Introduction to QCD

1. Fundamentals of QCD
2. **Jets and Fixed-Order QCD**
3. Monte Carlo Generators and Showers
4. Matching at LO and NLO
5. QCD in the Infrared

Note: slides posted at:
www.cern.ch/skands/slides

QCD

Lecture II

Factorization

Hadrons are composite, with time-dependent structure:

Partons within clouds of further partons, constantly emitted and absorbed

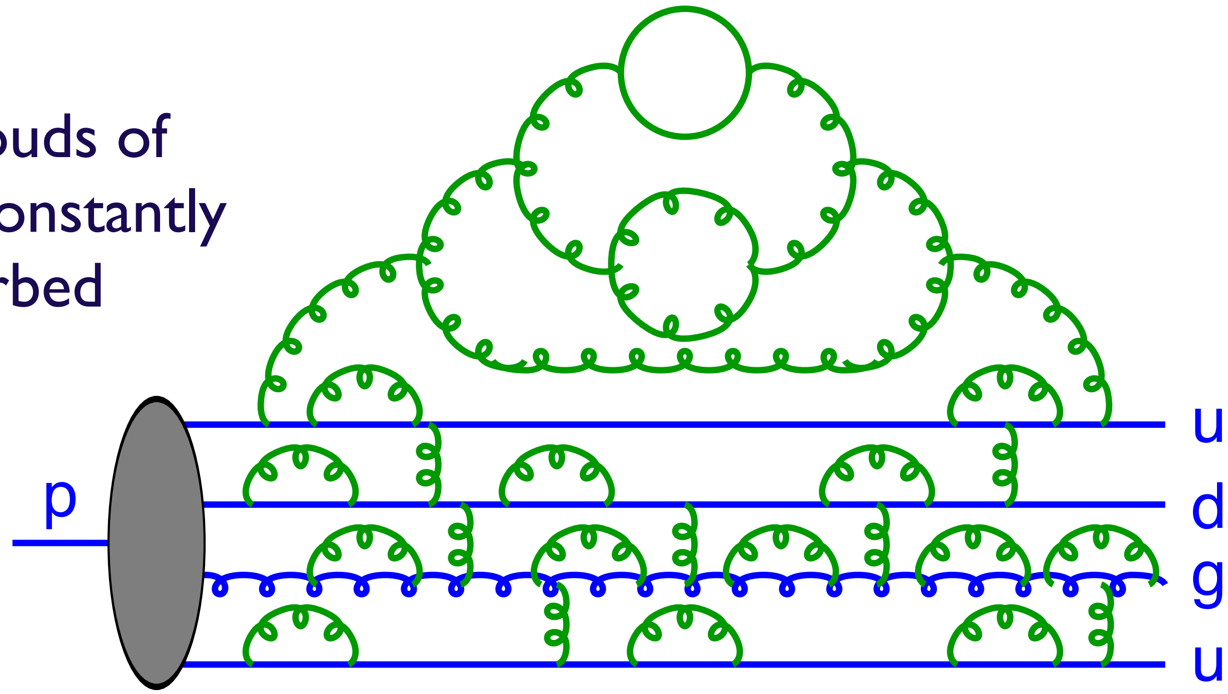

Illustration from T. Sjöstrand

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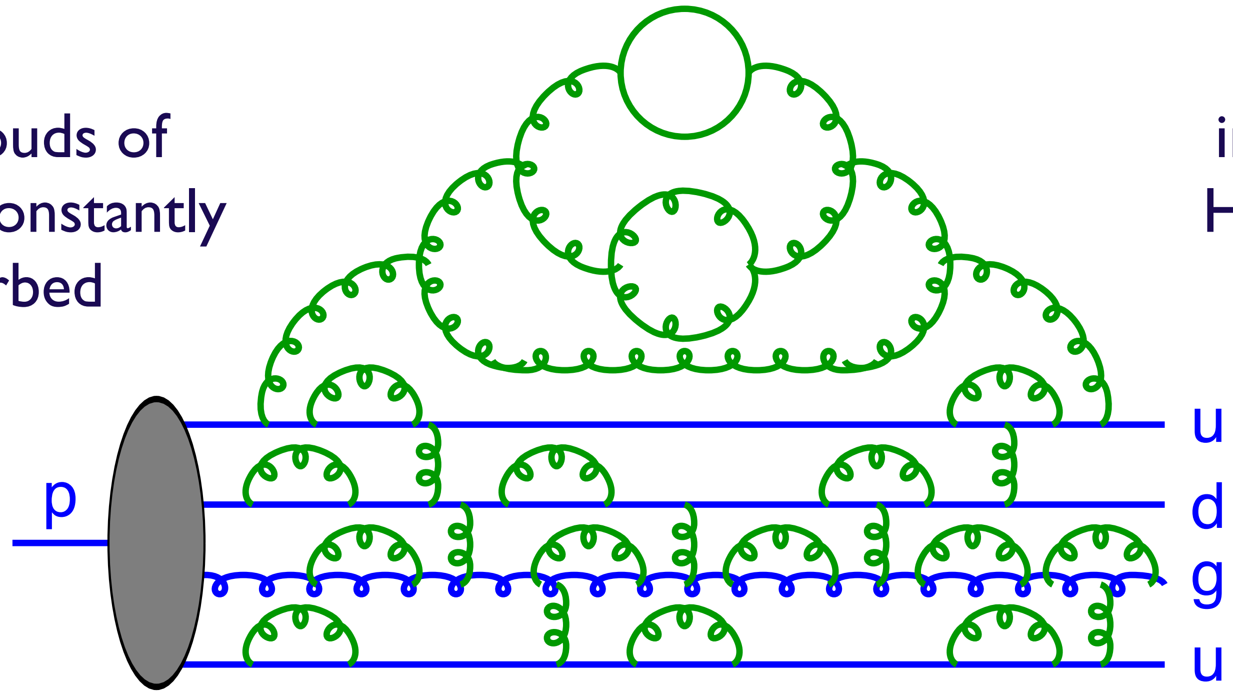

For hadron to remain intact, virtualities $k^2 < M_h^2$

High-virtuality fluctuations suppresed by powers of

$$\frac{\alpha_s M_h^2}{k^2}$$

M_h : mass of hadron

k^2 : virtuality of fluctuation

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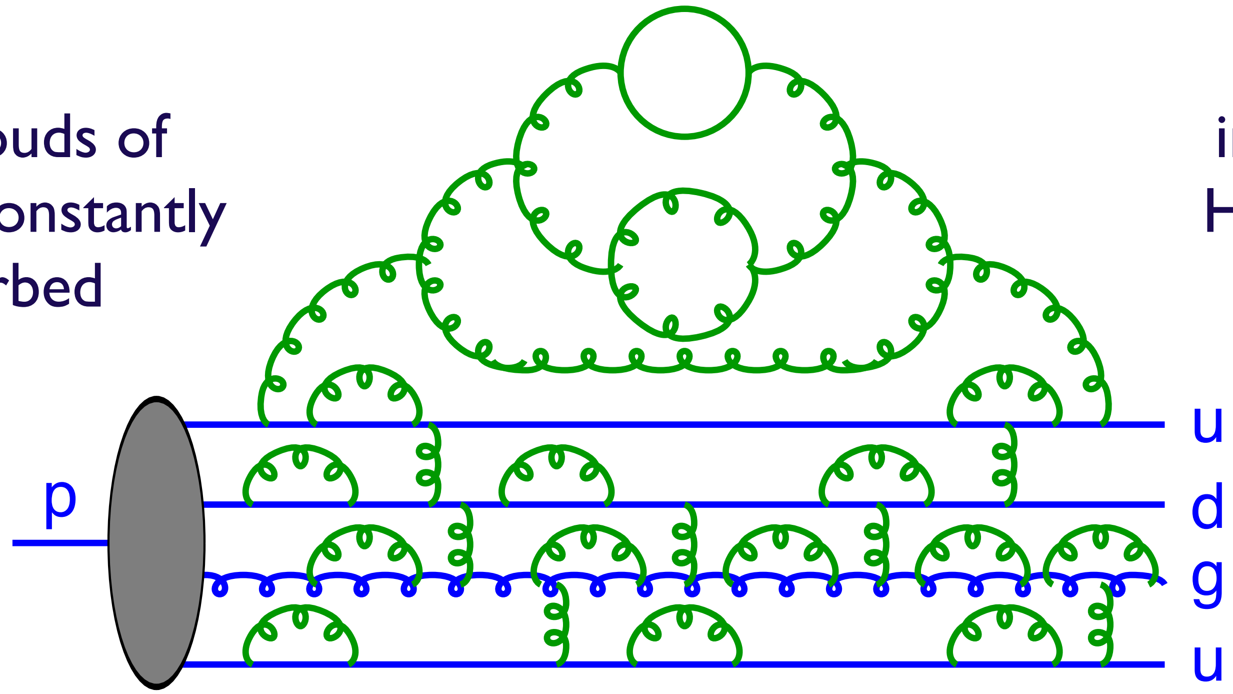

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→ Lifetime of fluctuations ~ $1/M_h$

Hard incoming probe interacts over much shorter time scale ~ 1/Q

On that timescale, partons ~ frozen

Hard scattering knows nothing of the target hadron apart from the fact that it contained the struck parton

Illustration from T. Sjöstrand

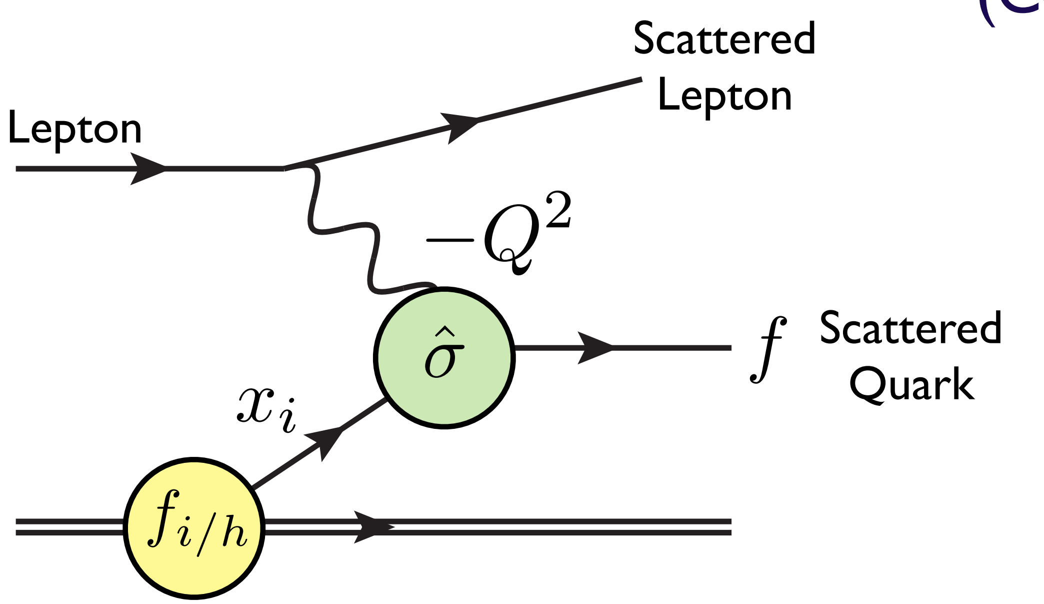
Factorization Theorem

In DIS, there is a formal proof of factorization

(Collins, Soper, 1987)

Deep Inelastic Scattering (DIS)

(By "deep", we mean $Q^2 >> M_h^2$)



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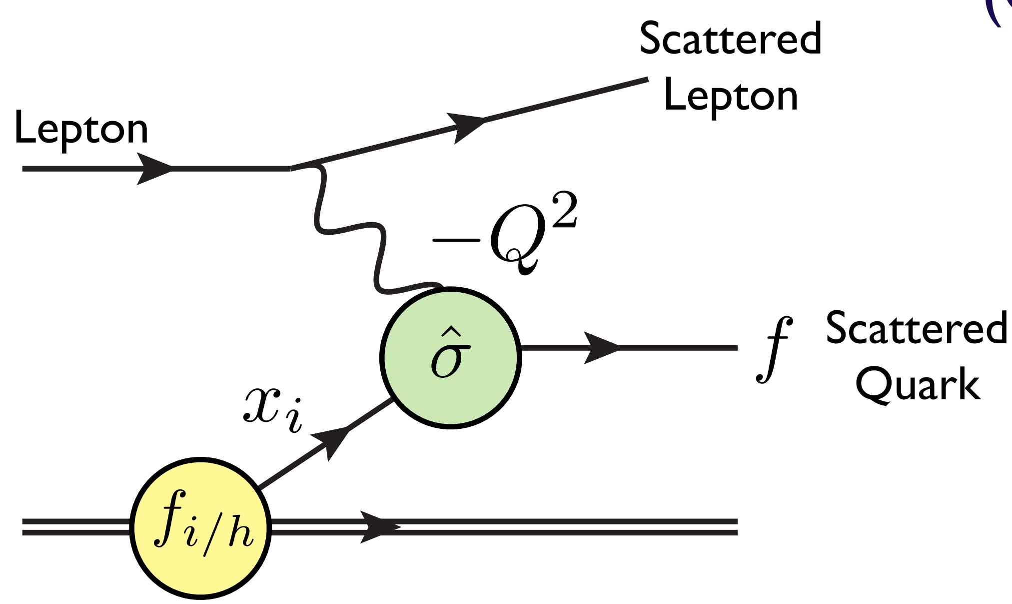

→ We really can write the cross section in factorized form :

$$\sigma^{\ell h} = \sum_i \sum_f \int dx_i \int d\Phi_f \; f_{i/h}(x_i, Q_F^2) \, \frac{d\hat{\sigma}^{\ell i \to f}(x_i, \Phi_f, Q_F^2)}{dx_i \, d\Phi_f}$$

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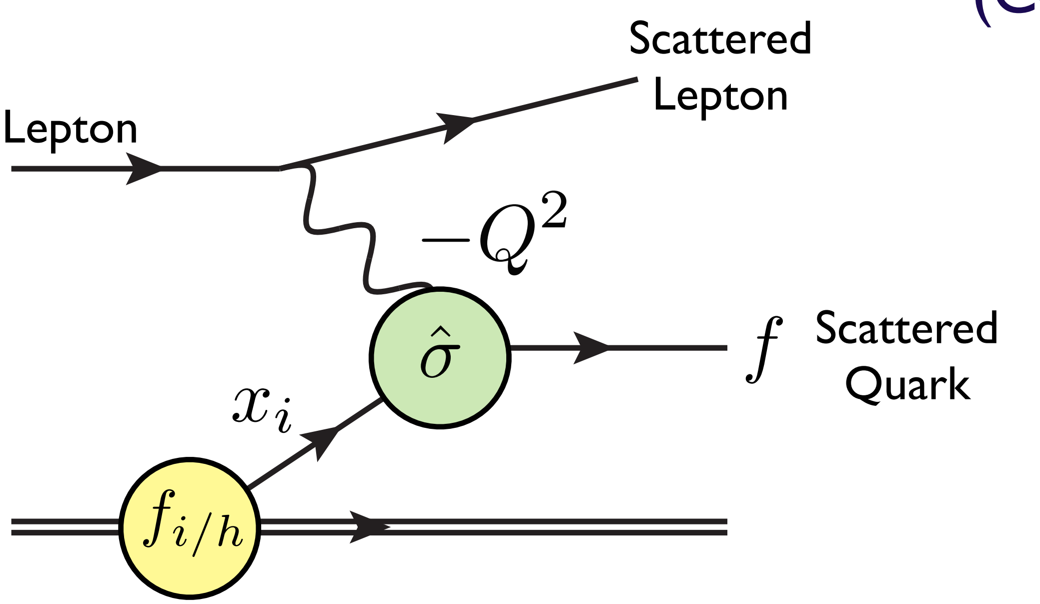

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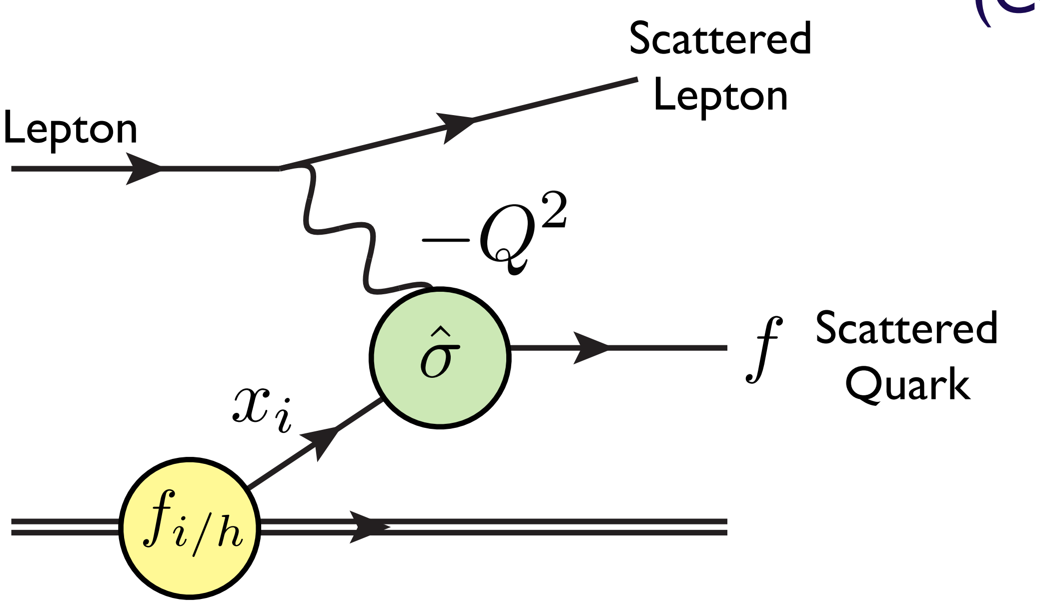

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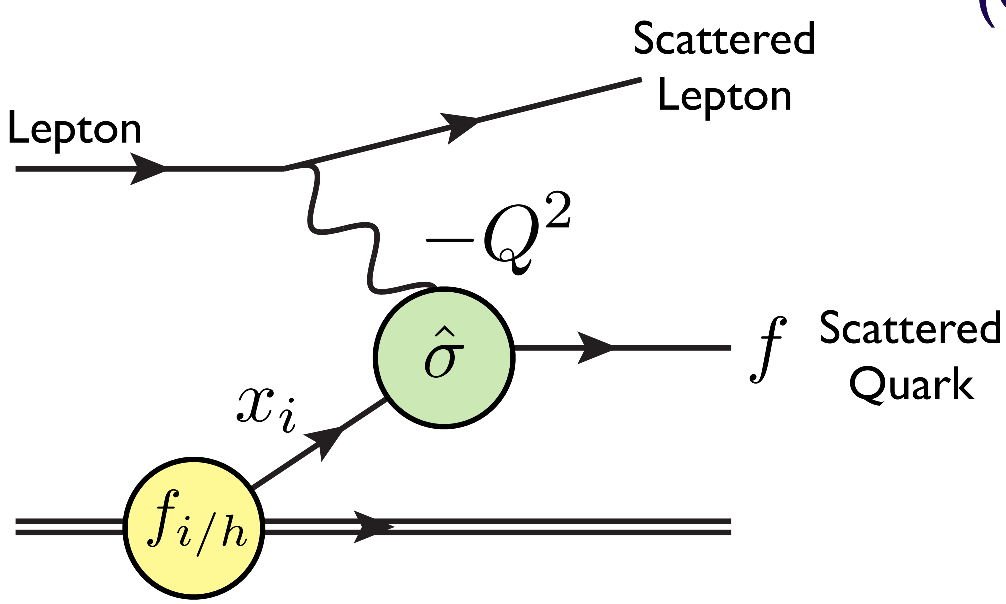

