\right|_{\rm Born} = \int d\Phi_H \; |M_H^{(0)}|^2 \; \delta(\mathcal{O} - \mathcal{O}(\{p\}_H))$

H = Hard process

{p} : partons

But instead of evaluating O directly on the Born final state, first insert a showering operator

The Shower Operator

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Born + shower $$\left.\frac{\mathrm{d}\sigma_H}{\mathrm{d}\mathcal{O}}\right|_{\mathcal{S}} = \int \mathrm{d}\Phi_H \; |M_H^{(0)}|^2 \; \mathcal{S}(\{p\}_H, \mathcal{O})$$

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S : showering operator

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Unitarity: to first order, S does nothing

$$\mathcal{S}(\{p\}_H, \mathcal{O}) = \delta\left(\mathcal{O} - \mathcal{O}(\{p\}_H)\right) \; + \; \mathcal{O}(\alpha_s)$$

The Shower Operator

To ALL Orders

$$S(\{p\}_X, \mathcal{O}) = \Delta(t_{\mathrm{start}}, t_{\mathrm{had}})\delta(\mathcal{O}-\mathcal{O}(\{p\}_X))$$

"Nothing Happens" → "Evaluate Observable"

$$-\int_{t_{\mathrm{start}}}^{t_{\mathrm{had}}} \mathrm{d}t \frac{\mathrm{d}\Delta(t_{\mathrm{start}}, t)}{\mathrm{d}t} S(\{p\}_{X+1}, \mathcal{O})$$

"Something Happens" → "Continue Shower"

All-orders Probability that nothing happens

$$\Delta(t_1, t_2) = \exp\left(-\int_{t_1}^{t_2} \mathrm{d}t \, \frac{\mathrm{d}\mathcal{P}}{\mathrm{d}t}\right)$$

(Exponentiation)
Analogous to nuclear decay
N(t) ≈ N(0) exp(-ct)

The Shower Operator

To ALL Orders

(Markov Chain)

$$S(\{p\}_X,\mathcal{O}) = \Delta(t_{\mathrm{start}},t_{\mathrm{had}})\delta(\mathcal{O}-\mathcal{O}(\{p\}_X))$$

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(Exponentiation)
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$N(t) \approx N(0)\exp(-ct)$

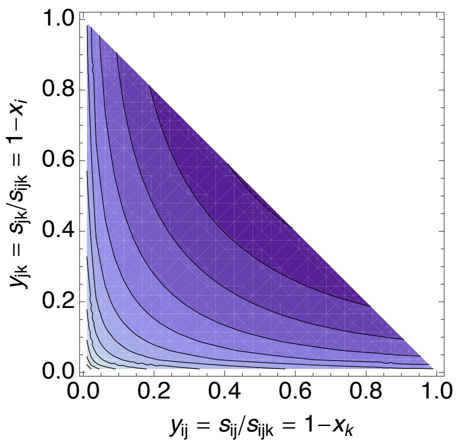
A Shower Algorithm

1. Generate Random Number, R ∈ [0,1]

Solve equation $R = \Delta(t_1, t)$ for t (with starting scale t_1)

Analytically for simple splitting kernels, else numerically (or by trial+veto)

→ t scale for next branching



QCD
Lecture III

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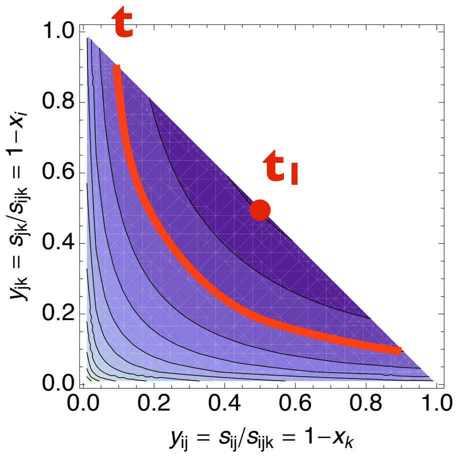

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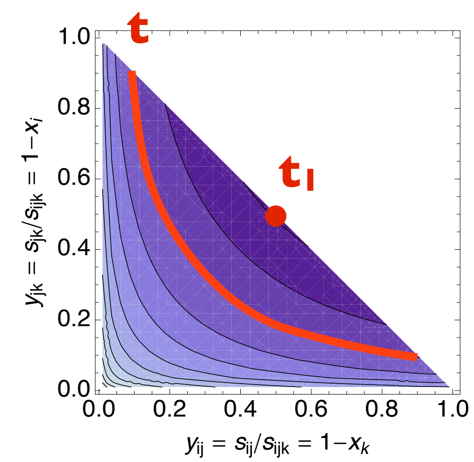

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Analytically for simple splitting kernels, else numerically (or by trial+veto)

→ *t scale for next branching*

2. Generate another Random Number, R_z ∈ [0,1]

To find second (linearly independent) phase-space invariant

Solve equation $R_z = \frac{I_z(z,t)}{I_z(z_{\max}(t),t)}$ for z (at scale t)

With the "primitive function" $I_z(z,t) = \int_{z_{\min}(t)}^{z} \mathrm{d}z \left. \frac{\mathrm{d}\Delta(t')}{\mathrm{d}t'} \right|_{t'=t}$

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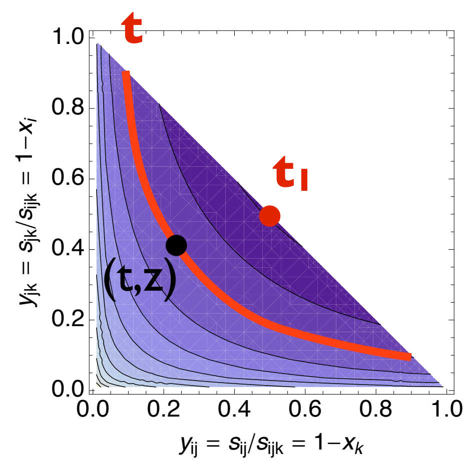

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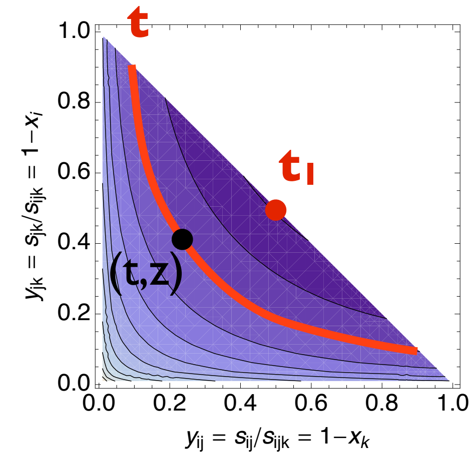

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With the "primitive function" $I_z(z,t) = \int_{z_{\min}(t)}^{z} \mathrm{d}z \left. \frac{\mathrm{d}\Delta(t')}{\mathrm{d}t'} \right|_{t'=t}$

3. Generate a third Random Number, R_φ ∈ [0,1]

Solve equation $R_\varphi = \varphi/2\pi$ for φ → Can now do 3D branching

Ambiguities

The final states generated by the shower algorithm will depend on

1. The choice of perturbative evolution variable(s) $t^{[i]}$.
2. The choice of phase-space mapping $\mathrm{d}\Phi_{n+1}^{[i]}/\mathrm{d}\Phi_n$.
3. The choice of radiation functions a_i, as a function of the phase-space variables.
4. The choice of renormalization scale function μ_R.
5. Choices of starting and ending scales.

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4. The choice of renormalization scale function μ_R.
5. Choices of starting and ending scales.

→ gives us additional handles for uncertainty estimates, beyond just μ_R

(Physics Consequences)

Subleading Issues

Hard Jet Substructure (showers approximate 1→3 by iterated 1→2, but full 1→3 kernels have additional structure. Iterated 1→2 only works when successive emissions are strongly ordered (dominant) but not when two or more emissions happen at ~ the same scale → hard substructure)

p_T kicks from recoil strategy (global vs local; 1→ 2 vs 2→3)

Gluon Splittings $g\to q\bar{q}$ (less well controlled than gluon emission)

Mass Effects (example: b-jet calibration vs light-jet)

Subleading coherence (e.g., angular-ordered parton showers vs p_T-ordered dipole ones, in particular initial-final connections…)

(Physics Consequences)

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Gluon Splittings $g \to q\bar{q}$ (less w

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ordered dipole ones, in particular initial-

Current "holy grail":

Include full higher-order splitting kernels
→ will reduce all these ambiguities

Active field of research.

For now, must do our best to estimate the uncertainties.

Tuning

1. Fragmentation Tuning

Perturbative: jet radiation, jet broadening, jet structure

Non-perturbative: hadronization modeling & parameters

2. Initial-State Tuning

Perturbative: initial-state radiation, initial-final interference

Non-perturbative: PDFs, primordial k_T

3. Underlying-Event & Min-Bias Tuning

Perturbative: Multi-parton interactions, rescattering

Non-perturbative: Multi-parton PDFs, Beam Remnant fragmentation, Color (re)connections, collective effects, impact parameter dependence, …

Example: pQCD Shower Tuning

Main pQCD Parameters

$\alpha_s(m_Z)$

The value of the strong coupling at the Z pole

Governs overall amount of radiation

α_s Running

Renormalization Scheme and Scale for α_s

1- / 2-loop running, MSbar / CMW scheme, $\mu_R \sim Q^2$ or p_T^2

Matching

Additional Matrix Elements included?

At tree level / one-loop level? Using what scheme?

Subleading Logs

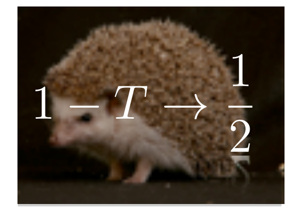
Ordering variable, coherence treatment, effective $1\to3$ (or $2\to4$), recoil strategy, etc

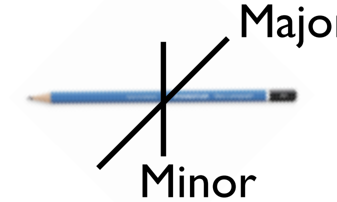
Need IR Corrections?

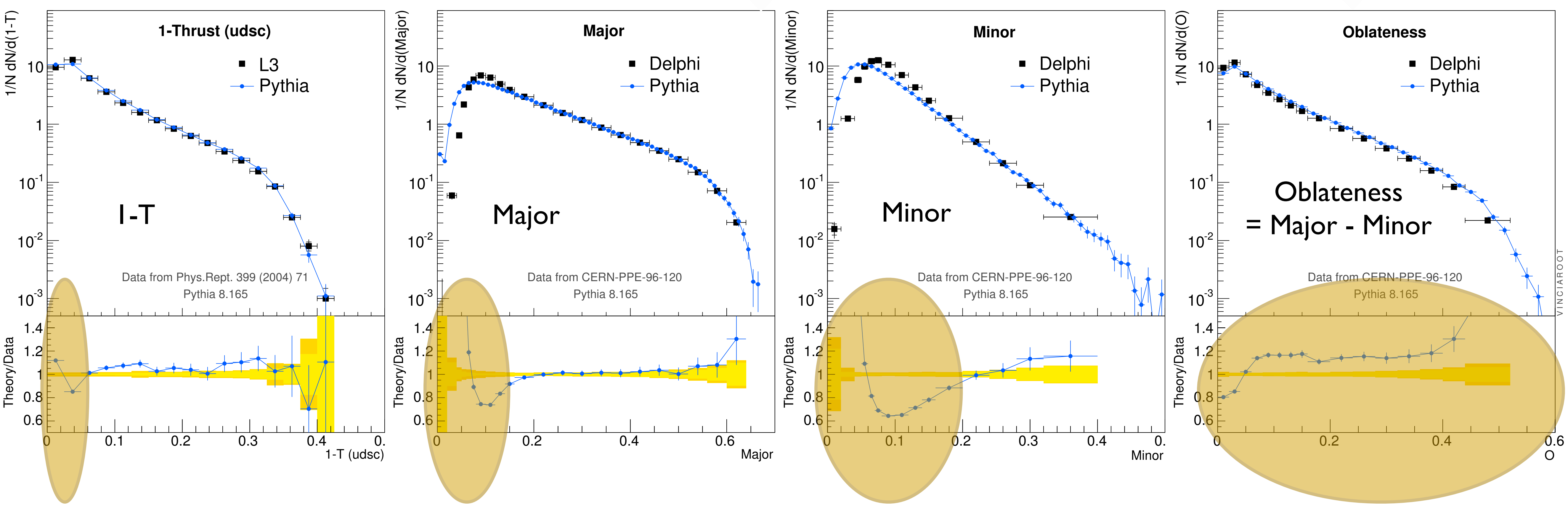
PYTHIA 8 (hadronization off) **vs LEP: Thrust**

$$T = \max_{\vec{n}} \left(\frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|} \right)$$

$1 - T \to 0$







Significant Discrepancies (>10%)

for T < 0.05, Major < 0.15, Minor < 0.2, and for all values of Oblateness

QCD

Lecture III

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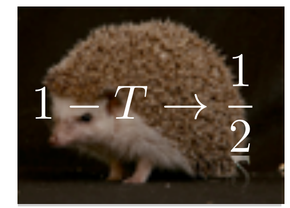