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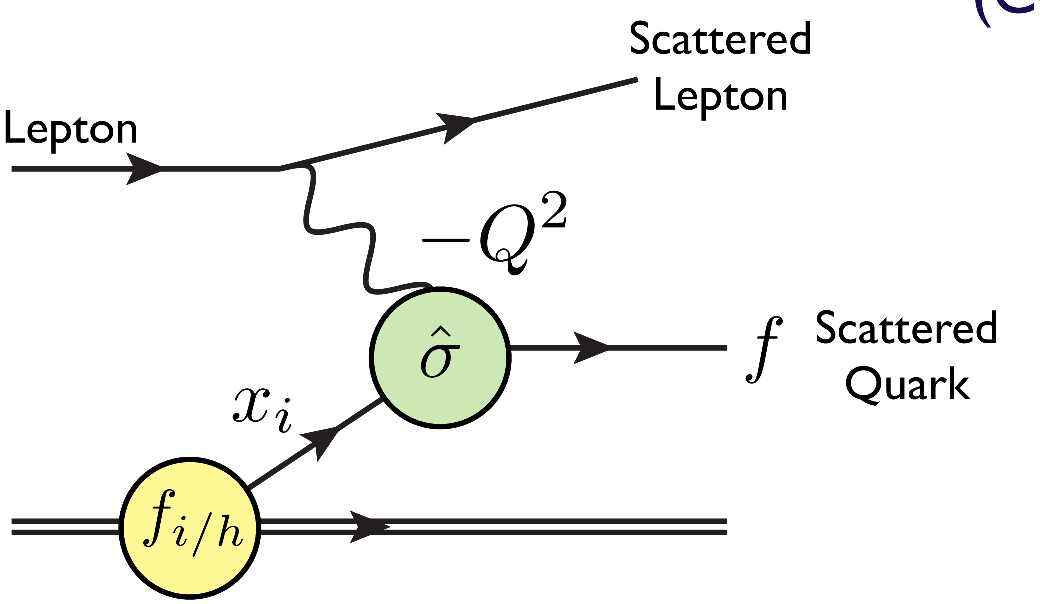

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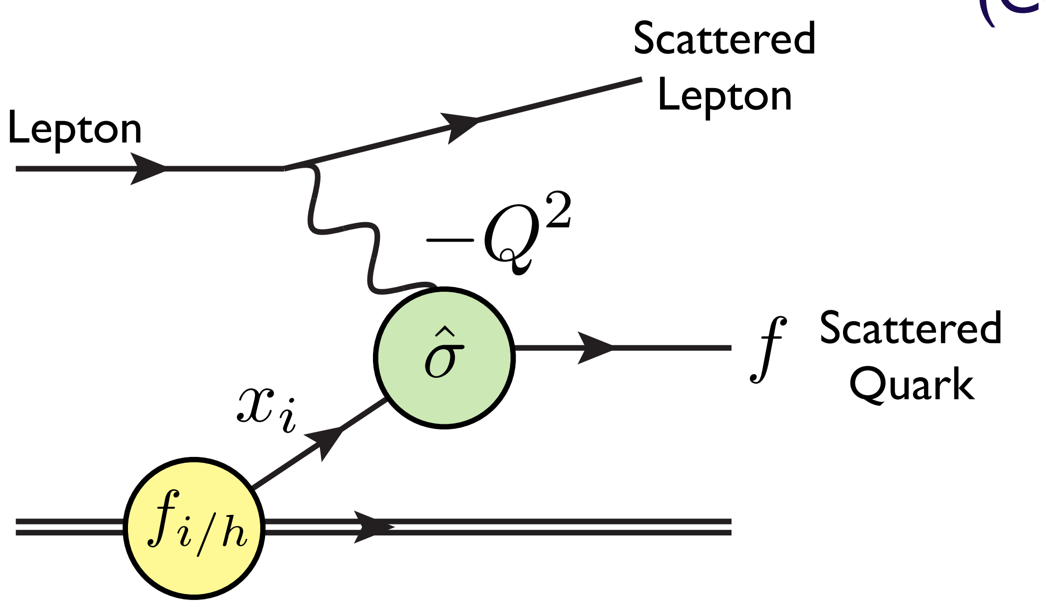

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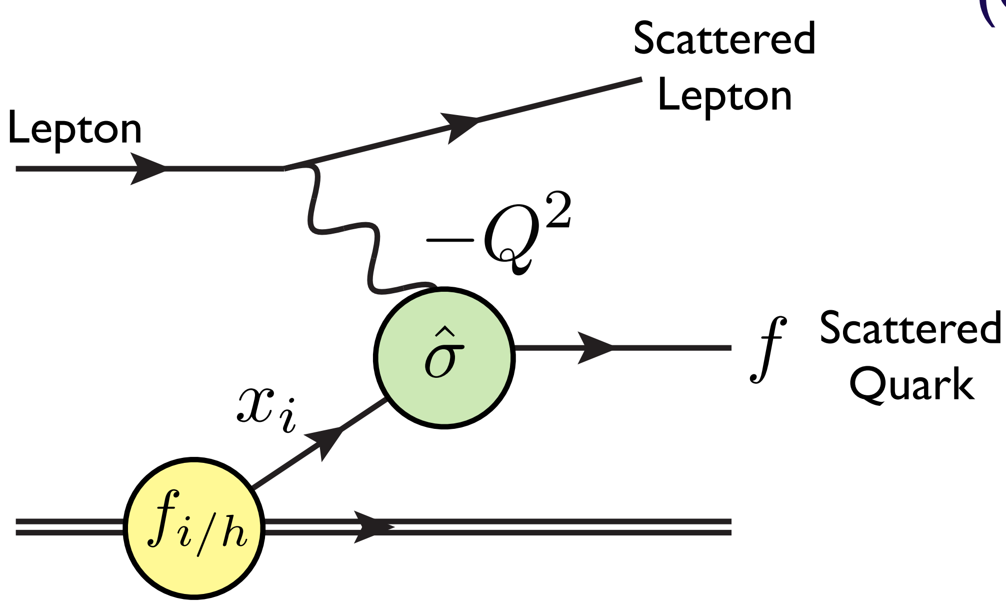

Note: Beyond LO, *f* can be more than one parton

Surprise Question: What's the color factor for DIS?

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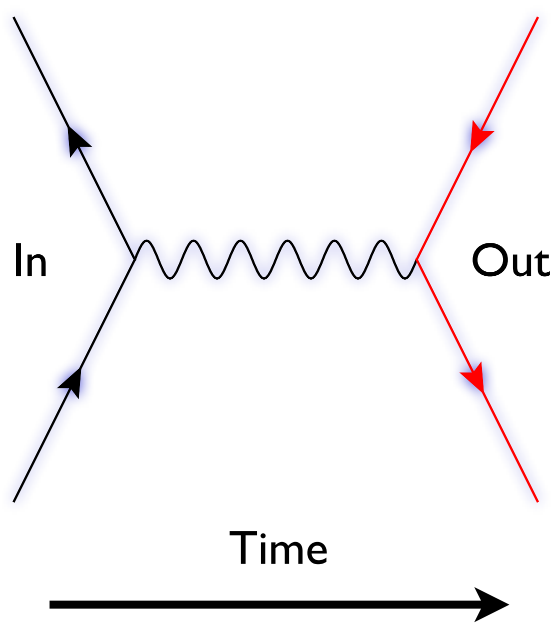
Differential partornic Hard-scattering Matrix Element(s)

It's just another crossing

$$e^+e^- \to \gamma^*/Z \to q\bar{q}$$

(Hadronic Z Decay)



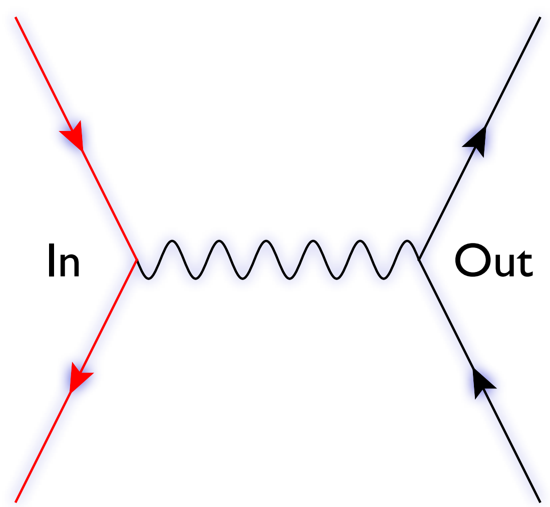
Color Factor:

$$\mathrm{Tr}[\delta_{ij}] = N_C$$

(see Lecture I)

$$q\bar{q} \to \gamma^*/Z \to \ell^+\ell^-$$

(Drell & Yan, 1970)



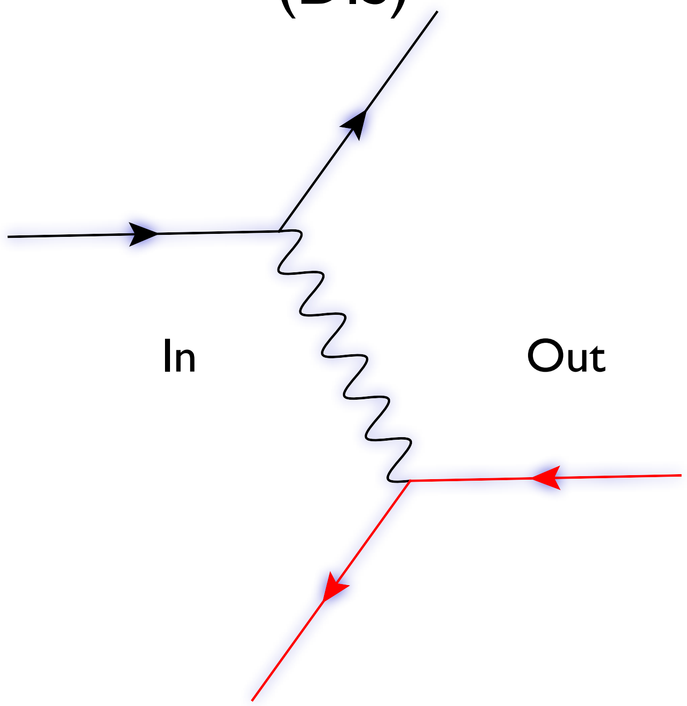
Color Factor:

$$\frac{1}{N_C^2}\mathrm{Tr}[\delta_{ij}] = \frac{1}{N_C}$$

(see Lecture I)

$$\ell q \stackrel{\gamma^*/Z}{\to} \ell q$$

(DIS)



Color Factor:

$$\frac{1}{N_C}\mathrm{Tr}[\delta_{ij}] = 1$$

(Factorization: Caveats)

1. The proof only includes the first term in an operator product expansion in "twist" = mass dimension - spin

→ Strictly speaking, only valid for $Q^2 \to \infty$. Neglects corrections of order

Higher Twist : $$\frac{\left[\ln\left(Q^2/\Lambda^2\right)\right]^{m<2n}}{Q^{2n}}$$ (n=2 for DIS)

2. The proof only applies to inclusive cross sections

In e^+e^- , in DIS, and in Drell-Yan. For everything else: factorization *ansatz*

3. Scheme dependence

In practice limited to MSbar + variations of Q_F

4. Interpretation of PDFs as parton number densities

Is only valid at Leading Order

QCD

Lecture II

Parton Densities

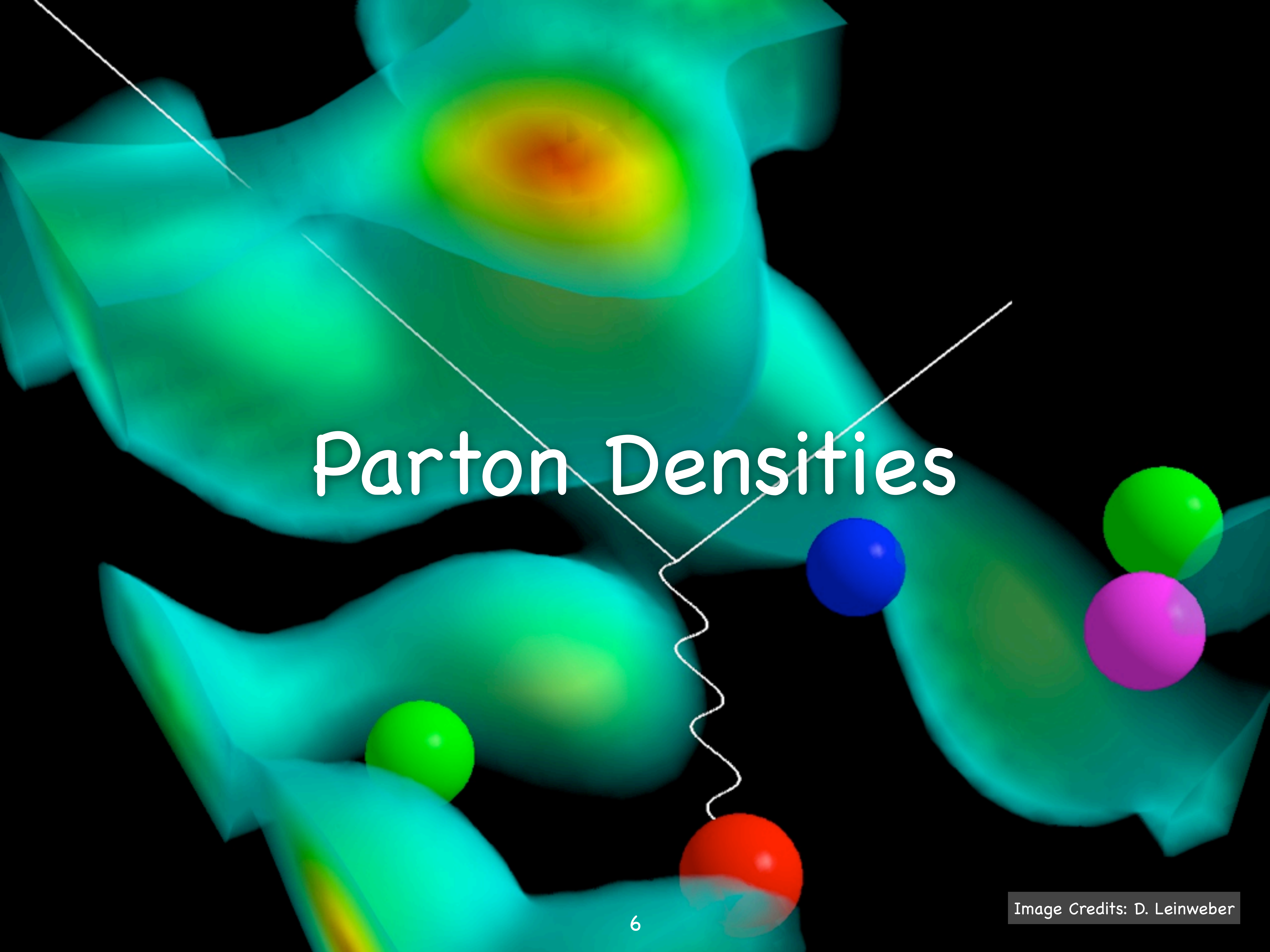


Image Credits: D. Leinweber

Parton Densities

$p_j = x P_{proton}$

$$\boxed{f_a(x_a, Q_i^2)}$$

Parton distribution functions (PDF)

- sum over long-wavelength histories leading to a with x_a at the scale Q_i^2 (ISR)

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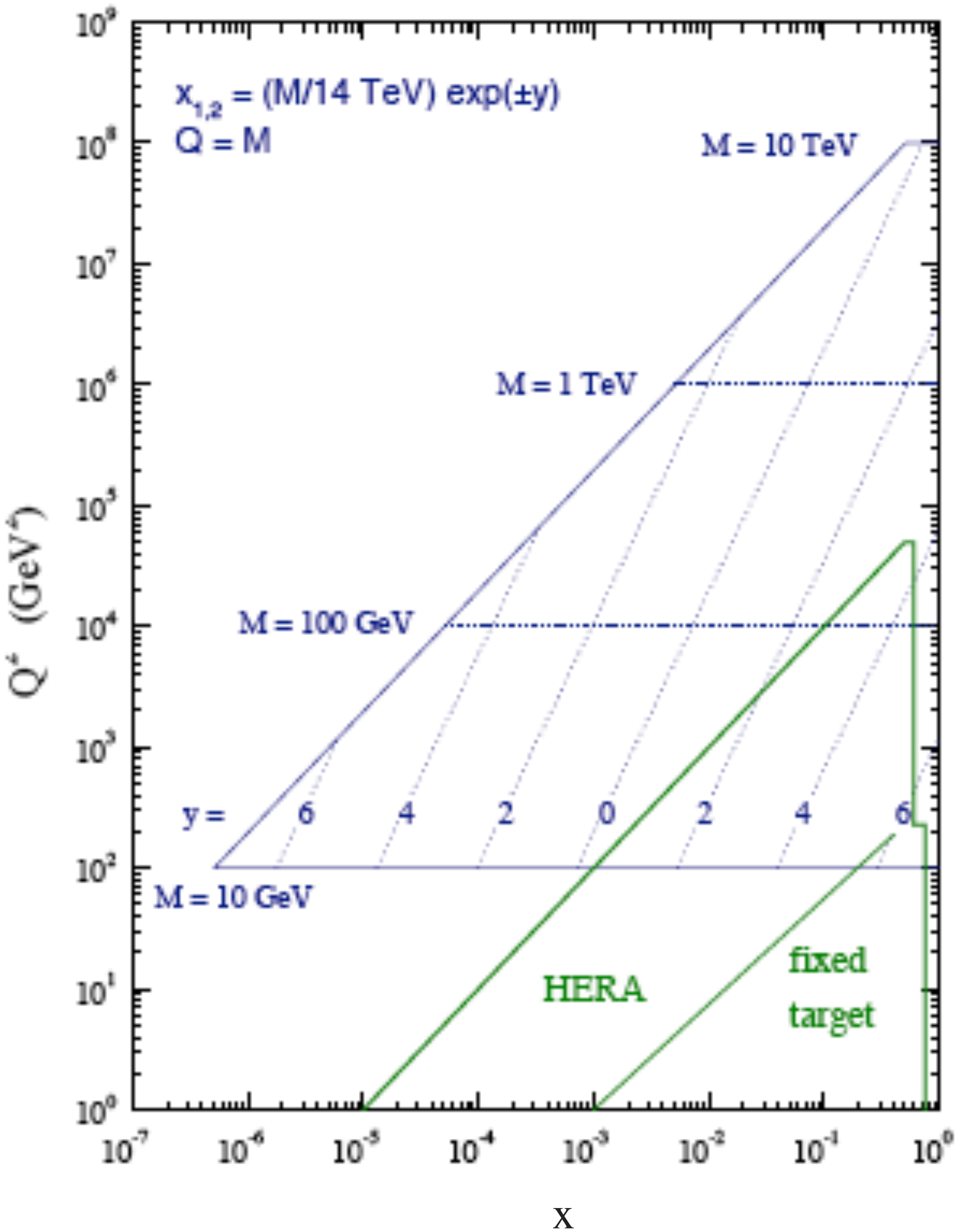
LHC Coverage
x and Q^2