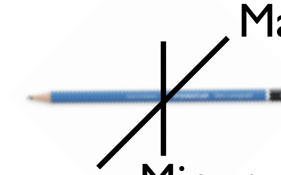

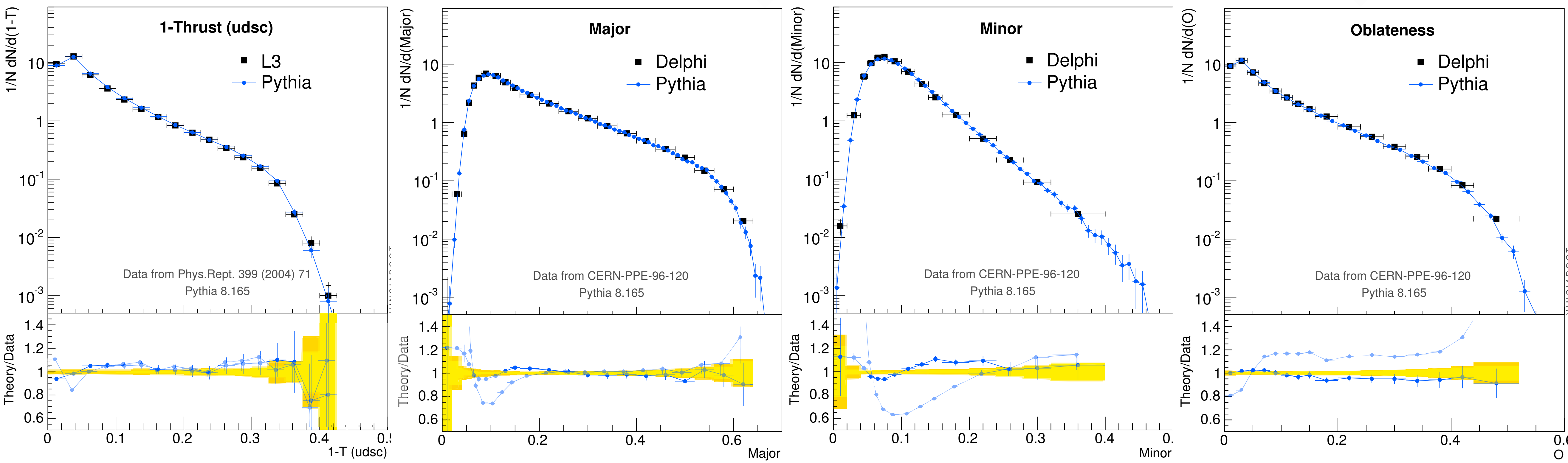

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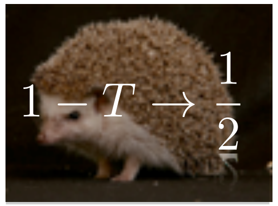

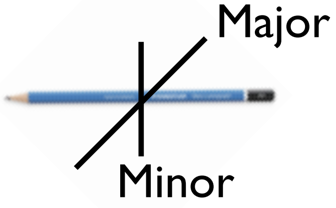

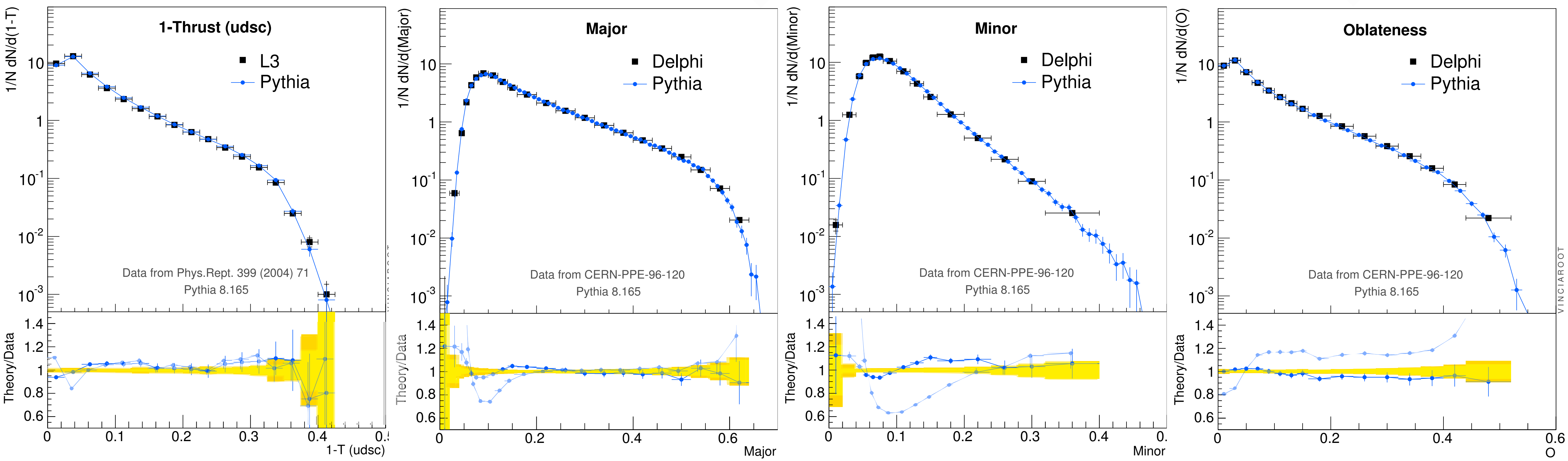

Note: Value of Strong coupling is
$\alpha_s(M_Z) = 0.14$

QCD
Lecture III

Value of Strong Coupling

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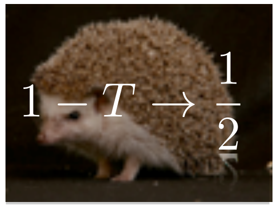

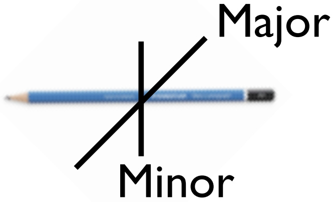

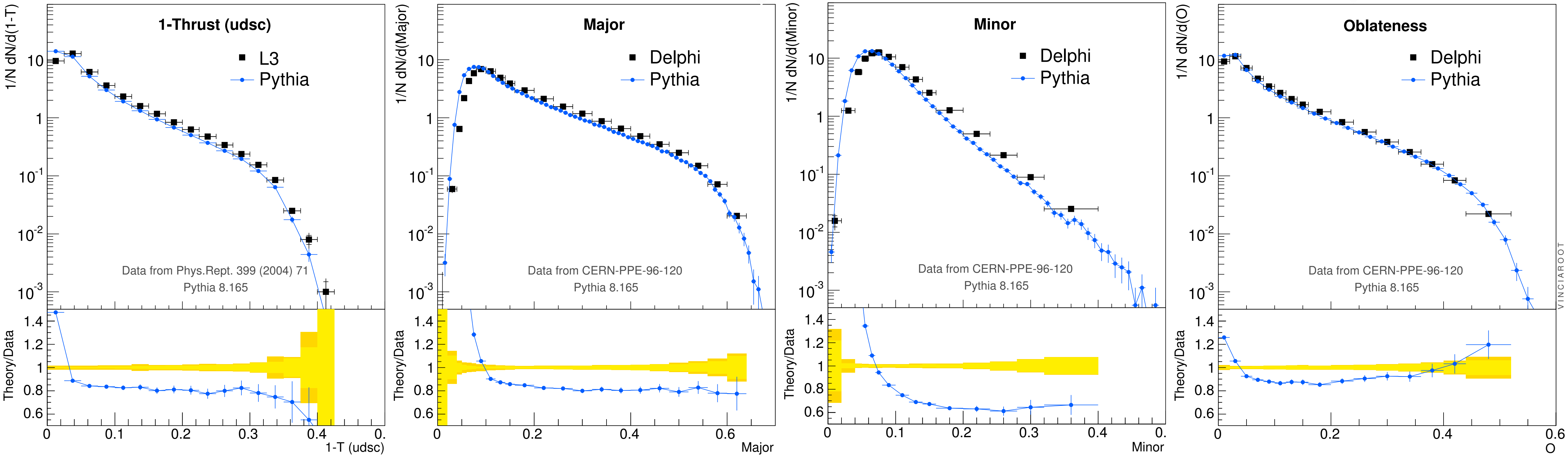

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$\alpha_s(M_Z) = 0.12$

QCD
Lecture III

Wait … is this Crazy?

Best result

Obtained with $\alpha_s(M_Z) \approx 0.14 \neq$ World Average = 0.1176 ± 0.0020

QCD

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Value of α_s

Depends on the order and scheme

MC ≈ Leading Order + LL resummation

Other leading-Order extractions of $\alpha_s \approx$ 0.13 - 0.14

Effective scheme interpreted as "CMW" → 0.13; 2-loop running → 0.127; NLO → 0.12 ?

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Tune/measure even pQCD parameters with the actual generator.

Sanity check = consistency with other determinations at a similar formal order, within the uncertainty at that order (including a CMW-like scheme redefinition to go to 'MC scheme')

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Improve → Matching at LO and NLO
Non-perturbative → Lecture on IR


Uncertainties

The Tyranny of Carlo

J. D. Bjorken

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QCD

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But it often happens that the physics simulations provided by the the MC generators carry the authority of data itself. They look like data and feel like data, and if one is not careful they are accepted as if they were data. All Monte Carlo codes come with a GIGO (garbage in, garbage out) warning label. But the GIGO warning label is just as easy for a physicist to ignore as that little message on a packet of cigarettes is for a chain smoker to ignore. I see nowadays experimental papers that claim agreement with QCD (translation: someone's simulation labeled QCD) and/or disagreement with an alternative piece of physics (translation: an unrealistic simulation), without much evidence of the inputs into those simulations."

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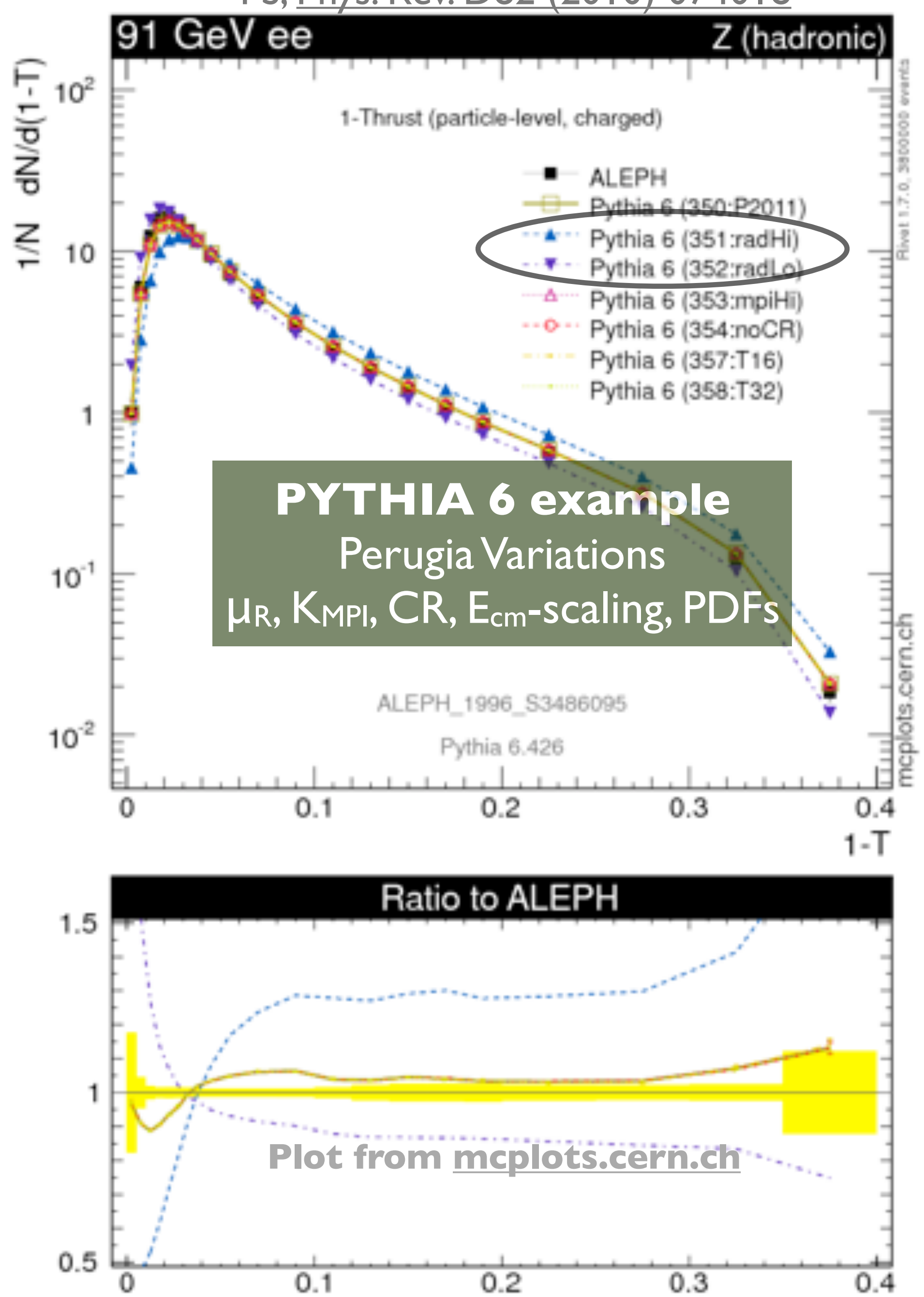
Account for parameters + pertinent cross-checks and validations
Do serious effort to estimate uncertainties, by salient variations

Uncertainty Estimates

a) Authors provide specific “tune variations”