QCD
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Shape of $f(x)$ unknown
(non-perturbative)

Different groups (CTEQ, MSTW, NNPDF, etc) use different ansätze

→ fit to measurements

Evolve to fixed small reference scale
$Q \approx m_{proton}$

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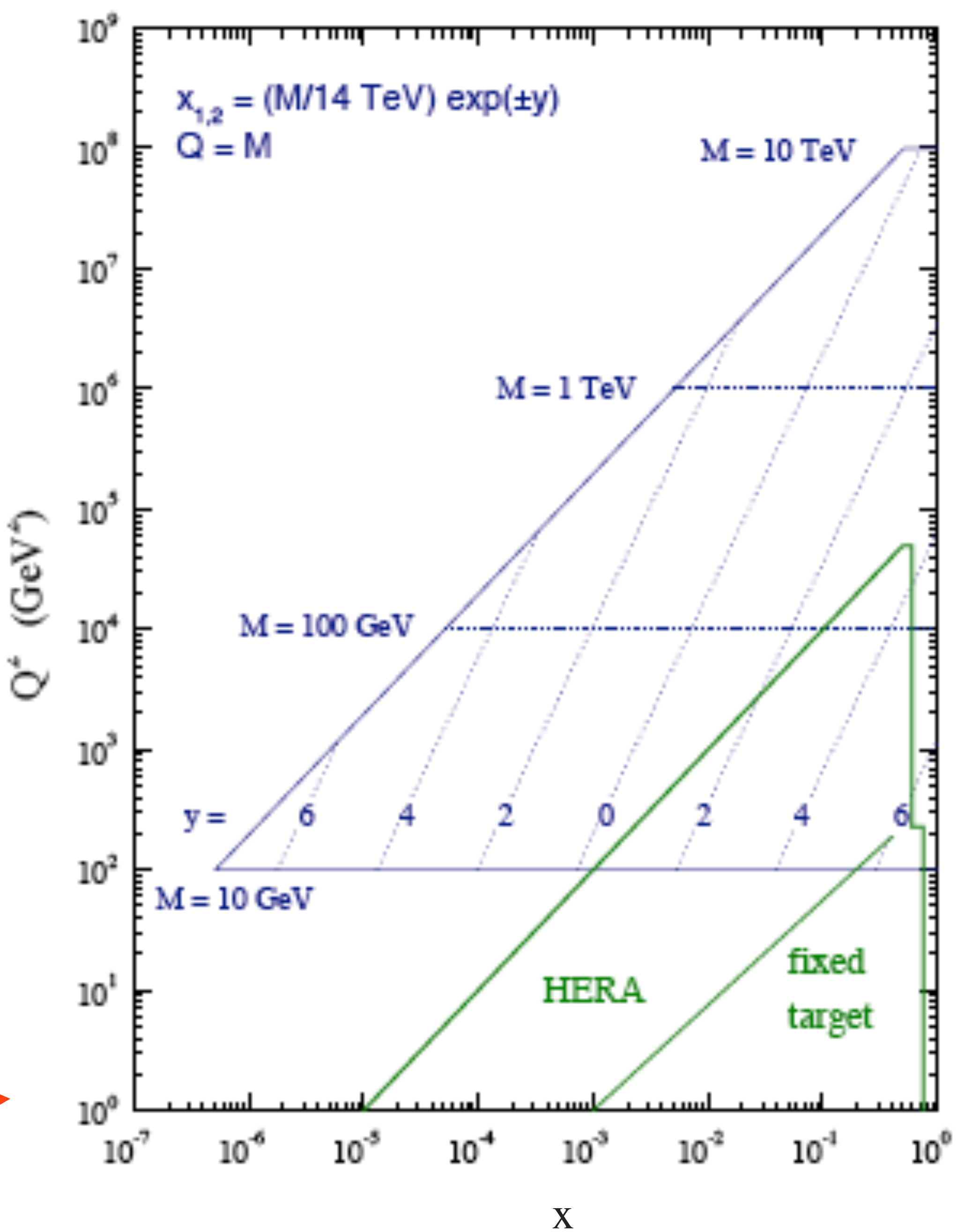


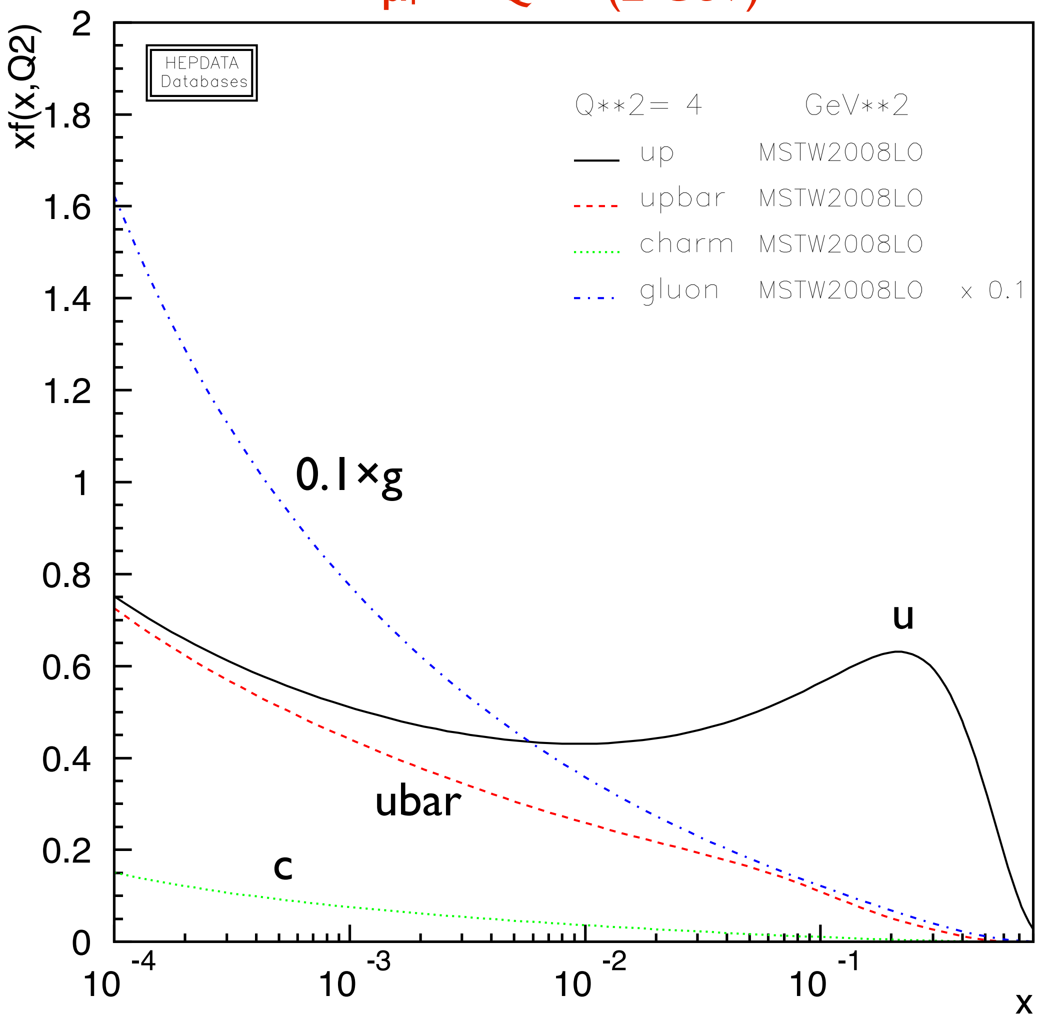
Evolution in Q^2 by DGLAP

(Dokshitzer-Gribov-Lipatov-Altarelli-Parisi)

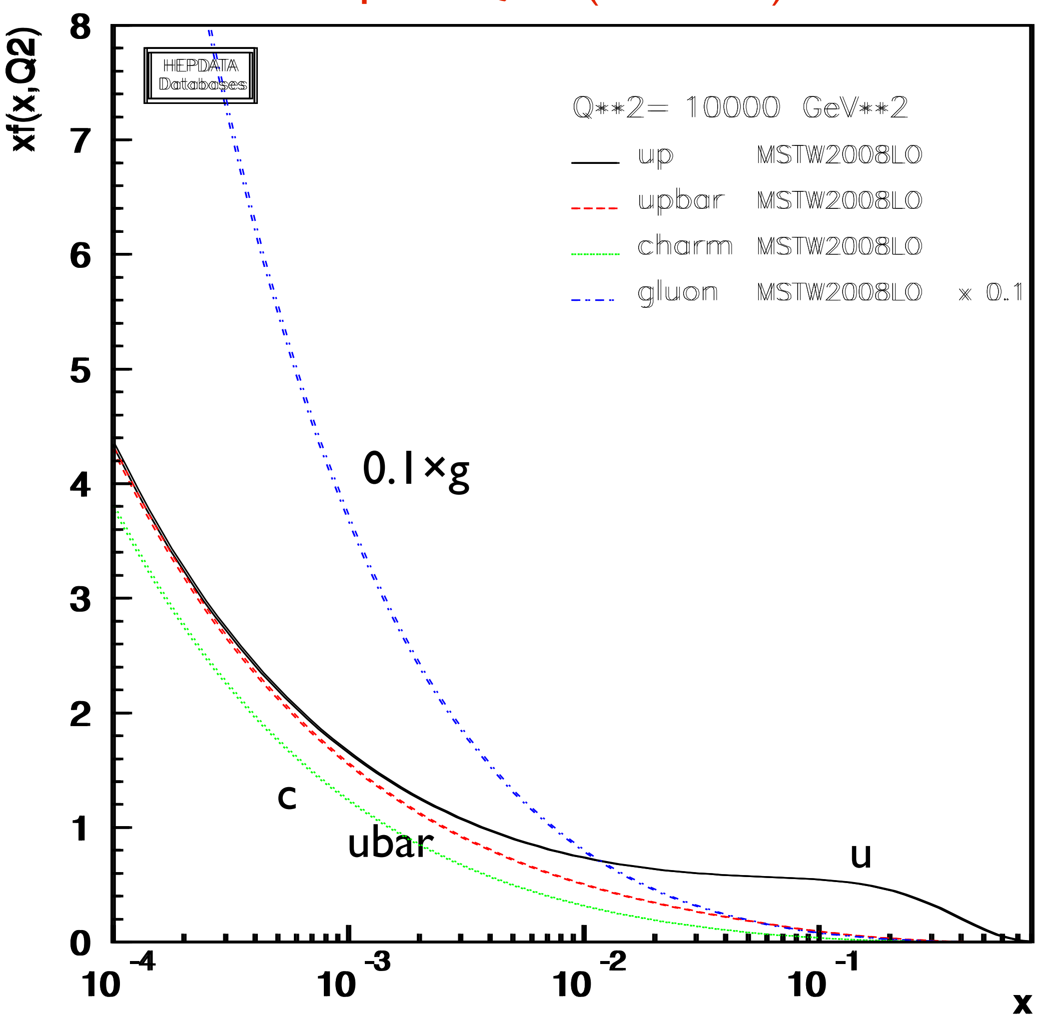
Require cross section independent of μ_F (at calculated order) → RGE

$$\frac{df_i(x_i,\mu_F^2)}{d\ln\mu_F^2} = \ldots$$

$\mu_F^2 = Q^2 = (2\text{ GeV})^2$



$\mu_F^2 = Q^2 = (100\text{ GeV})^2$



LO vs NLO

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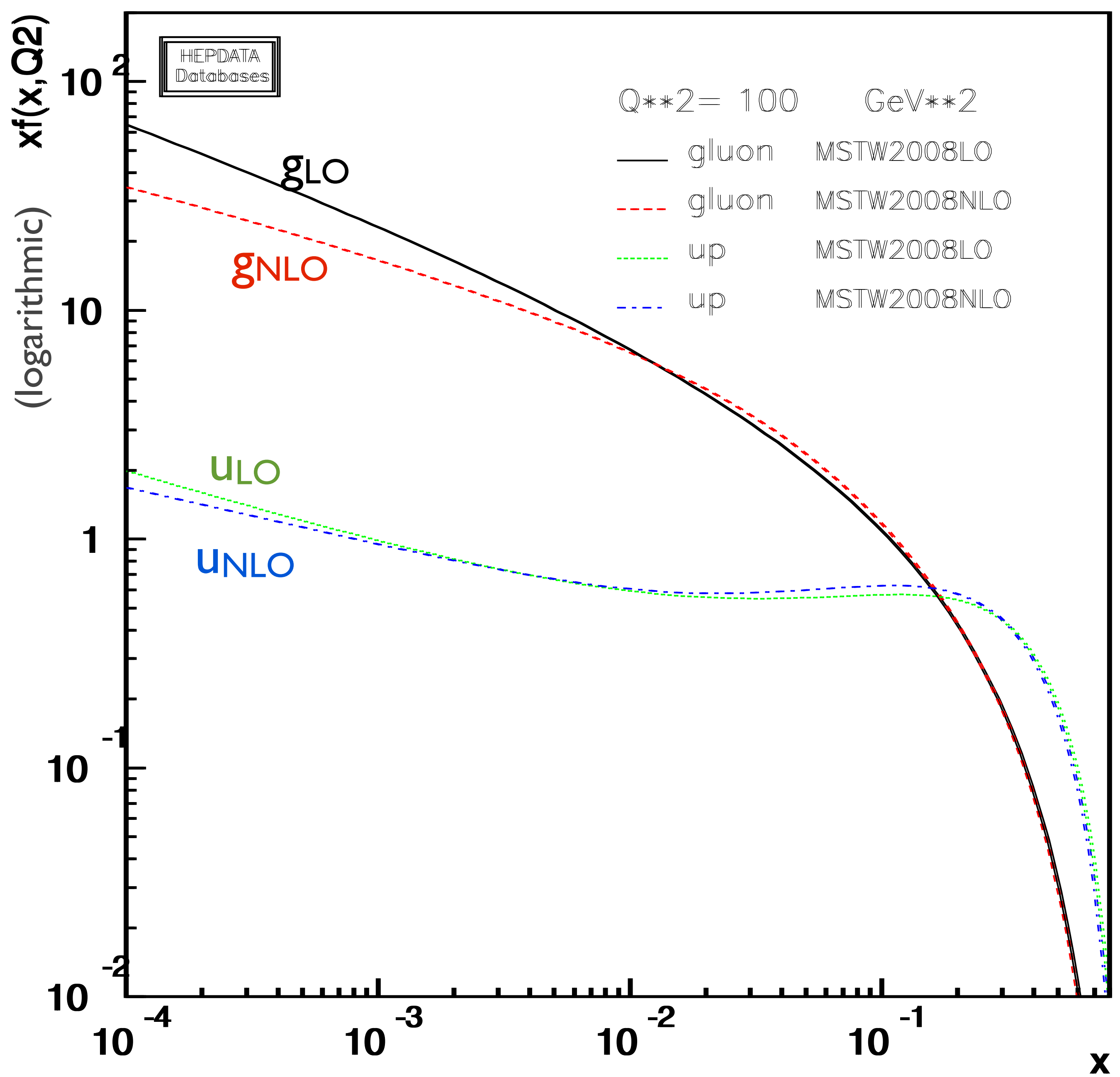

The "best fit" depends on the matrix elements you use when doing the fit

NLO matrix elements

contain low-x enhancements (they are larger than LO×DGLAP)

→ need less low-x PDFs

(+ momentum conservation

→ more partons at high x

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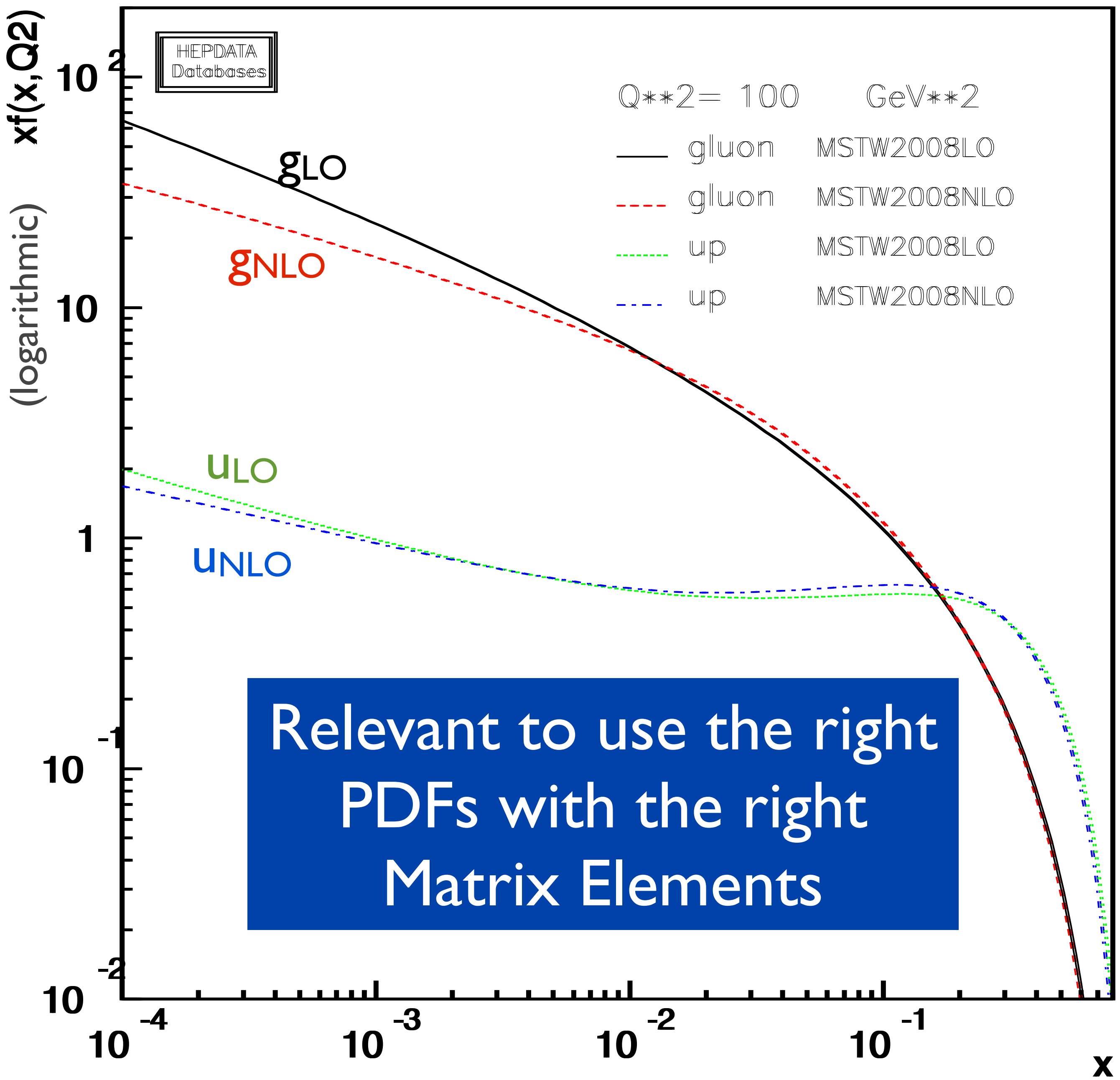