Run once for each variation → envelope

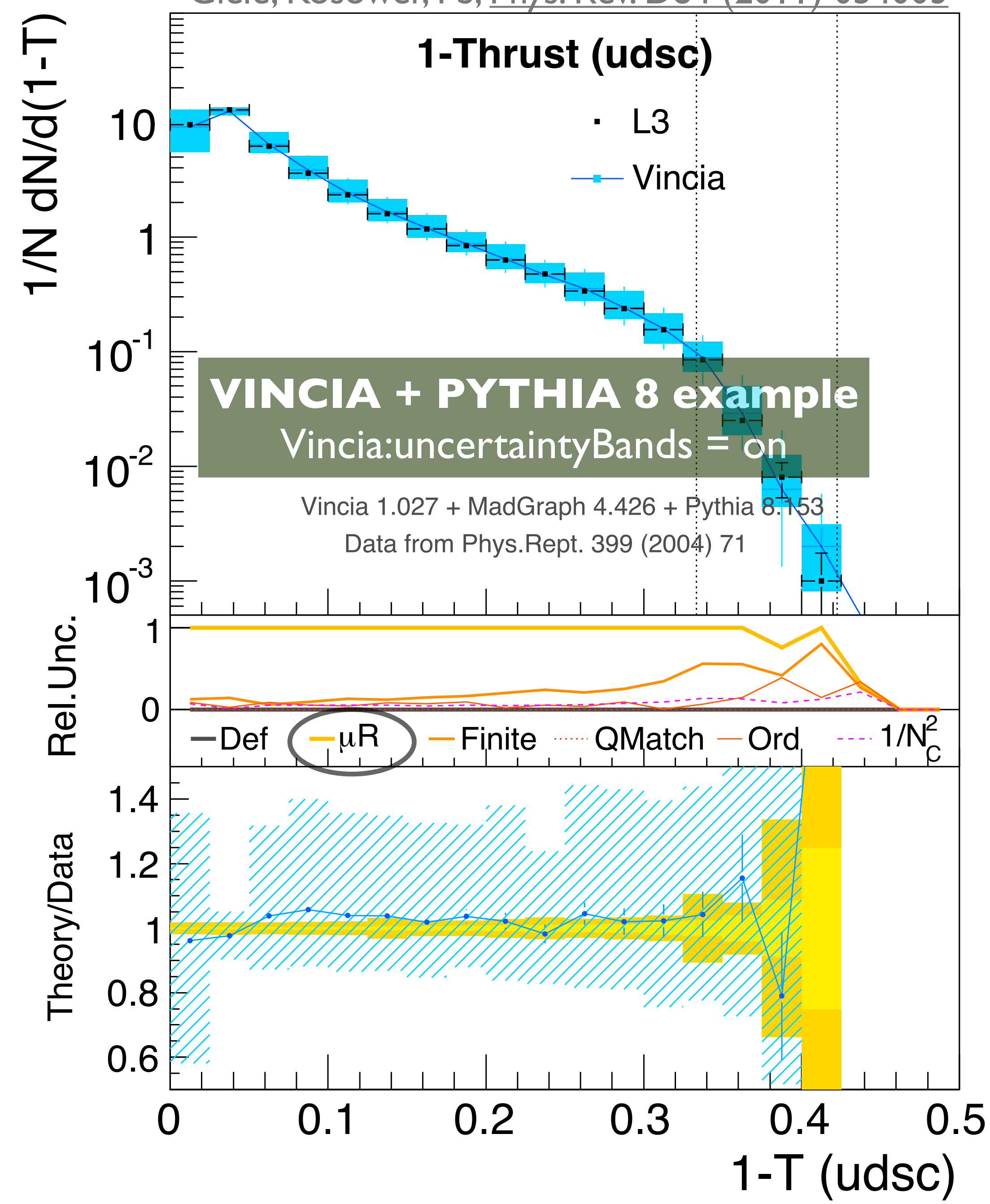
PS, Phys. Rev. D82 (2010) 074018



b) **One** shower run

+ unitarity-based uncertainties → envelope

Giele, Kosower, PS; Phys. Rev. D84 (2011) 054003



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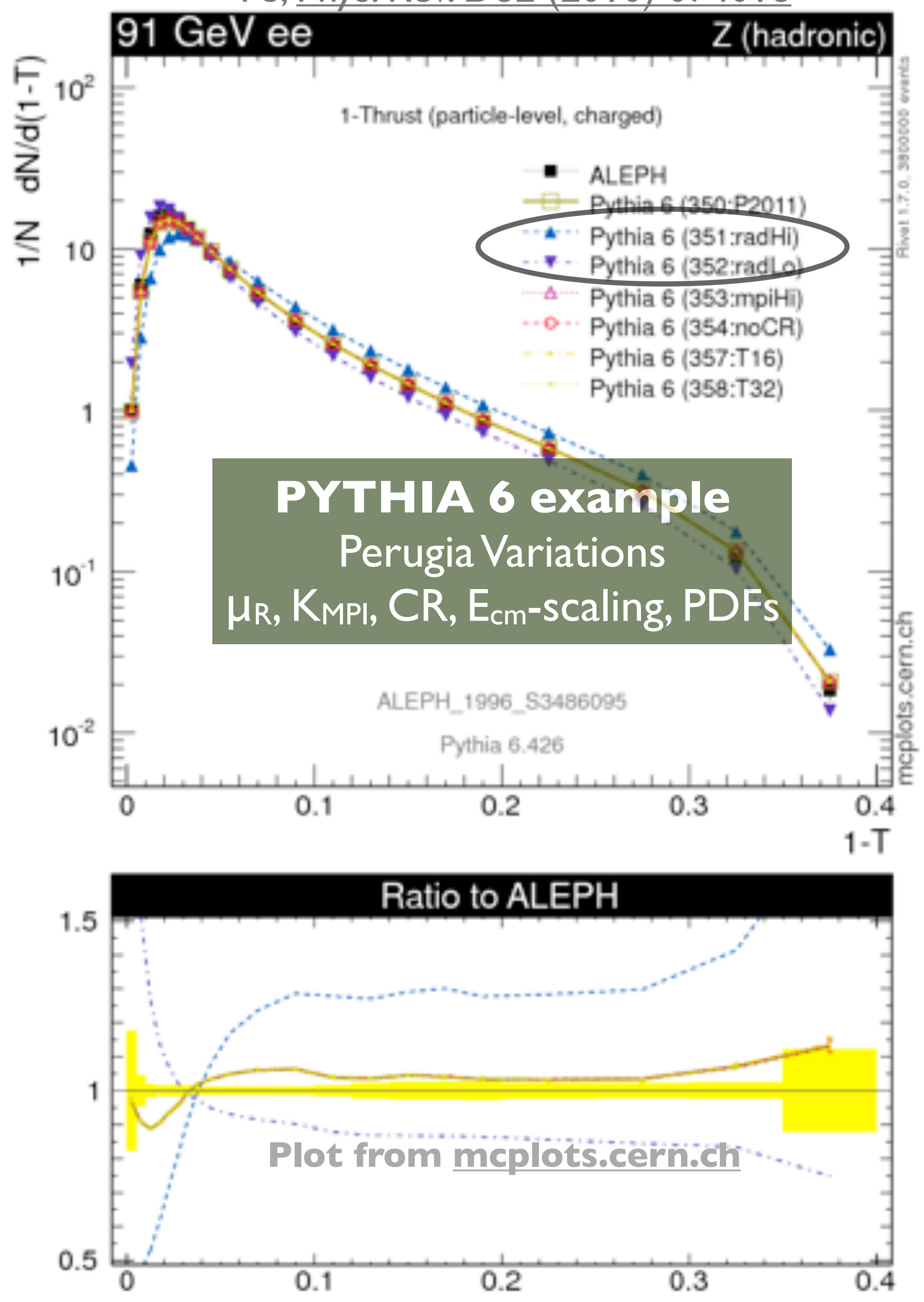