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(Advanced) PDF Uncertainties

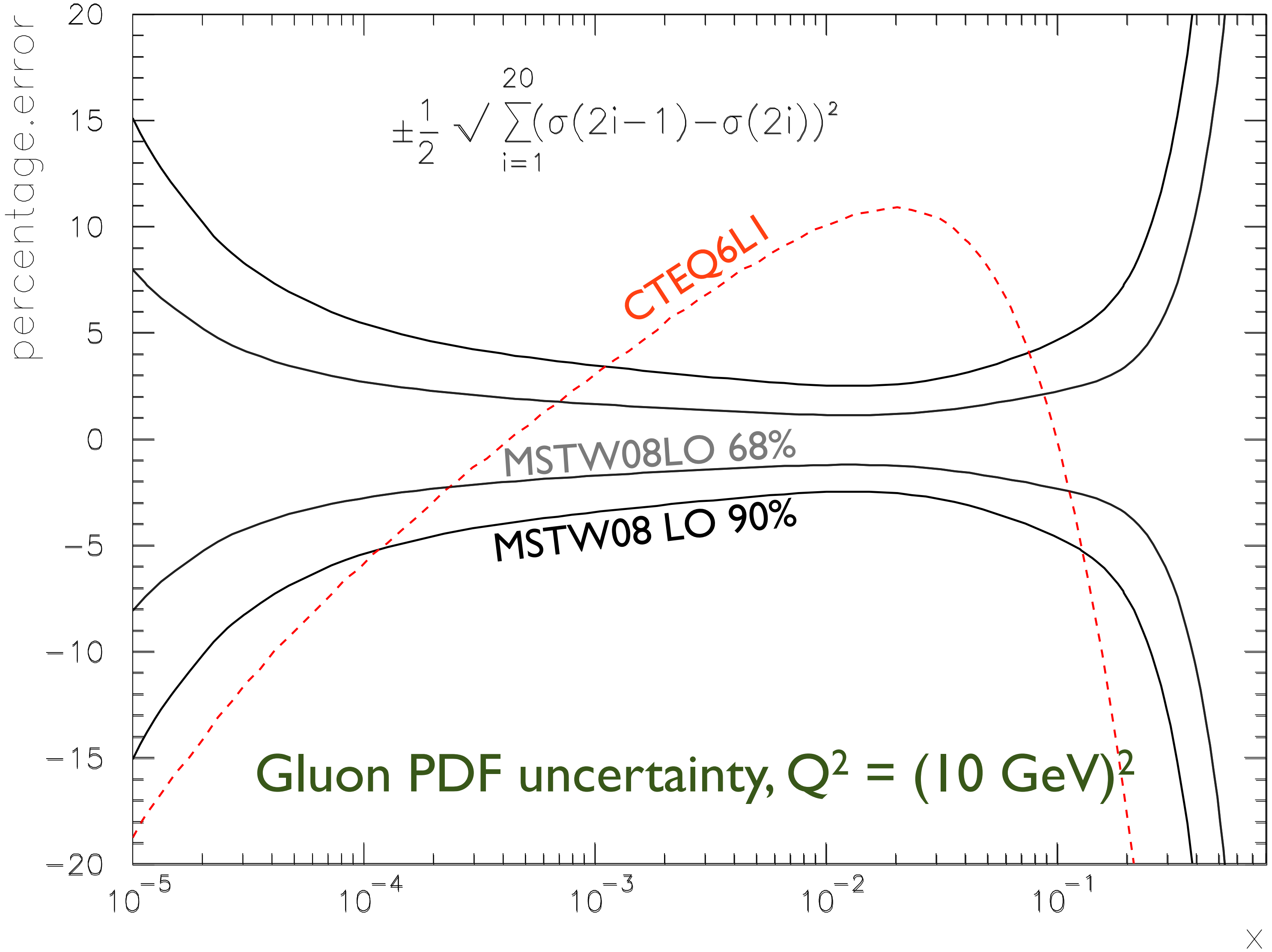
Much debate recently on PDF errors

Attempt to propagate experimental errors through PDF fits → 68% CL

But "tensions" between different data sets

→ 90%, or something else?

+ Different groups (CTEQ, MSTW, NNPDF, etc) use different ansätze for shape of f(x) at low-Q boundary



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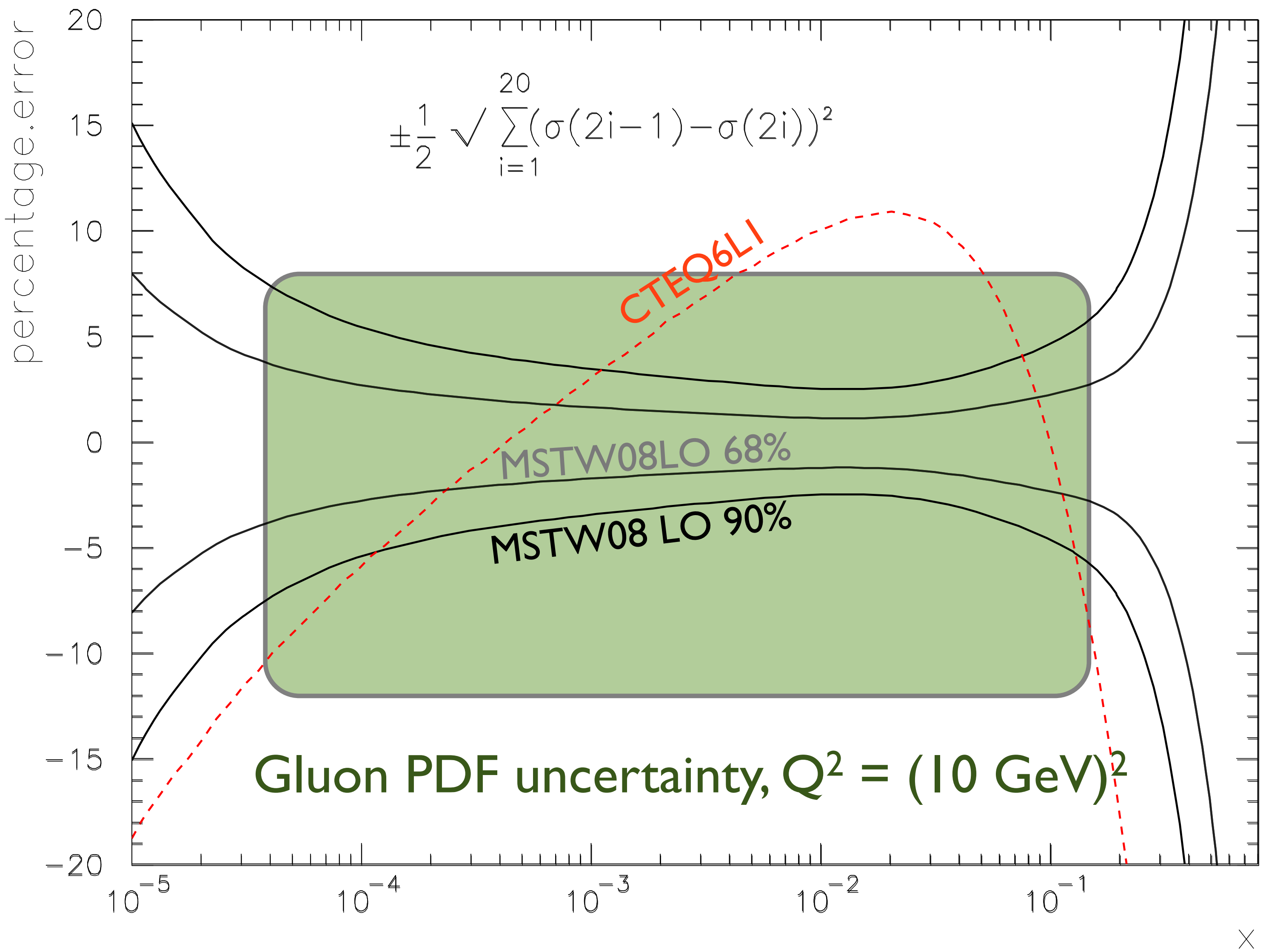

Still, good to ≈ 10% even for LO gluon in $10^{-4} < x < 10^{-1}$ (bigger errors at lower Q^2)

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Factorization: expresses the independence of long-wavelength (soft) emission on the nature of the hard (short-distance) process.

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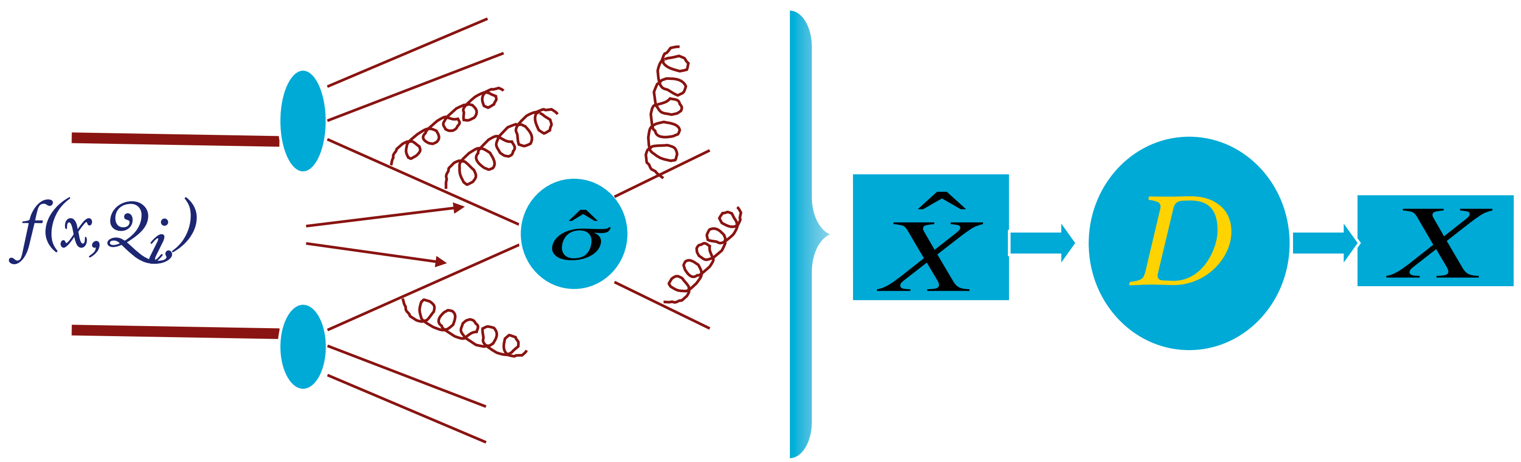

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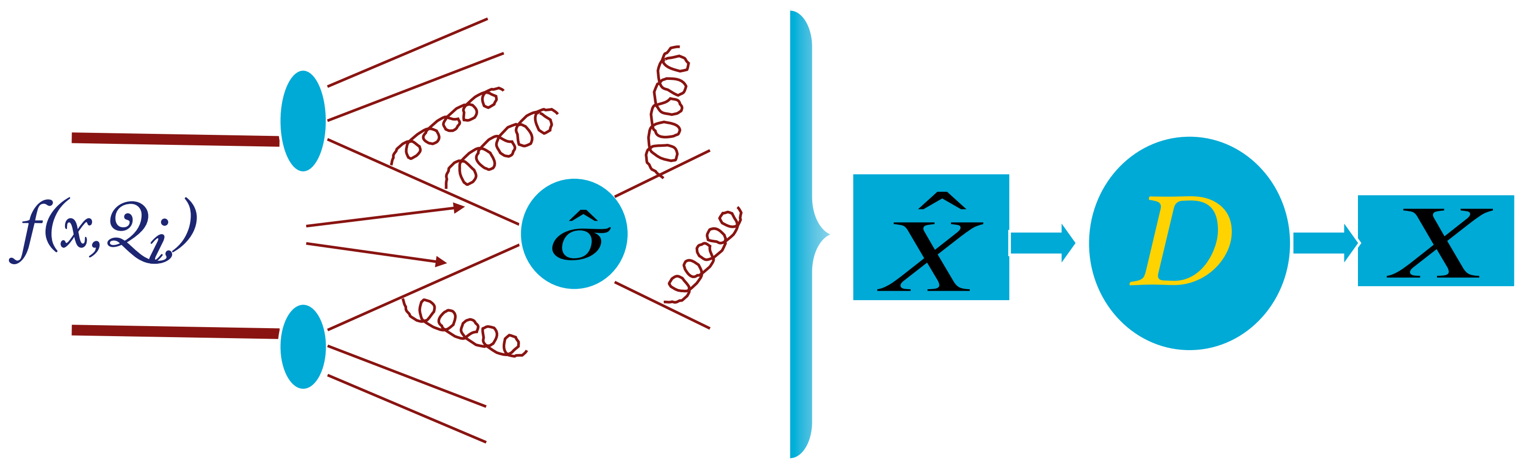

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$$D(\hat{X}_f \to X, Q_i^2, Q_f^2)$$ Fragmentation Function (FF)

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+ (At H.O. each of these defined in a specific scheme, usually $\overline{\mathrm{MS}}$)

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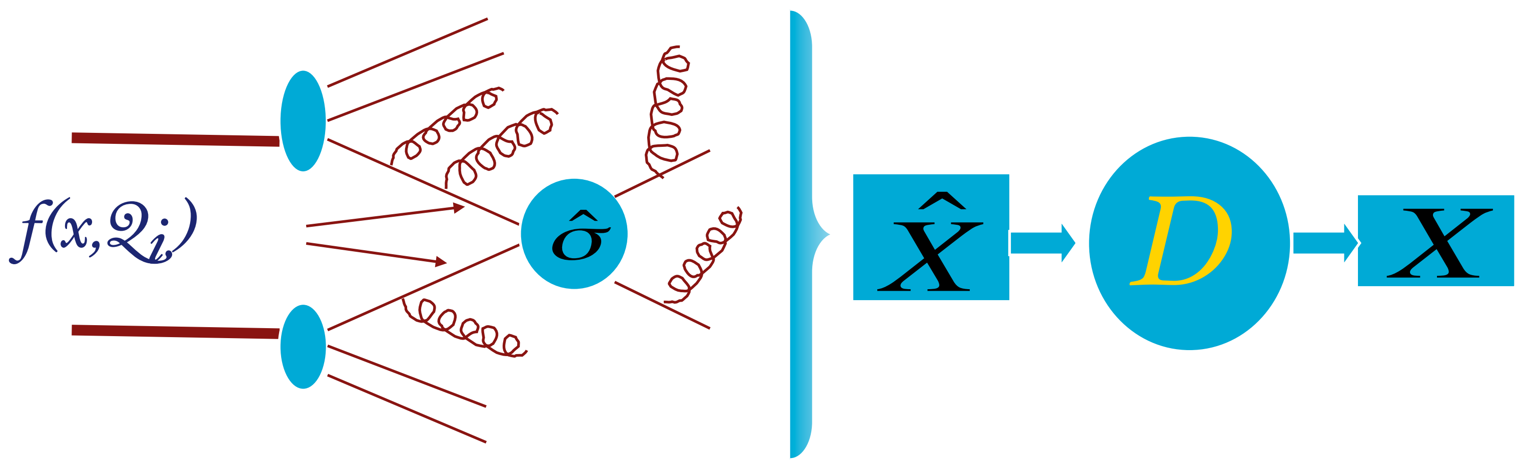

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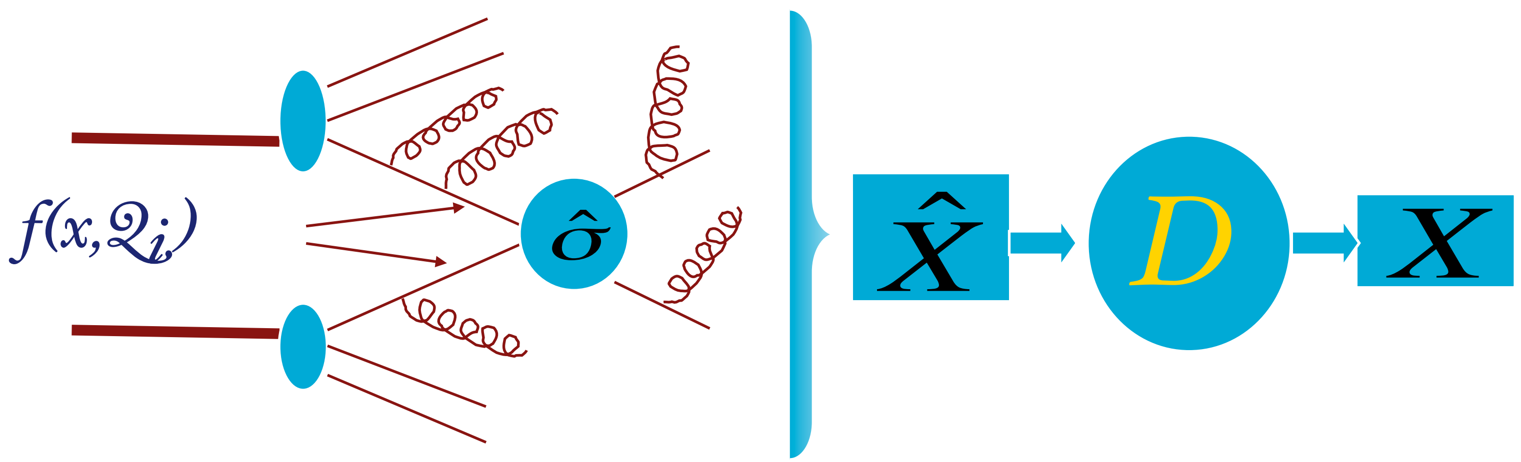

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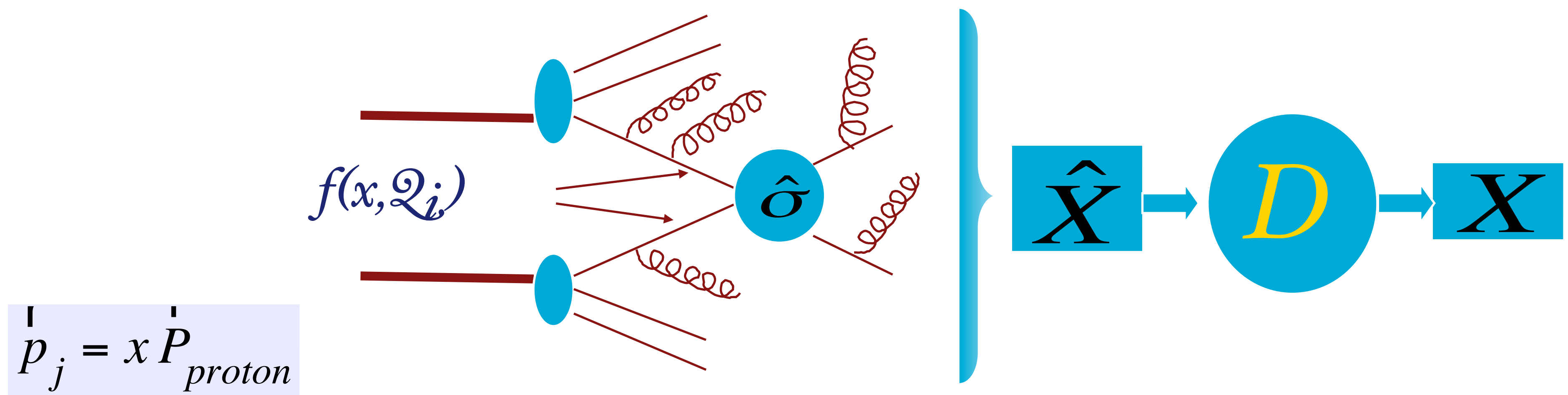

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QCD at Fixed Order

Distribution of observable: *O*

In production of X + anything

Fixed Order
(All Orders)

$$\left.\frac{d\sigma}{d\mathcal{O}}\right|_{\mathrm{ME}} = \sum_{k=0} \int \mathrm{d}\Phi_{X+k} \left| \sum_{\ell=0} M^{(\ell)}_{X+k} \right|^2 \delta\left(\mathcal{O} - \mathcal{O}(\{p\}_{X+k})\right)$$

Sum over identical amplitudes, then square

Momentum configuration

Phase Space

Cross Section differentially in *O*

Sum over "anything" ≈ legs

Matrix Elements for X+k at (l) loops

Evaluate observable → differential in O

QCD at Fixed Order

Distribution of observable: *O*

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Fixed Order
(All Orders)

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Momentum configuration

Phase Space

Cross Section differentially in *O*

Sum over "anything" ≈ legs

Matrix Elements for X+k at (l) loops

Evaluate observable → differential in O

Truncate at $k = 0, \ell = 0$,
→ **Born Level = First Term**
Lowest order at which X happens

QCD at Fixed Order

Distribution of observable: *O*

In production of X + anything

Fixed Order
(All Orders)

$$\left.\frac{d\sigma}{d\mathcal{O}}\right|_{\mathrm{ME}} = \sum_{k=0} \int \mathrm{d}\Phi_{X+k} \left| \sum_{\ell=0} M^{(\ell)}_{X+k} \right|^2 \delta\left(\mathcal{O} - \mathcal{O}(\{p\}_{X+k})\right)$$

Sum over identical amplitudes, then square

Momentum configuration

Phase Space

Cross Section differentially in *O*

Sum over "anything" ≈ legs

Matrix Elements for X+k at (l) loops

Evaluate observable → differential in O

Truncate at $k = n, \ell = 0$,
→ **Leading Order** for X + n
Lowest order at which X + n happens

QCD at Fixed Order

Distribution of observable: *O*

In production of X + anything

Fixed Order
(All Orders)

$$\left.\frac{d\sigma}{d\mathcal{O}}\right|_{\mathrm{ME}} = \sum_{k=0} \int \mathrm{d}\Phi_{X+k} \left| \sum_{\ell=0} M_{X+k}^{(\ell)} \right|^2 \delta\left(\mathcal{O} - \mathcal{O}(\{p\}_{X+k})\right)$$

Sum over identical amplitudes, then square

Momentum configuration

Phase Space

Cross Section differentially in *O*

Sum over "anything" ≈ legs

Matrix Elements for X+k at (l) loops

Evaluate observable → differential in O

Truncate at $k + \ell = n$,
→ **N^{n}LO** for X
Includes **N^{n-1}LO** for X+1, **N^{n-2}LO** for X+2, …

Loops and Legs

Another representation

Loops					
	$X^{(2)}$	$X+1^{(2)}$	…		
	$X^{(1)}$	$X+1^{(1)}$	$X+2^{(1)}$	$X+3^{(1)}$	…
	Born	$X+1^{(0)}$	$X+2^{(0)}$	$X+3^{(0)}$	…
	Legs				

Another representation

Born

(1882-1970)
Nobel Prize 1954

Loops

$X^{(2)}$ $X{+}1^{(2)}$ …

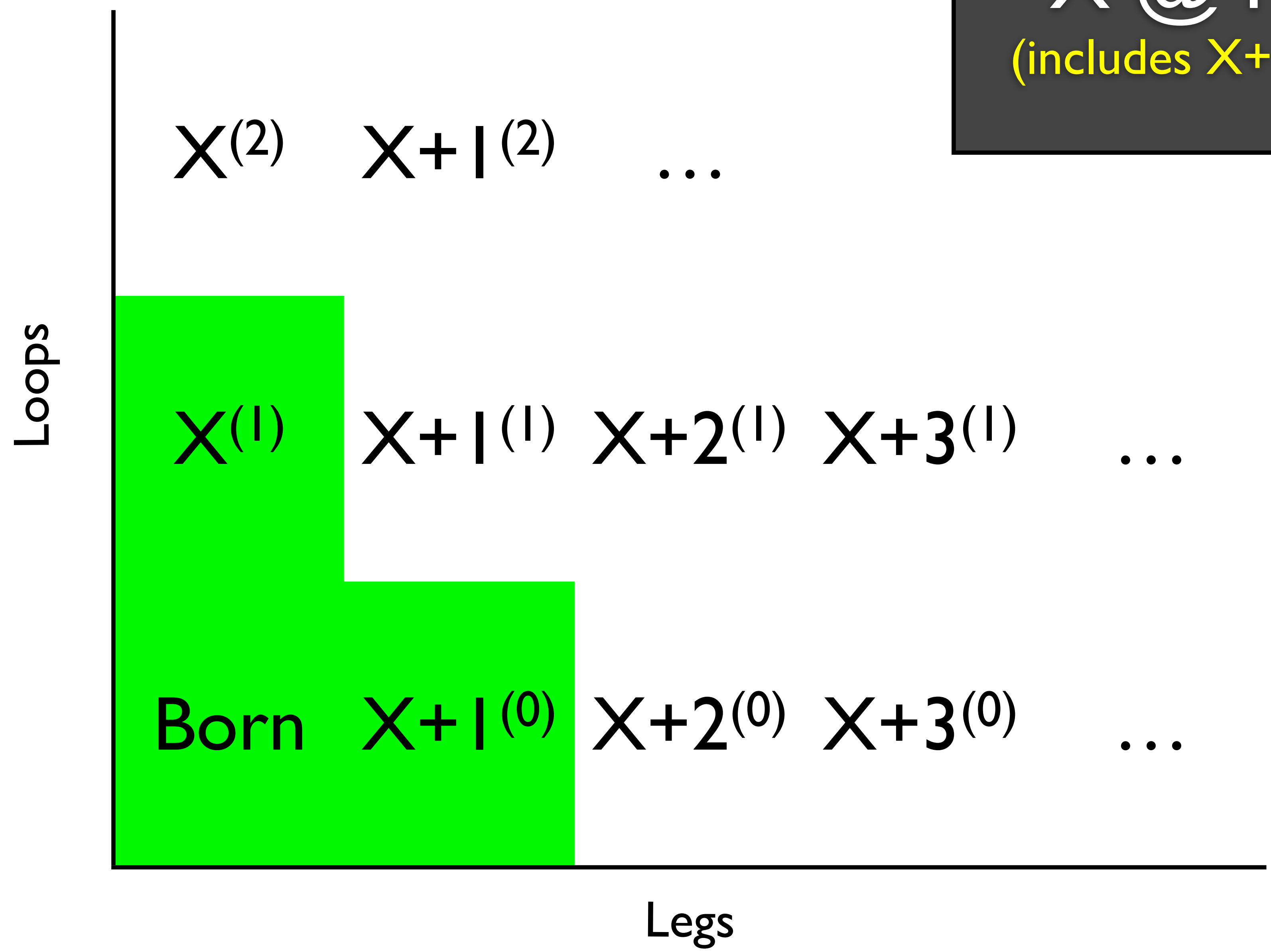
$X^{(1)}$ $X{+}1^{(1)}$ $X{+}2^{(1)}$ $X{+}3^{(1)}$ …

Born $X{+}1^{(0)}$ $X{+}2^{(0)}$ $X{+}3^{(0)}$ …

Legs

Loops and Legs

Another representation

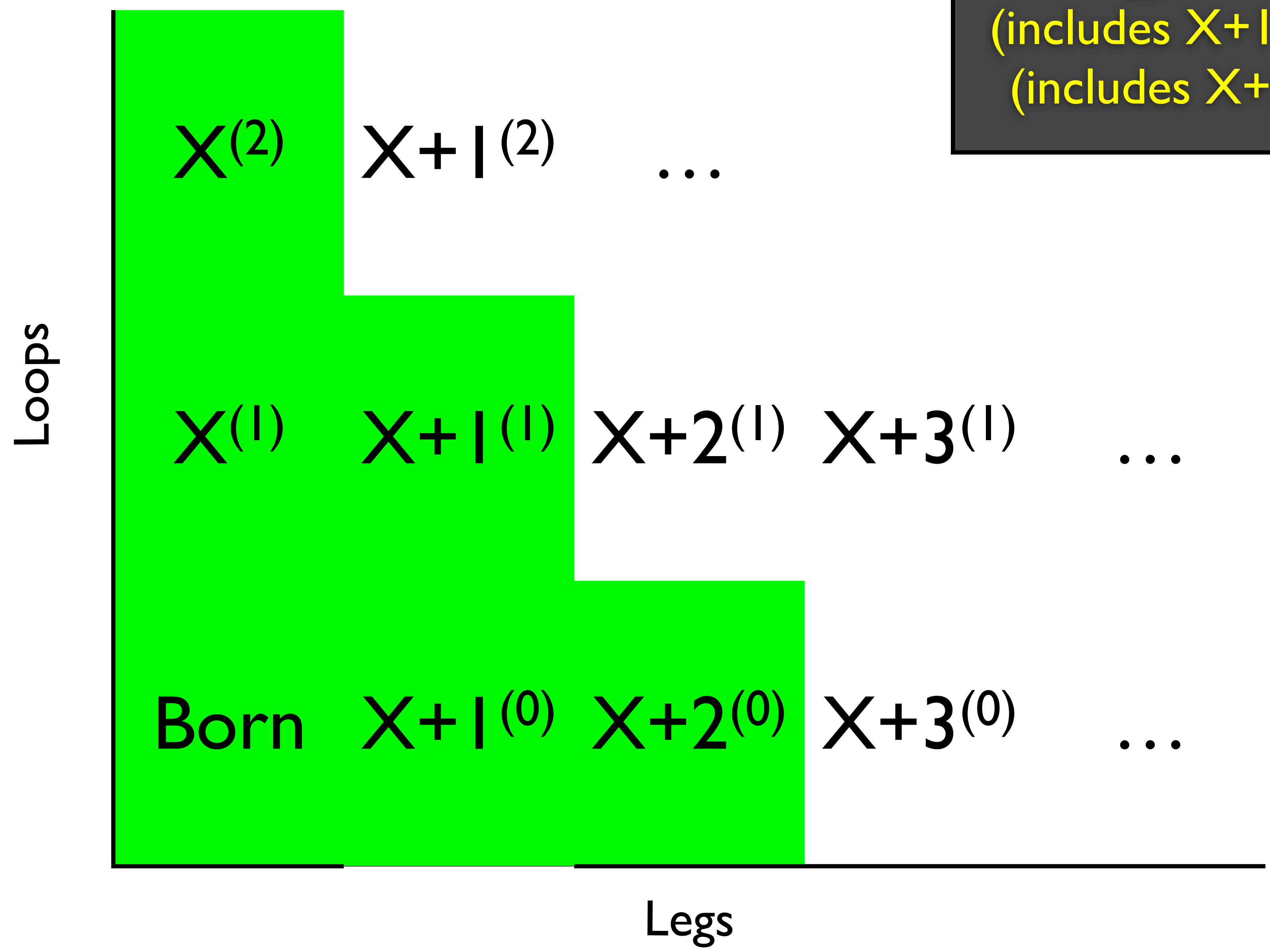


X @ NLO
(includes X+1 @ LO)

Note: X+1 jet observables only correct at LO

Loops and Legs

Another representation



X @ NNLO
(includes X+1 @ NLO)
(includes X+2 @ LO)

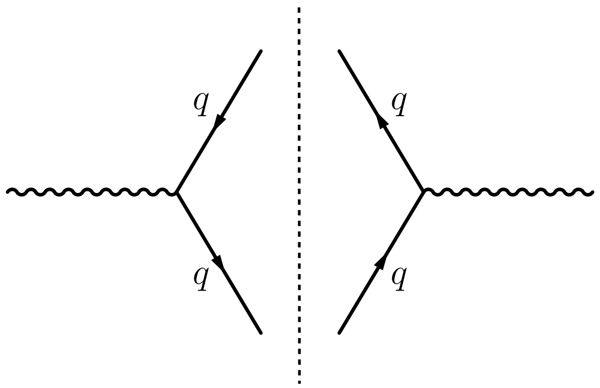
Note: X+1 jet observables only correct at NLO

Note: X+2 jet observables only correct at LO

Cross sections at LO

Born:

$$\sigma_{\mathrm{Born}} = \int |M_X^{(0)}|^2$$



$X^{(2)}$	$X+1^{(2)}$	…
$X^{(1)}$	$X+1^{(1)}$	…
Born	$X+1^{(0)}$	$X+2^{(0)}$

Born:

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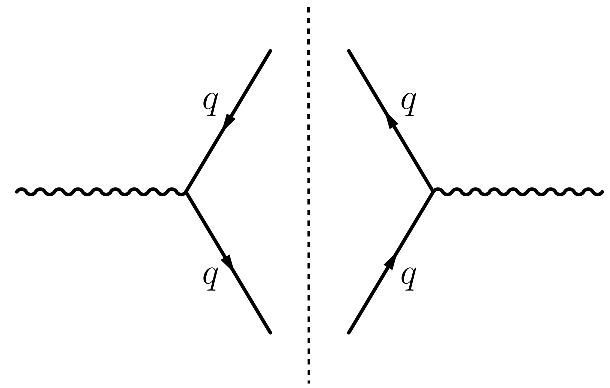
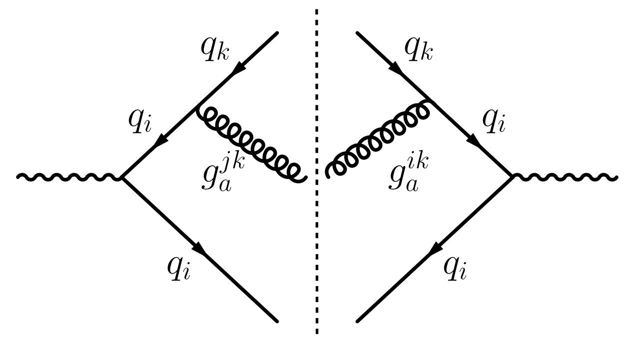


$X^{(2)}$	$X+1^{(2)}$	...
$X^{(1)}$	$X+1^{(1)}$	...
Born	$X+1^{(0)}$	$X+2^{(0)}$

Born + n

$$\sigma^{\mathrm{LO}}_{\mathrm{X+1}}(R) = \int_R |M_{X+1}^{(0)}|^2$$



$X^{(2)}$	$X+1^{(2)}$	...
$X^{(1)}$	$X+1^{(1)}$	...
Born	$X+1^{(0)}$	$X+2^{(0)}$

Infrared divergent → Must be regulated

R = some Infrared Safe phase space region

(Often a cut on $p_\perp > n$ GeV)

Careful not to take it too low!