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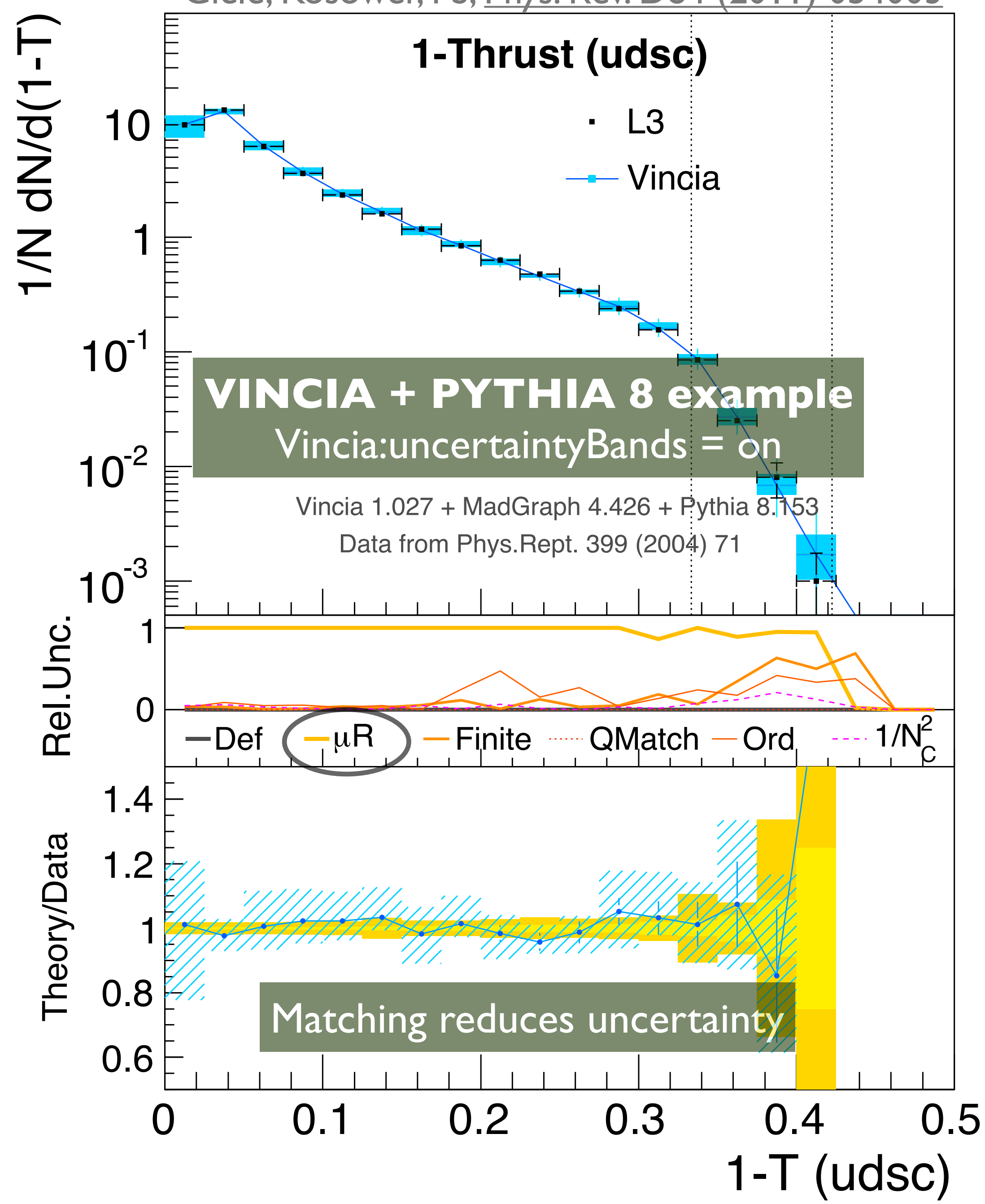

Automatic Uncertainty Estimates

One shower run (VINCIA + PYTHIA)

+ unitarity-based uncertainties → envelope

Giele, Kosower, PS; Phys. Rev. D84 (2011) 054003

```
*-------  PYTHIA Event and Cross Section Statistics  -------------------------------------------------------------*
|                                                                                                                  |
| Subprocess                                    Code |            Number of events       |      sigma +- delta    |
|                                                    |       Tried   Selected   Accepted |     (estimated) (mb)   |
|                                                    |                                   |                        |
|------------------------------------------------------------------------------------------------------------------|
|                                                    |                                   |                        |
| f fbar -> gamma*/Z0                            221 |       10511      10000      10000 |  4.143e-05  0.000e+00  |
|                                                    |                                   |                        |
| sum                                                |       10511      10000      10000 |  4.143e-05  0.000e+00  |
|                                                                                                                  |
*-------  End PYTHIA Event and Cross Section Statistics ----------------------------------------------------------*

*-------  VINCIA Statistics  ------------------------------------------------------------------------------------*
|                                                                                                                  |
|                                                                                                                  |
| Number of nonunity-weight events                                   =      none                                   |
| Number of negative-weight events                                   =      none                                   |
|                                                                                                                  |
|                                  weight(i)          Avg Wt    Avg Dev  rms(dev)     kUnwt     Expected effUnw    |
| This run                           i =      IsUnw     <w>      <w-1>                1/<w>    Max Wt  <w>/MaxWt   |
|    User settings                       0    yes     1.000      0.000         -      1.000         -         -    |
|    Var : VINCIA defaults               1    yes     1.000      0.000         -      1.000     1.000     1.000    |
|    Var : AlphaS-Hi                     2     no     0.996  -3.89e-03         -      1.004    22.414  4.44e-02    |
|    Var : AlphaS-Lo                     3     no     1.020   1.99e-02         -      0.981    43.099  2.37e-02    |
|    Var : Antennae-Hi                   4     no     1.000   2.61e-04         -      1.000     5.417     0.185    |
|    Var : Antennae-Lo                   5     no     0.996  -4.33e-03         -      1.004    10.753  9.26e-02    |
|    Var : NLO-Hi                        6    yes     1.000      0.000         -      1.000     1.000     1.000    |
|    Var : NLO-Lo                        7    yes     1.000      0.000         -      1.000     1.000     1.000    |
|    Var : Ordering-Stronger             8     no     1.004   4.48e-03         -      0.996    14.225  7.06e-02    |
|    Var : Ordering-mDaughter            9     no     1.033   3.25e-02         -      0.968    55.954  1.85e-02    |
|    Var : Subleading-Color-Hi          10     no     1.001   7.37e-04         -      0.999     1.505     0.665    |
|    Var : Subleading-Color-Lo          11     no     1.006   6.44e-03         -      0.994     5.283     0.191    |
|                                                                                                                  |
*-------  End VINCIA Statistics ---------------------------------------------------------------------------------*
```

Introduction to QCD

1. Fundamentals of QCD
2. Jets and Fixed-Order QCD
3. **Monte Carlo Generators and Showers**
4. **Matching at LO and NLO**
5. **QCD in the Infrared**

Note: Teach-yourself PYTHIA tutorial posted at:
www.cern.ch/skands/slides

Supplementary Slides

Hard Processes

Slide from T. Sjöstrand

Wide spectrum from "general-purpose" to "one-issue", see e.g.

`http://www.cedar.ac.uk/hepcode/`

Free for all as long as Les-Houches-compliant output.

I) General-purpose, leading-order:

- MadGraph/MadEvent (amplitude-based, $\leq$ 7 outgoing partons):

  `http://madgraph.physics.uiuc.edu/`
- CompHEP/CalcHEP (matrix-elements-based, $\sim\leq$ 4 outgoing partons)
- Comix: part of SHERPA (Behrends-Giele recursion)
- HELAC–PHEGAS (Dyson-Schwinger)

Hard Processes

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II) Special processes, leading-order:

- ALPGEN: $\mathrm{W/Z}+\leq 6\mathrm{j}$, $n\mathrm{W}+m\mathrm{Z}+k\mathrm{H}+\leq 3\mathrm{j}, \ldots$
- AcerMC: $\mathrm{t\bar{t}b\bar{b}}, \ldots$
- VECBOS: $\mathrm{W/Z}+\leq 4\mathrm{j}$

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- VECBOS: $\mathrm{W/Z} + \leq 4\mathrm{j}$

III) Special processes, next-to-leading-order:

- MCFM: NLO $\mathrm{W/Z} + \leq 2\mathrm{j}$, WZ, WH, $\mathrm{H} + \leq 1\mathrm{j}$
- GRACE+Bases/Spring