Cross sections at LO

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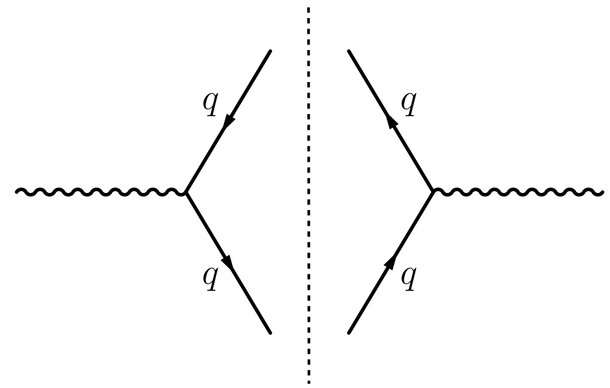
$$\sigma_{\mathrm{Born}} = \int |M_X^{(0)}|^2$$



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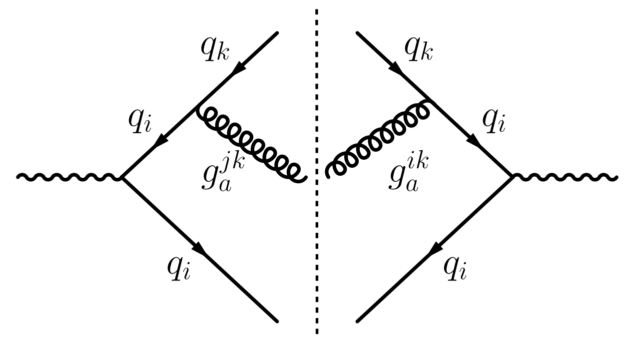

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(Often a cut on $p_\perp > n$ GeV)

Careful not to take it too low!

if σ(X+n) ≈ σ(X) you got a problem
perturbative expansion not reliable

Recall: Conformal QCD

Naively, brems suppressed by $\alpha_s \approx 0.1$

Truncate at fixed order = LO, NLO, …

But beware the jet-within-a-jet-within-a-jet …

→ More on this in lectures on Monte Carlo & Matching

Example:

100 GeV can be "soft" at the LHC

SUSY pair production at 14 TeV, with $M_{SUSY} \approx 600$ GeV

LHC - sps1a - m~600 GeV

Plehn, Rainwater, PS PLB645(2007)217

FIXED ORDER pQCD	$\sigma_{\rm tot}$[pb]	$\tilde{g}\tilde{g}$	$\tilde{u}_L\tilde{g}$	$\tilde{u}_L\tilde{u}_L^*$	$\tilde{u}_L\tilde{u}_L$	TT
$p_{T,j} >$ 100 GeV	σ_{0j}	4.83	5.65	0.286	0.502	1.30
inclusive X + 1 "jet" →	σ_{1j}	2.89	2.74	0.136	0.145	0.73
inclusive X + 2 "jets" →	σ_{2j}	1.09	0.85	0.049	0.039	0.26
$p_{T,j} >$ 50 GeV	σ_{0j}	4.83	5.65	0.286	0.502	1.30
	σ_{1j}	5.90	5.37	0.283	0.285	1.50
	σ_{2j}	4.17	3.18	0.179	0.117	1.21

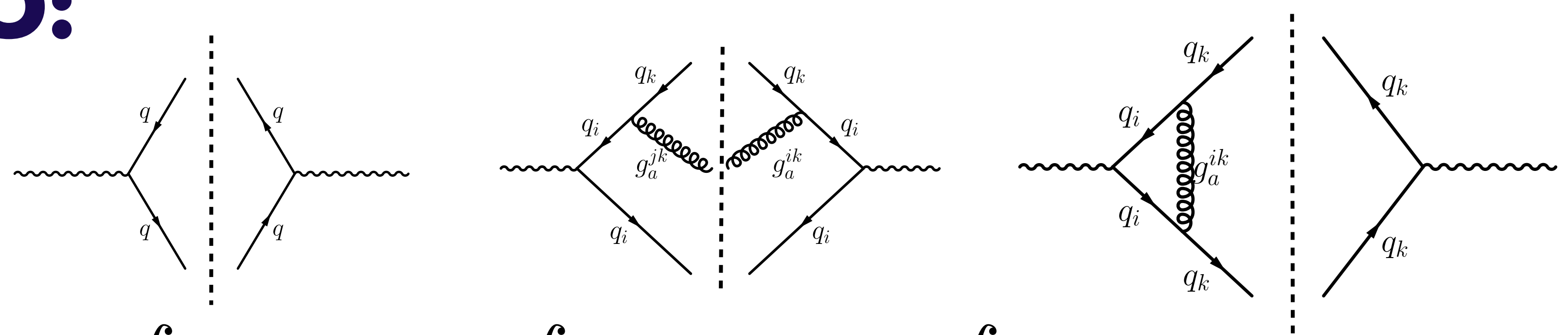
(Computed with SUSY-MadGraph)

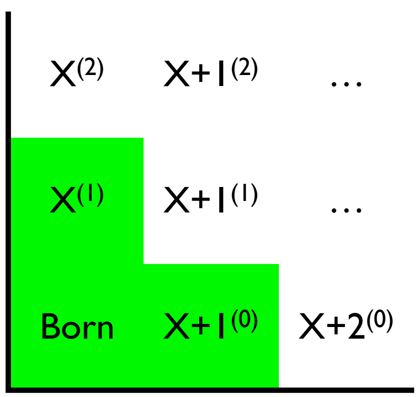
σ for X + jets much larger than naive estimate

σ for 50 GeV jets ≈ larger than total cross section → not under control

Cross sections at NLO

NLO:



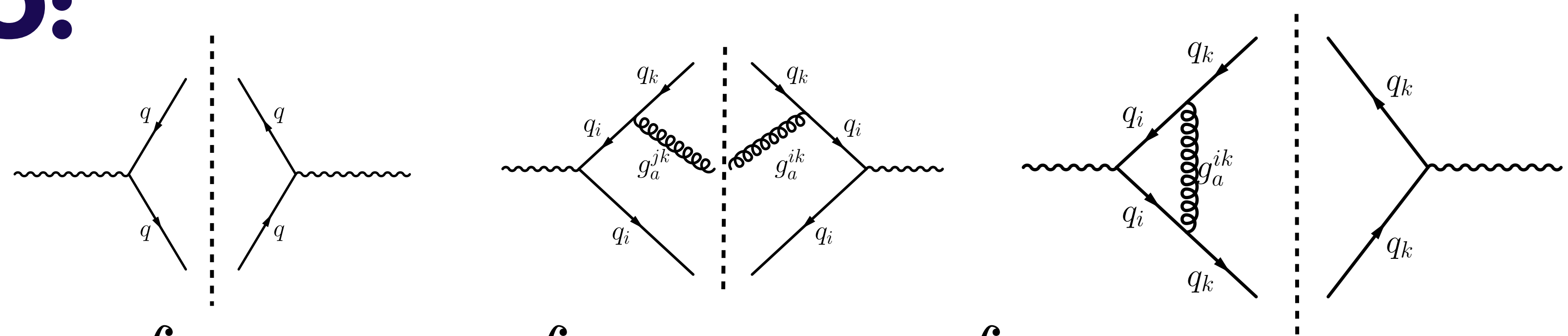


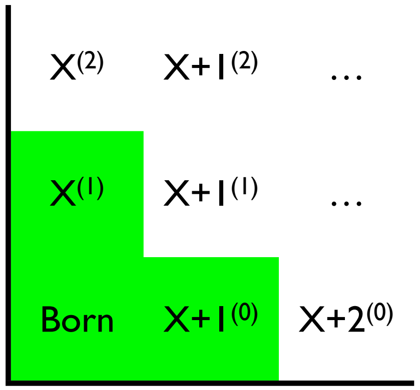
$$\sigma_{\mathrm{X}}^{\mathrm{NLO}} = \int |M_X^{(0)}|^2 + \int |M_{X+1}^{(0)}|^2 + \int 2\mathrm{Re}[M_X^{(1)} M_X^{(0)*}]$$

(note: this is *not* the 1-loop diagram squared)

Cross sections at NLO

NLO:





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(note: this is *not* the 1-loop diagram squared)

KLN Theorem (Kinoshita-Lee-Nauenberg)

Singularities cancel at complete order (only finite terms left over)

$$= \sigma_{\mathrm{Born}} + \mathrm{Finite}\left\{\int |M_{X+1}^{(0)}|^2\right\} + \mathrm{Finite}\left\{\int 2\mathrm{Re}[M_X^{(1)} M_X^{(0)*}]\right\}$$

$$\sigma_{\mathrm{NLO}}(e^+e^- \to q\bar{q}) = \sigma_{\mathrm{LO}}(e^+e^- \to q\bar{q}) \left(1 + \frac{\alpha_s(E_{\mathrm{CM}})}{\pi} + \mathcal{O}(\alpha_s^2)\right)$$

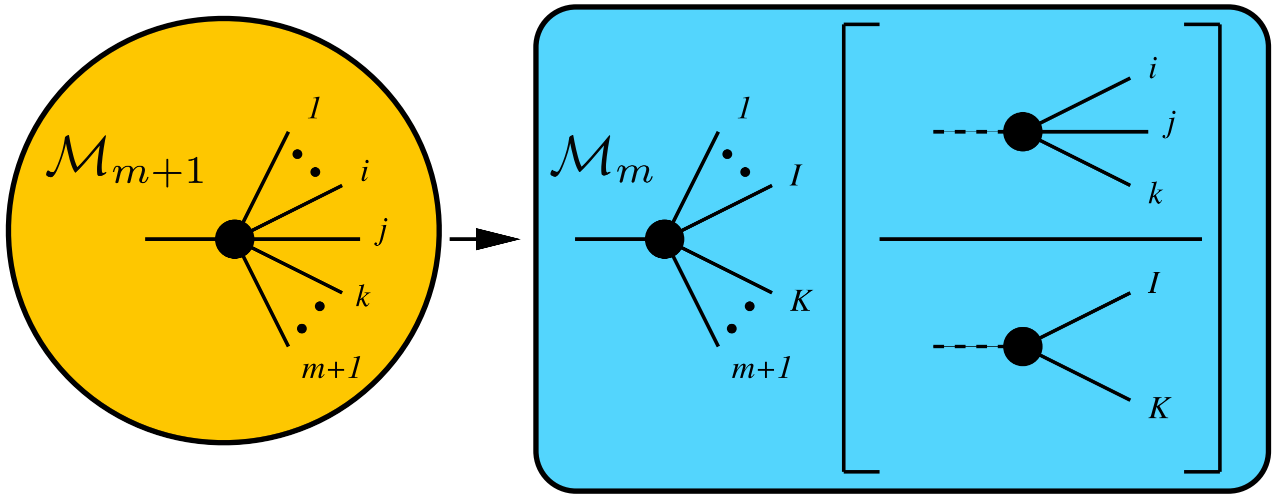
The Subtraction Idea

How do I get finite{Real} **and** finite{Virtual} **?**

First step: classify IR singularities using universal functions

EXAMPLE: factorization of amplitudes in the soft limit

Soft Limit ($E_j \to 0$):



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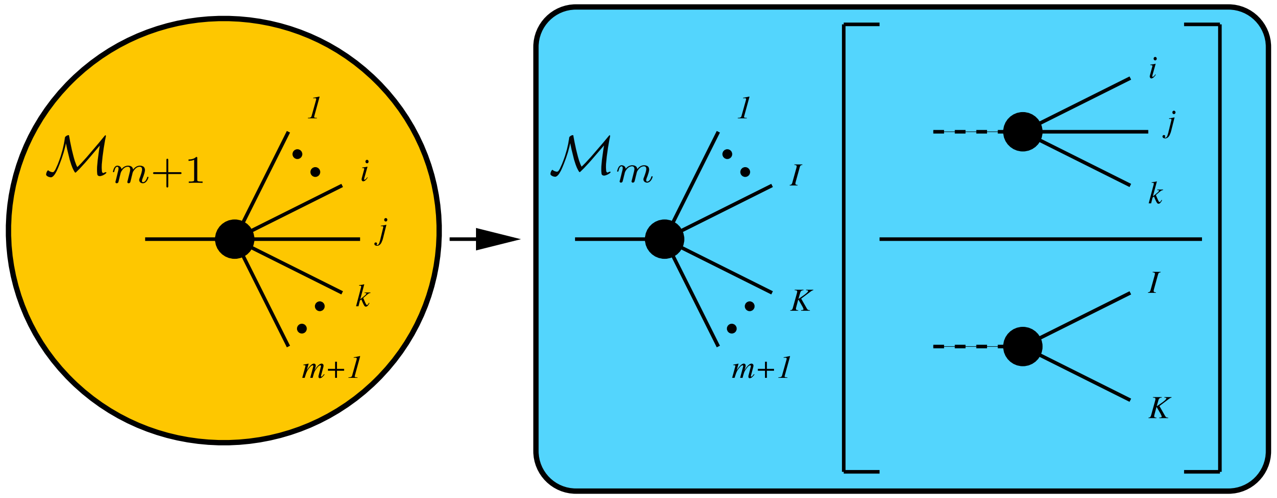

$$|\mathcal{M}_{n+1}(1,\cdots,i,j,k,\cdots,n+1)|^2 \xrightarrow{j_g \to 0} g_s^2 \mathcal{C}_{ijk}\, S_{ijk}\, |\mathcal{M}_n(1,\cdots,i,k,\cdots,n+1)|^2$$

Universal
"Soft Eikonal"

$$S_{ijk}(m_I, m_K) = \frac{2s_{ik}}{s_{ij}s_{jk}} - \frac{2m_I^2}{s_{ij}^2} - \frac{2m_K^2}{s_{jk}^2} \qquad s_{ij} \equiv 2p_i \cdot p_j$$

The Subtraction Idea

Add and subtract IR limits (SOFT and COLLINEAR)

$$\mathrm{d}\sigma_{NLO} = \underbrace{\int_{\mathrm{d}\Phi_{m+1}} \left(\mathrm{d}\sigma^R_{NLO} - \mathrm{d}\sigma^S_{NLO}\right)}_{\text{Finite by Universality}} + \underbrace{\left[\int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\sigma^S_{NLO} + \int_{\mathrm{d}\Phi_m} \mathrm{d}\sigma^V_{NLO}\right]}_{\text{Finite by KLN}}$$

Dipoles (Catani-Seymour)

Global Antennae (Gehrmann, Gehrmann-de Ridder, Glover)

Sector Antennae (Kosower)

…

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Choice of subtraction terms:

Singularities mandated by gauge theory

Non-singular terms: up to you (added and subtracted, so vanish)

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- Non-singular terms: up to you (added and subtracted, so vanish)

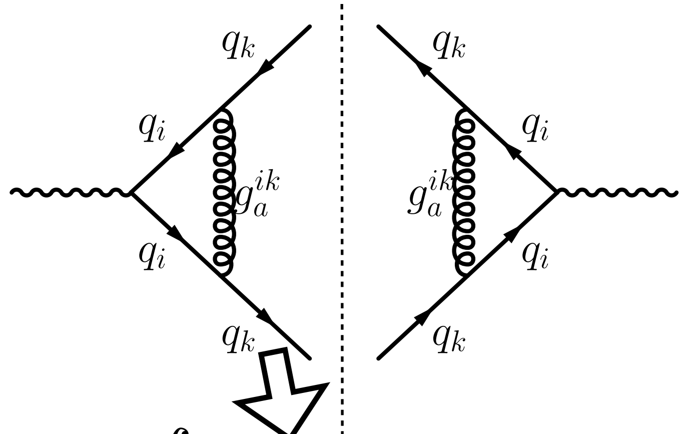
$$\frac{|\mathcal{M}(Z^0 \to q_i g_j \bar{q}_k)|^2}{|\mathcal{M}(Z^0 \to q_I \bar{q}_K)|^2} = g_s^2\, 2C_F \left[\overbrace{\frac{2s_{ik}}{s_{ij}s_{jk}}}^{\text{SOFT}} + \overbrace{\frac{1}{s_{IK}}\left(\frac{s_{ij}}{s_{jk}} + \frac{s_{jk}}{s_{ij}} \right)}^{\text{COLLINEAR}} \right]$$

$$\frac{|\mathcal{M}(H^0 \to q_i g_j \bar{q}_k)|^2}{|\mathcal{M}(H^0 \to q_I \bar{q}_K)|^2} = g_s^2\, 2C_F \left[\underbrace{\frac{2s_{ik}}{s_{ij}s_{jk}}}_{\text{SOFT}} + \underbrace{\frac{1}{s_{IK}}\left(\frac{s_{ij}}{s_{jk}} + \frac{s_{jk}}{s_{ij}} \right.}_{\text{COLLINEAR}} \underbrace{\left. + 2 \right)}_{\text{+F}} \right]$$

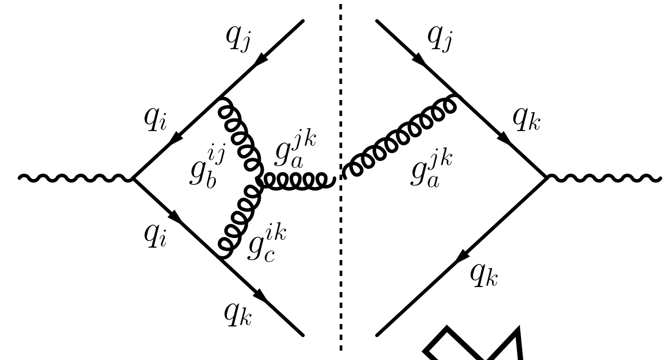
Structure of σ(NNLO)

NNLO

1-Loop × 1-Loop



1-Loop × Real (X+1)



$X^{(2)}$	$X+1^{(2)}$	…
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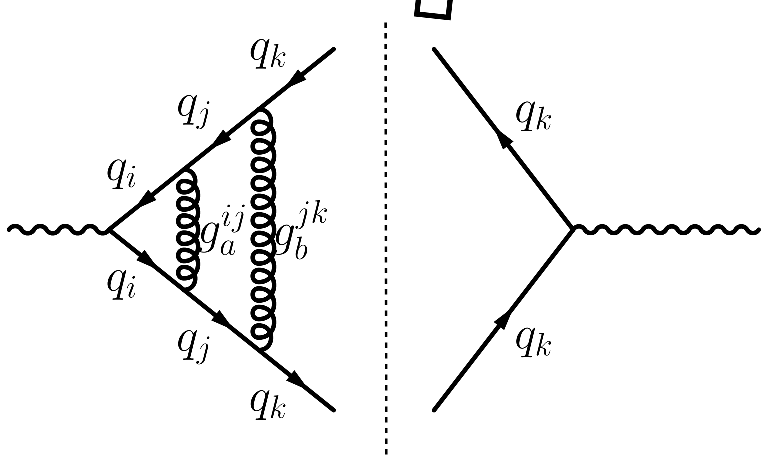
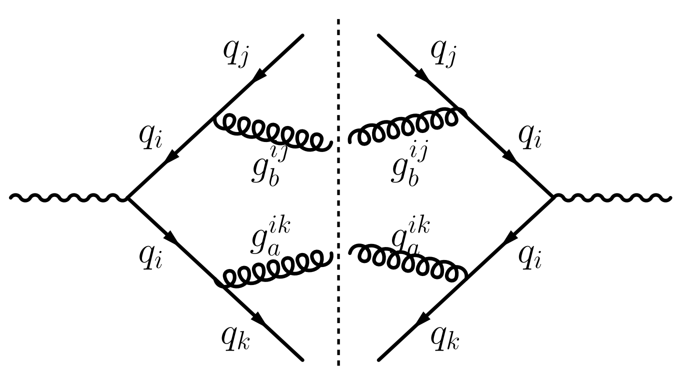
Born	$X+1^{(0)}$	$X+2^{(0)}$

$$\sigma_X^{\mathrm{NNLO}} = \sigma_X^{\mathrm{NLO}} + \int \left(|M_X^{(1)}|^2 + 2\mathrm{Re}[M_X^{(2)} M_X^{(0)*}] \right) + \int 2\mathrm{Re}[M_{X+1}^{(1)} M_{X+1}^{(0)*}] + \int |M_{X+2}^{(0)}|^2$$



Two-Loop × Born Interference



Real × Real (X+2)

QCD

Lecture II

Recall: Scale Dependence

Regularization of IR and UV divergences

E.g., using dimensional reg → unphysical scales μ_F and μ_R

These are artificial scales **we** introduce, to handle the calculation

At infinite order (+ non-pert), all dependence on them must vanish

But in a fixed-order truncation, a residual dependence remains

Recall: Scale Dependence

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E.g., using dimensional reg → unphysical scales μ_F and μ_R

These are artificial scales **we** introduce, to handle the calculation

At infinite order (+ non-pert), all dependence on them must vanish

But in a fixed-order truncation, a residual dependence remains

Why scale variation ~ uncertainty?

Any scale dependence of calculated orders must be canceled by the contribution from the uncalculated orders (+ non-pert)

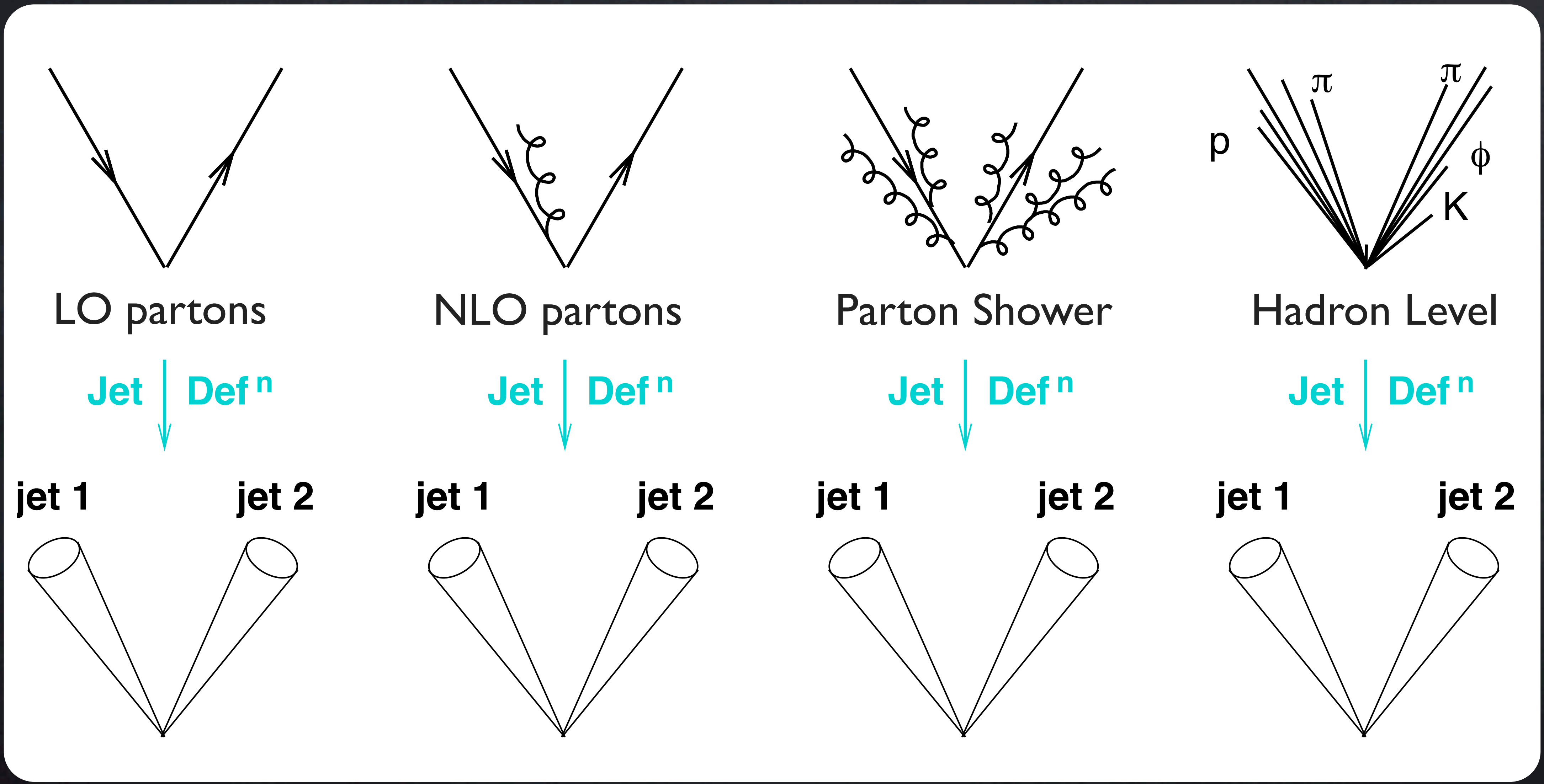
$$\left(\alpha_s(Q'^2) - \alpha_s(Q^2)\right) |M|^2 = \alpha_s^2(Q^2)|M|^2 + \ldots$$

Next order must include *at least* a term that exactly cancels this one (in addition to whatever *other* terms the next order also contains)

Jets

Image Credits: Richard Seaman

Jets as Projections



Projections to jets provides a universal view of event

Illustrations by G. Salam

There is no unique or “best” jet definition

YOU decide how to look at event

The construction of jets is inherently ambiguous

1. *Which particles get grouped together?*

   JET ALGORITHM (+ parameters)

2. *How will you combine their momenta?*

   RECOMBINATION SCHEME (e.g., ‘E’ scheme: add 4-momenta)

Jet Definition

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    RECOMBINATION SCHEME (e.g., ‘E’ scheme: add 4-momenta)

⇨ **Jet Definition**

Ambiguity complicates life, but gives flexibility in one’s view of events → Jets non-trivial!

Types of Algorithms

1. Sequential Recombination

(you'll hear more in lectures on Jet Substructure)

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Take your 4-vectors. Combine the ones that have the lowest 'distance measure'

Different names for different distance measures

Durham k_T : $\Delta R_{ij}^2 \times \min(k_{Ti}^2, k_{Tj}^2)$

Cambridge/Aachen : ΔR_{ij}^2

Anti-k_T : $\Delta R_{ij}^2 / \max(k_{Ti}^2, k_{Tj}^2)$

ArClus $(3\to2)$: $p_\perp^2 = s_{ij} s_{jk} / s_{ijk}$

$$k_{Ti}^2 = E_i^2(1 - \cos\theta_{ij})$$

$$\Delta R_{ij}^2 = (\eta_i - \eta_j)^2 + \Delta\phi_{ij}^2$$

+ Prescription for how to combine 2 momenta into 1

(or 3 momenta into 2)

→ New set of (n-1) 4-vectors

QCD

Lecture II

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Iterate until A or B (you choose which):

A: all distance measures larger than something

B: you reach a specified number of jets

Look at event at:

specific resolution

specific n_{jets}

Why k_T (or p_T or ΔR)?

Attempt to (approximately) capture universal jet-within-jet-witin-jet... behavior

Approximate full matrix element

$$\frac{|M^{(0)}_{X+1}(s_{i1}, s_{1k}, s)|^2}{|M^{(0)}_X(s)|^2} = 4\pi\alpha_s C_F \left(\frac{2s_{ik}}{s_{i1}s_{1k}} + \ldots \right)$$

"Eikonal"
(universal, always there)

by Leading-Log limit of QCD → universal dominant terms

$$\frac{\mathrm{d}s_{i1}\mathrm{d}s_{1k}}{s_{i1}s_{1k}} \to \frac{\mathrm{d}p_\perp^2}{p_\perp^2}\frac{\mathrm{d}z}{z(1-z)} \to \frac{\mathrm{d}E_1}{\min(E_i, E_1)}\frac{\mathrm{d}\theta_{i1}}{\theta_{i1}} \quad (E_1 \ll E_i,\ \theta_{i1} \ll 1) \qquad ,\ldots$$

Rewritings in soft/collinear limits

"smallest" k_T (or p_T or θ_{ij}, or …) → largest Eikonal

Types of Algorithms

2. "Cone" type

Take your 4-vectors. Select a procedure for which "test cones" to draw

- Different names for different procedures
  - Seeded : start from hardest 4-vectors (and possibly combinations thereof, e.g., CDF midpoint algorithm) = "seeds"
  - Unseeded : smoothly scan over entire event, trying everything
- Sum momenta inside test cone → new test cone direction
- Iterate until stable (test cone direction = momentum sum direction)

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Warning: seeded algorithms are INFRARED UNSAFE

Infrared Safety

Definition

An observable is infrared safe if it is <u>in</u>sensitive to

SOFT radiation:

Adding any number of infinitely soft particles (zero-energy) should not change the value of the observable

COLLINEAR radiation:

Splitting an existing particle up into two comoving particles (conserving the total momentum and energy) should not change the value of the observable

Safe vs Unsafe Jets

May look pretty similar in experimental environment …

But it's not nice to your theory friends …

Unsafe: badly divergent in pQCD → large IR corrections:

$$\text{IR Sensitive Corrections} \quad \propto \quad \alpha_s^n \log^m \left(\frac{Q^2_{\mathrm{UV}}}{Q^2_{\mathrm{IR}}} \right) \quad , \quad m \leq 2n$$

Even if we have a hadronization model with which to compute these corrections, the dependence on it → larger uncertainty

Safe → IR corrections power suppressed:

$$\text{IR Safe Corrections} \quad \propto \quad \frac{Q^2_{\mathrm{IR}}}{Q^2_{\mathrm{UV}}}$$

Can still be computed (MC) but can also be neglected (pure pQCD)

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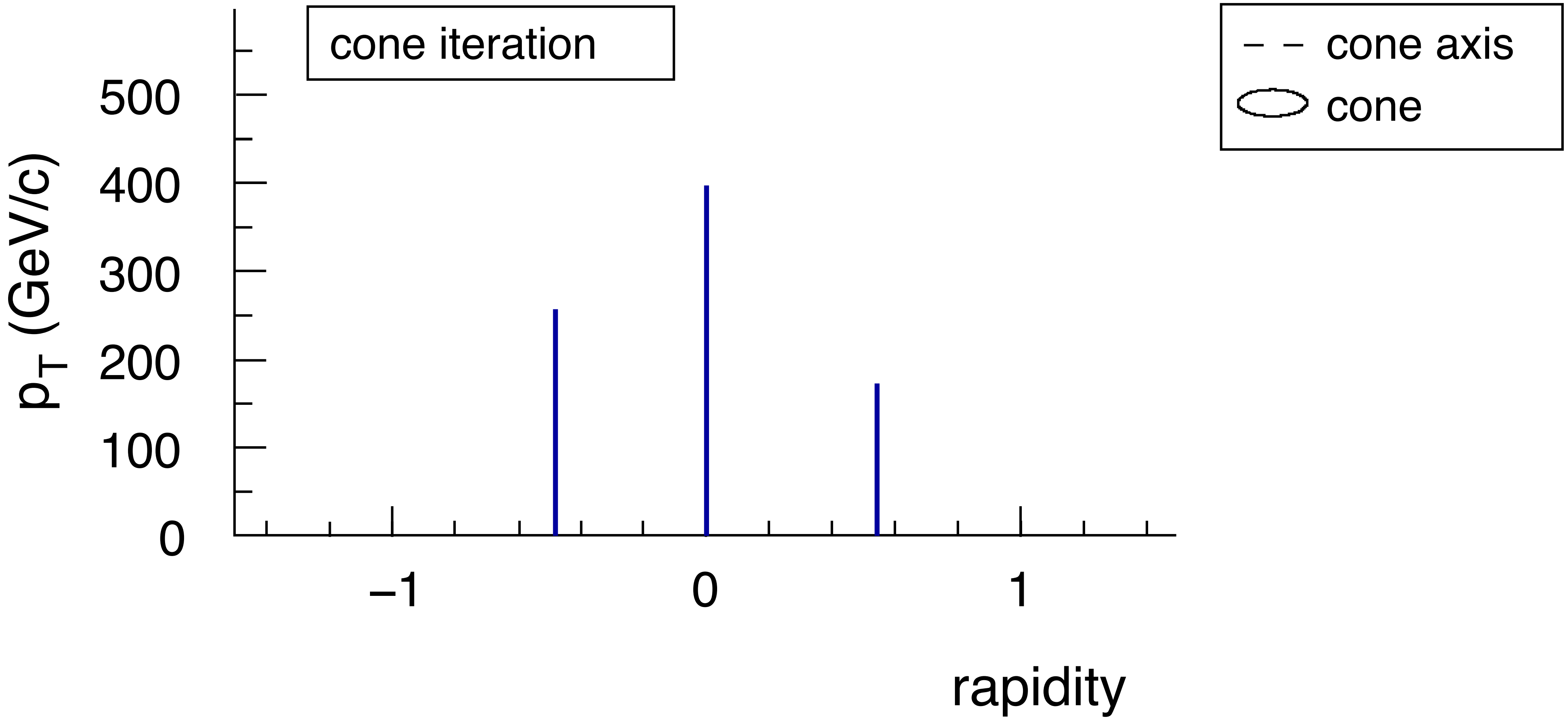
Let's look at a specific example …

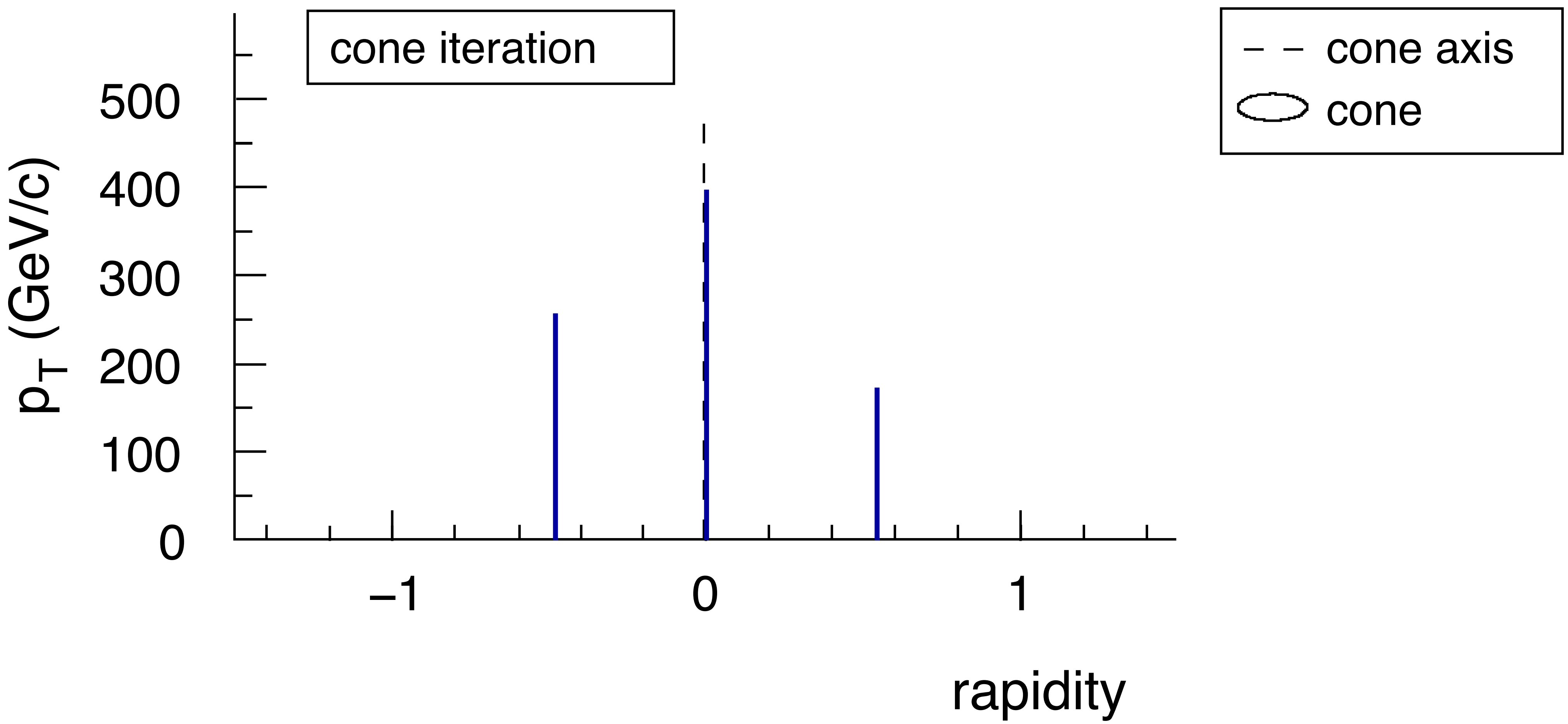
ICPR iteration issue

Iterative Cone Progressive Removal



Collinear splitting can modify the hard jets: ICPR algorithms are collinear unsafe $\Longrightarrow$ perturbative calculations give ∞

Slides from G. Salam



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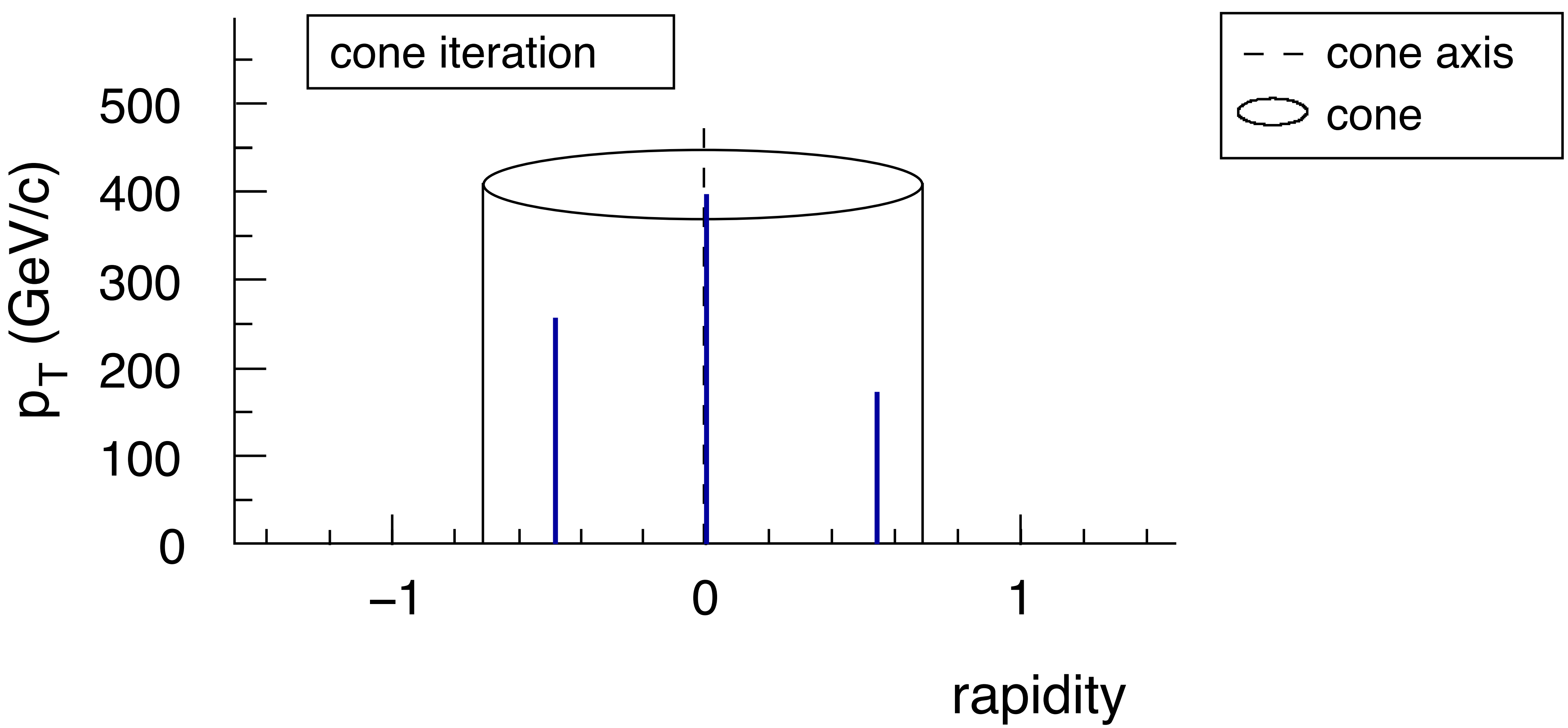
ICPR iteration issue