Note: NLO codes not yet generally interfaced to shower MCs

QCD

Lecture III

Splitting Functions

Altarelli-Parisi
(E.g., PYTHIA)

$$\mathrm{d}\mathcal{P}_a = \sum_{b,c} \frac{\alpha_{abc}}{2\pi} P_{a\to bc}(z)\,\mathrm{d}t\,\mathrm{d}z\ .$$

$$\begin{aligned}
P_{\mathrm{q}\to\mathrm{qg}}(z) &= C_F \frac{1+z^2}{1-z}\ , \\
P_{\mathrm{g}\to\mathrm{gg}}(z) &= N_C \frac{(1-z(1-z))^2}{z(1-z)}\ , \\
P_{\mathrm{g}\to\mathrm{q}\bar{\mathrm{q}}}(z) &= T_R\,(z^2+(1-z)^2)\ , \\
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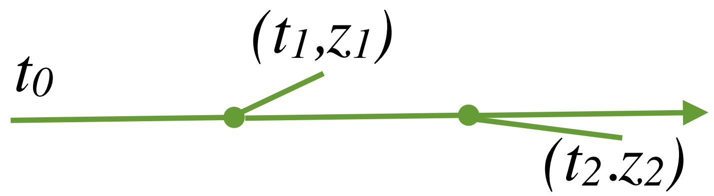
\end{aligned}$$



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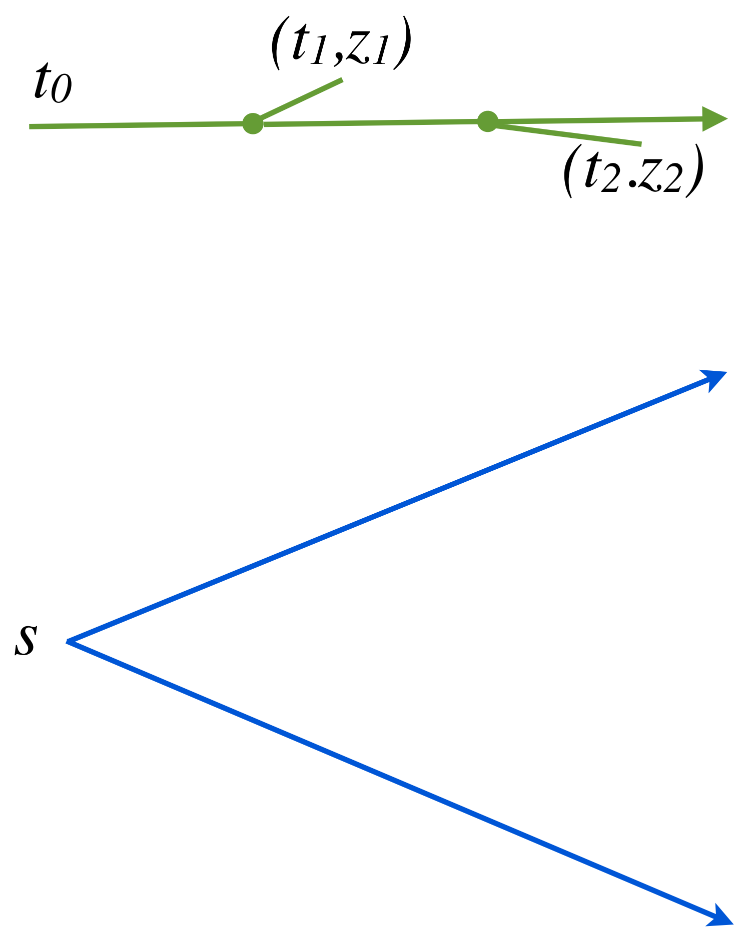

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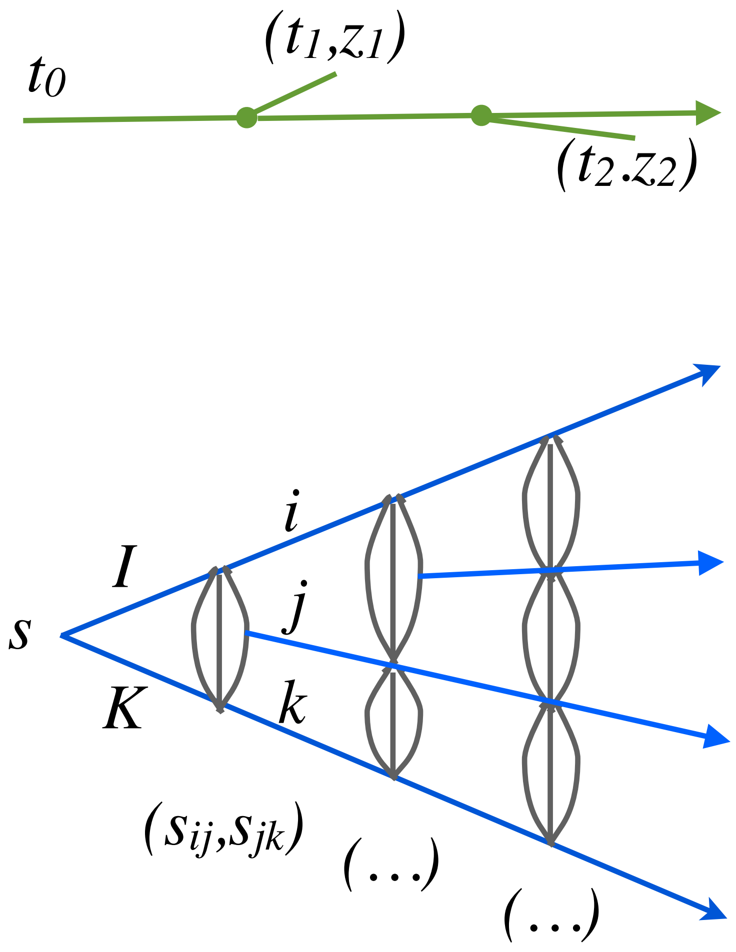

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Dipole-Antennae
(E.g., ARIADNE, VINCIA)

$$\mathrm{d}\mathcal{P}_{IK\to ijk} = \frac{\mathrm{d}s_{ij}\mathrm{d}s_{jk}}{16\pi^2 s}\ a(s_{ij}, s_{jk})$$

$$a_{q\bar{q}\to qg\bar{q}} = \frac{2C_F}{s_{ij}s_{jk}}\left(2s_{ik}s + s_{ij}^2 + s_{jk}^2\right)$$

$$a_{qg\to qgg} = \frac{C_A}{s_{ij}s_{jk}}\left(2s_{ik}s + s_{ij}^2 + s_{jk}^2 - s_{ij}^3\right)$$

$$a_{gg\to ggg} = \frac{C_A}{s_{ij}s_{jk}}\left(2s_{ik}s + s_{ij}^2 + s_{jk}^2 - s_{ij}^3 - s_{jk}^3\right)$$

$$a_{qg\to q\bar{q}'q'} = \frac{T_R}{s_{jk}}\left(s - 2s_{ij} + 2s_{ij}^2\right)$$

$$a_{gg\to g\bar{q}'q'} = a_{qg\to q\bar{q}'q'}$$

… + non-singular terms

Splitting Functions

Altarelli-Parisi
(E.g., PYTHIA)

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\end{aligned}$$

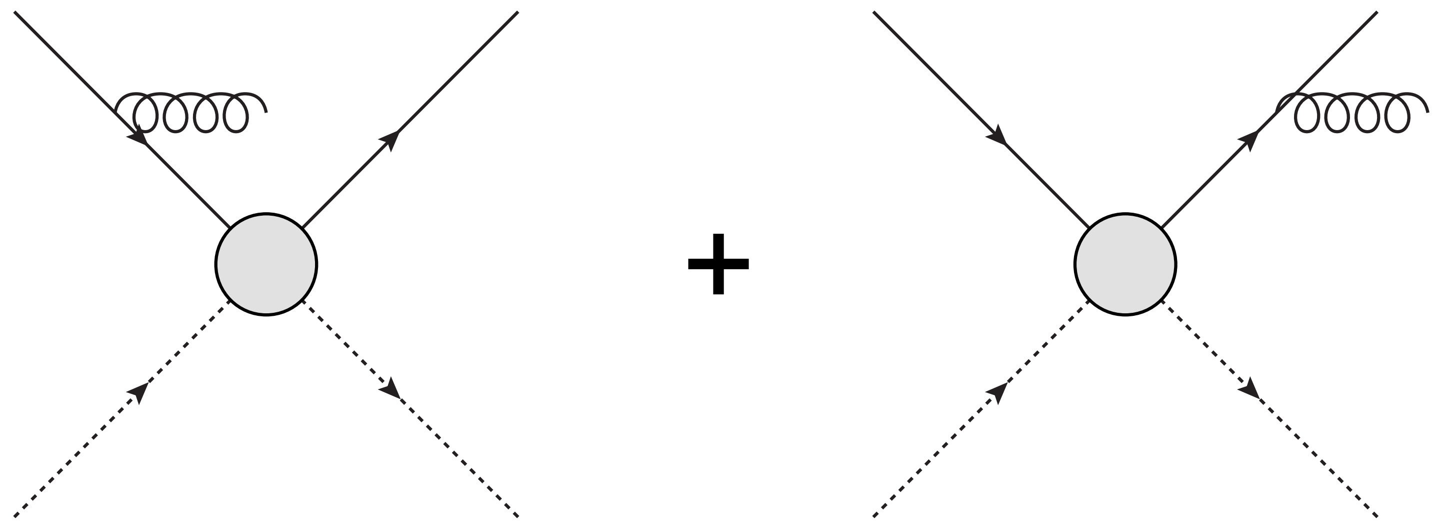
… + non-singular terms

NB: Also others, e.g., Catani-Seymour (SHERPA), Sector Antennae, ….

QCD Lecture III

Initial-Final Interference

Who emitted that gluon?

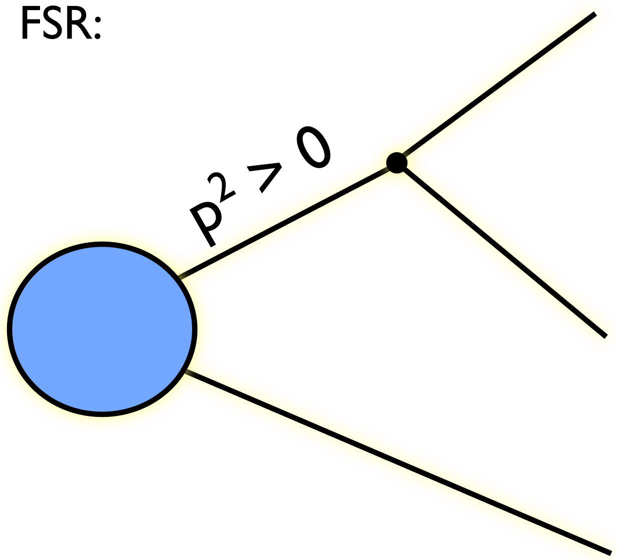


Real QFT = sum over amplitudes, then square → interference (IF coherence)
Respected by dipole/antenna languages (and by angular ordering), but not by conventional DGLAP

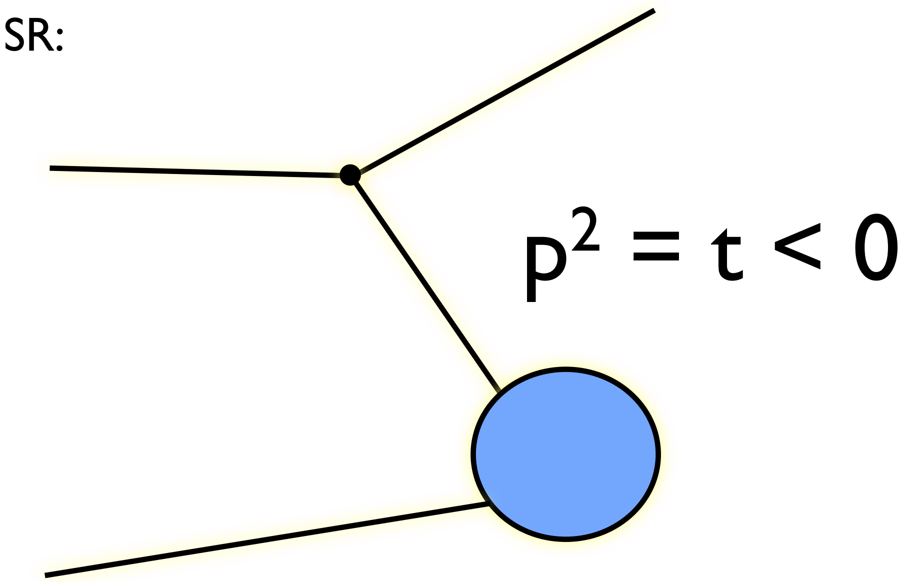
Separation meaningful for collinear radiation, but not for soft …

Initial-State vs Final-State Evolution

FSR:



ISR:



Virtualities are Timelike: $p^2>0$

Start at $Q^2 = Q_F^2$
"Forwards evolution"

Virtualities are Spacelike: $p^2<0$

Start at $Q^2 = Q_F^2$
Constrained backwards evolution towards boundary condition = proton

Separation meaningful for collinear radiation, but not for soft …

(Initial-State Evolution)

DGLAP for Parton Density

$$\frac{\mathrm{d}f_b(x,t)}{\mathrm{d}t} = \sum_{a,c} \int \frac{\mathrm{d}x'}{x'} f_a(x',t) \frac{\alpha_{abc}}{2\pi} P_{a\to bc}\left(\frac{x}{x'}\right)$$

→ Sudakov for ISR

$$\Delta(x, t_{\max}, t) = \exp\left\{ - \int_t^{t_{\max}} \mathrm{d}t' \sum_{a,c} \int \frac{\mathrm{d}x'}{x'} \frac{f_a(x',t')}{f_b(x,t')} \frac{\alpha_{abc}(t')}{2\pi} P_{a\to bc}\left(\frac{x}{x'}\right) \right\}$$

$$= \exp\left\{ - \int_t^{t_{\max}} \mathrm{d}t' \sum_{a,c} \int \mathrm{d}z \frac{\alpha_{abc}(t')}{2\pi} P_{a\to bc}(z) \frac{x' f_a(x',t')}{x f_b(x,t')} \right\} ,$$

(Initial-State Evolution)

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QCD

Lecture III