Iterative Cone Progressive Removal



Collinear splitting can modify the hard jets: ICPR algorithms are collinear unsafe $\Longrightarrow$ perturbative calculations give ∞

Slides from G. Salam

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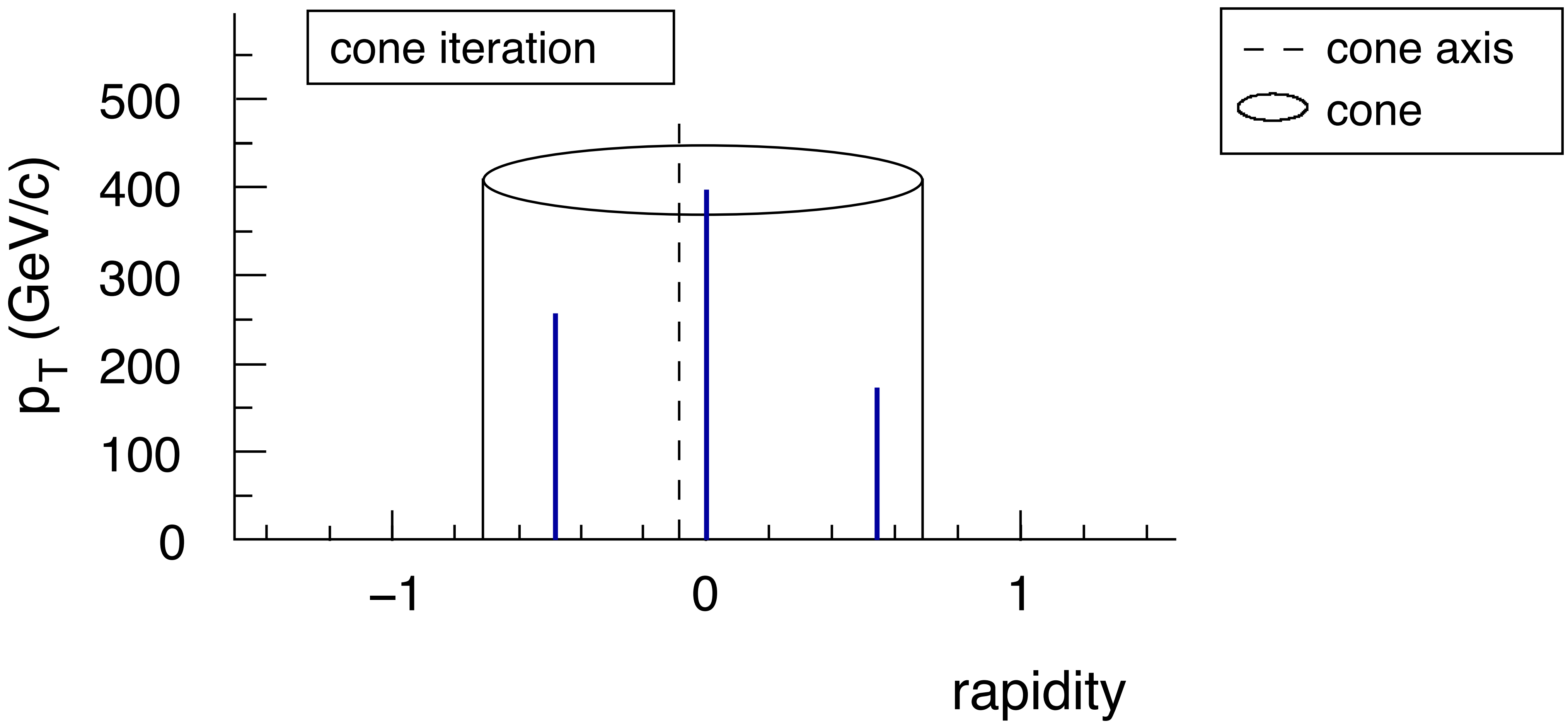

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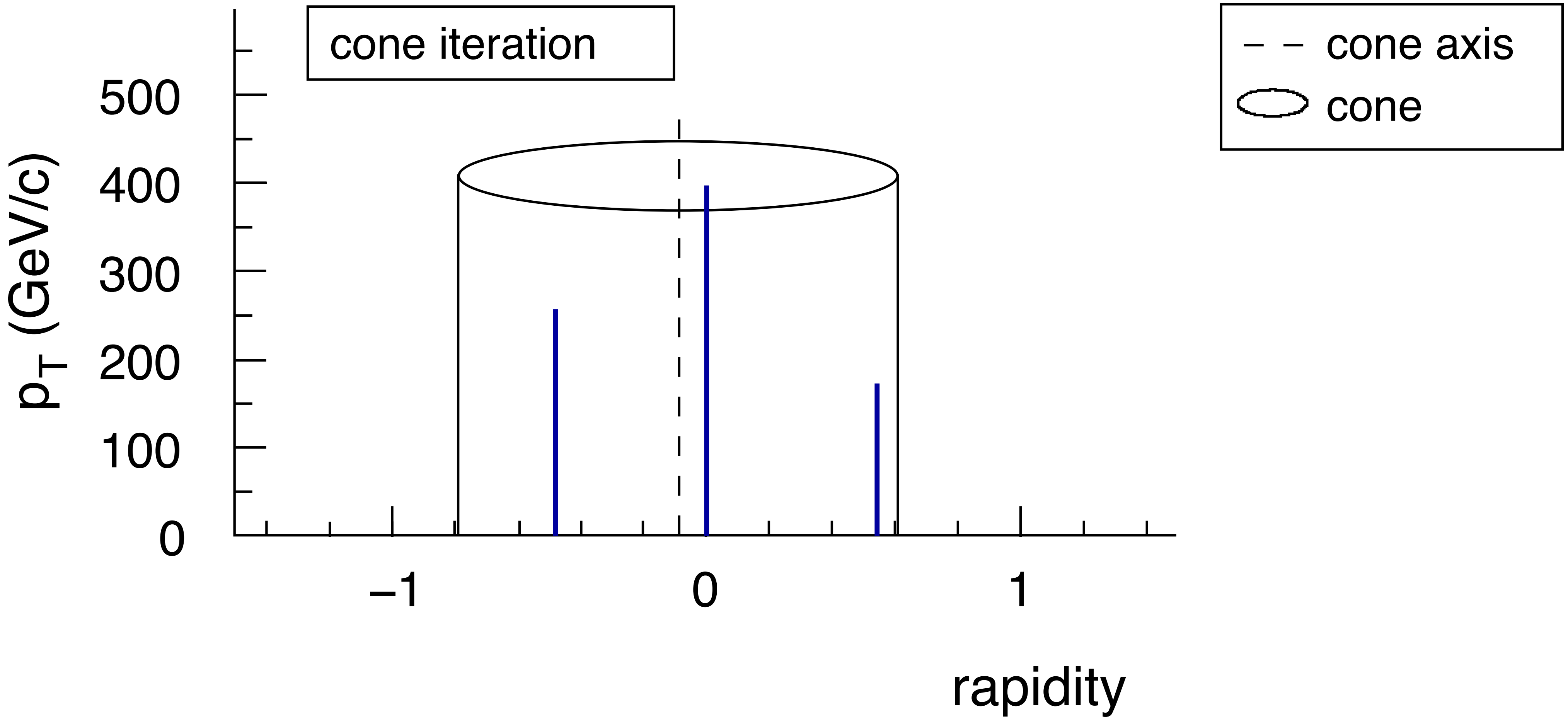

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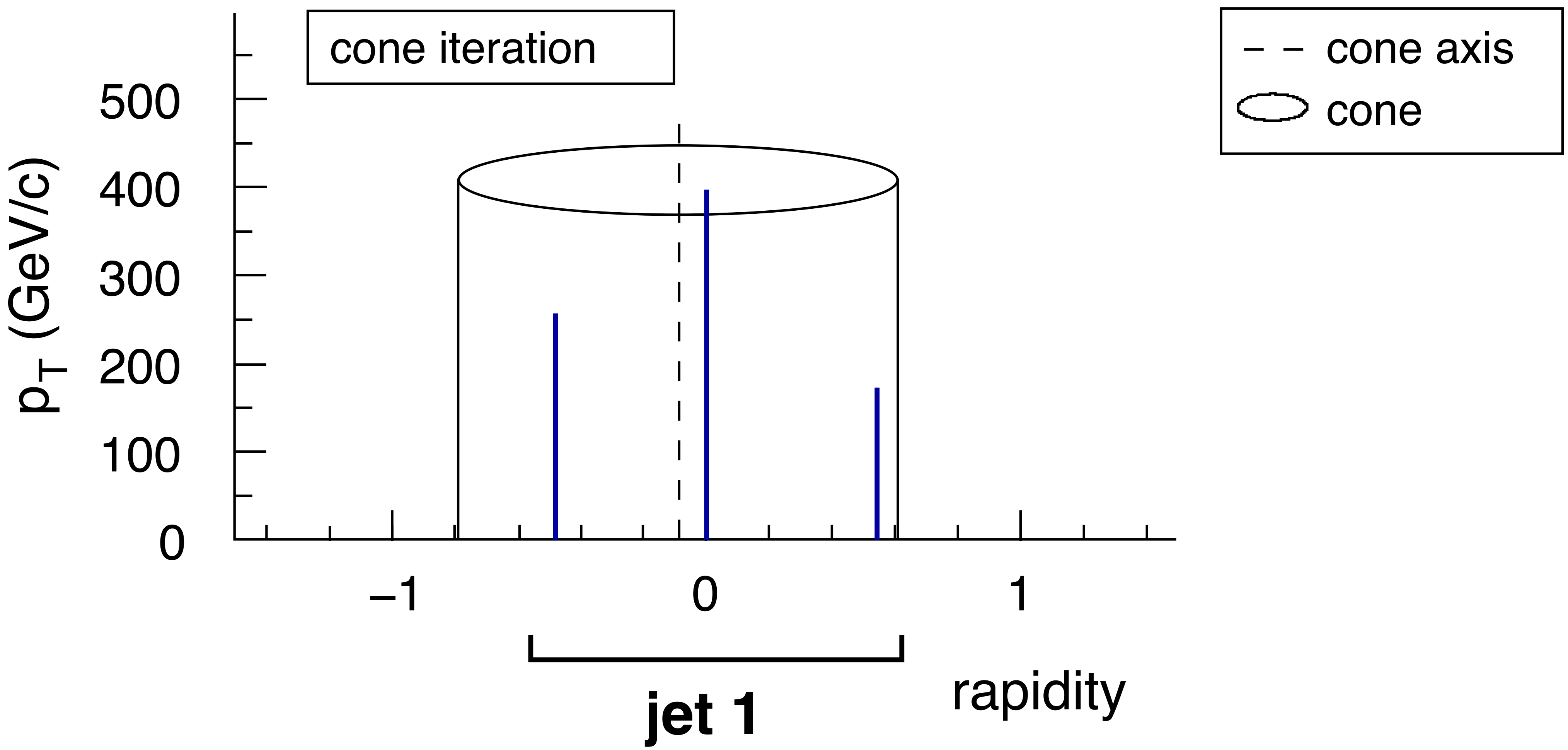

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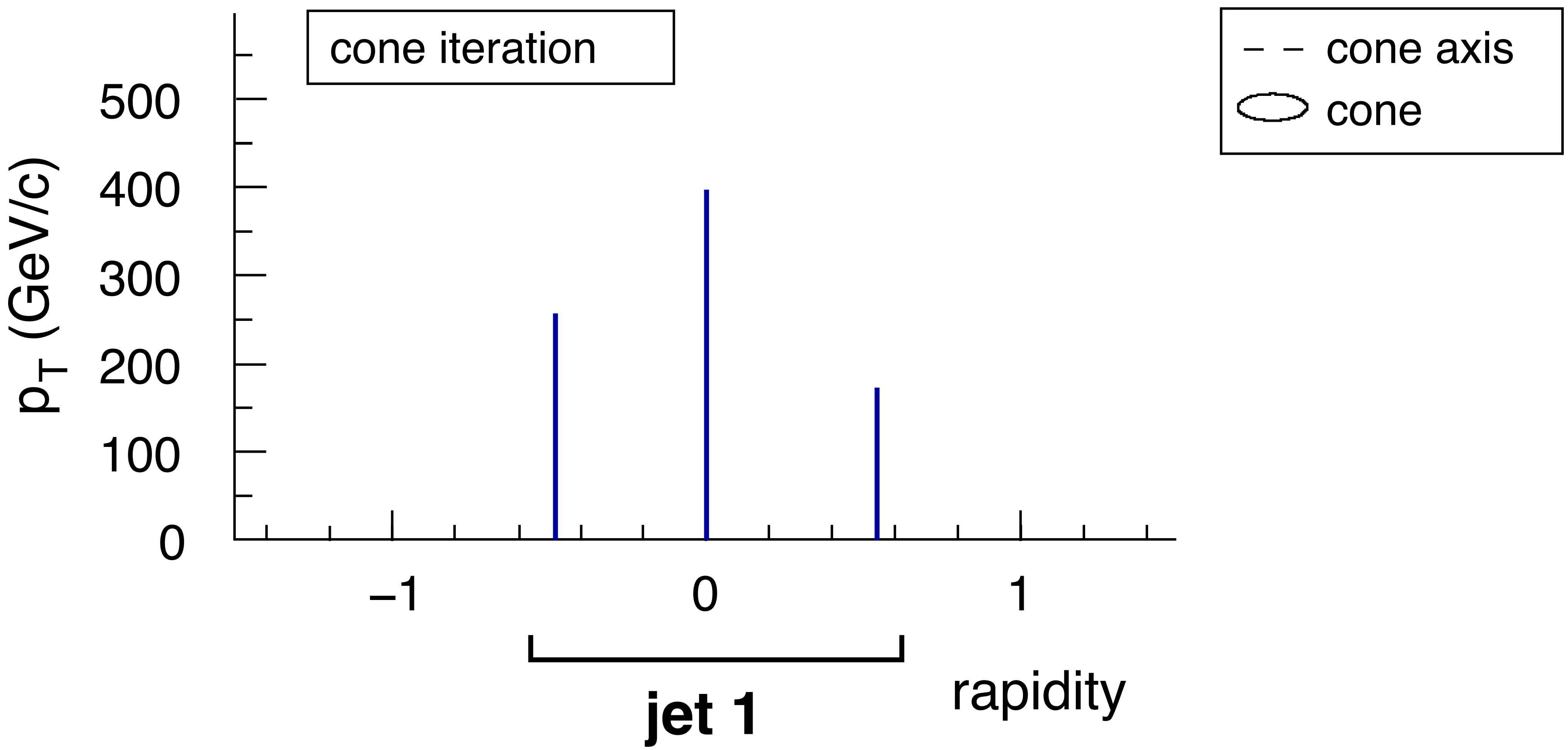

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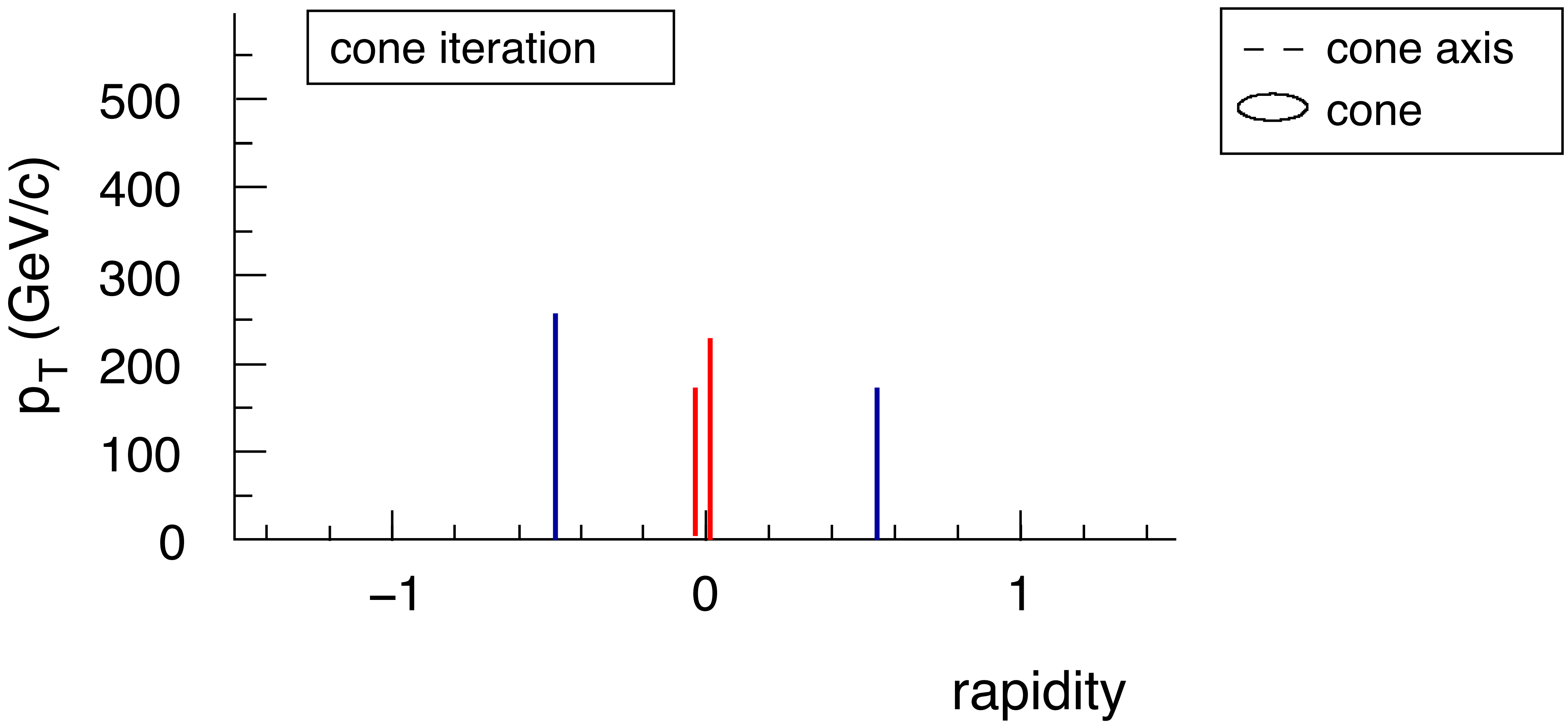

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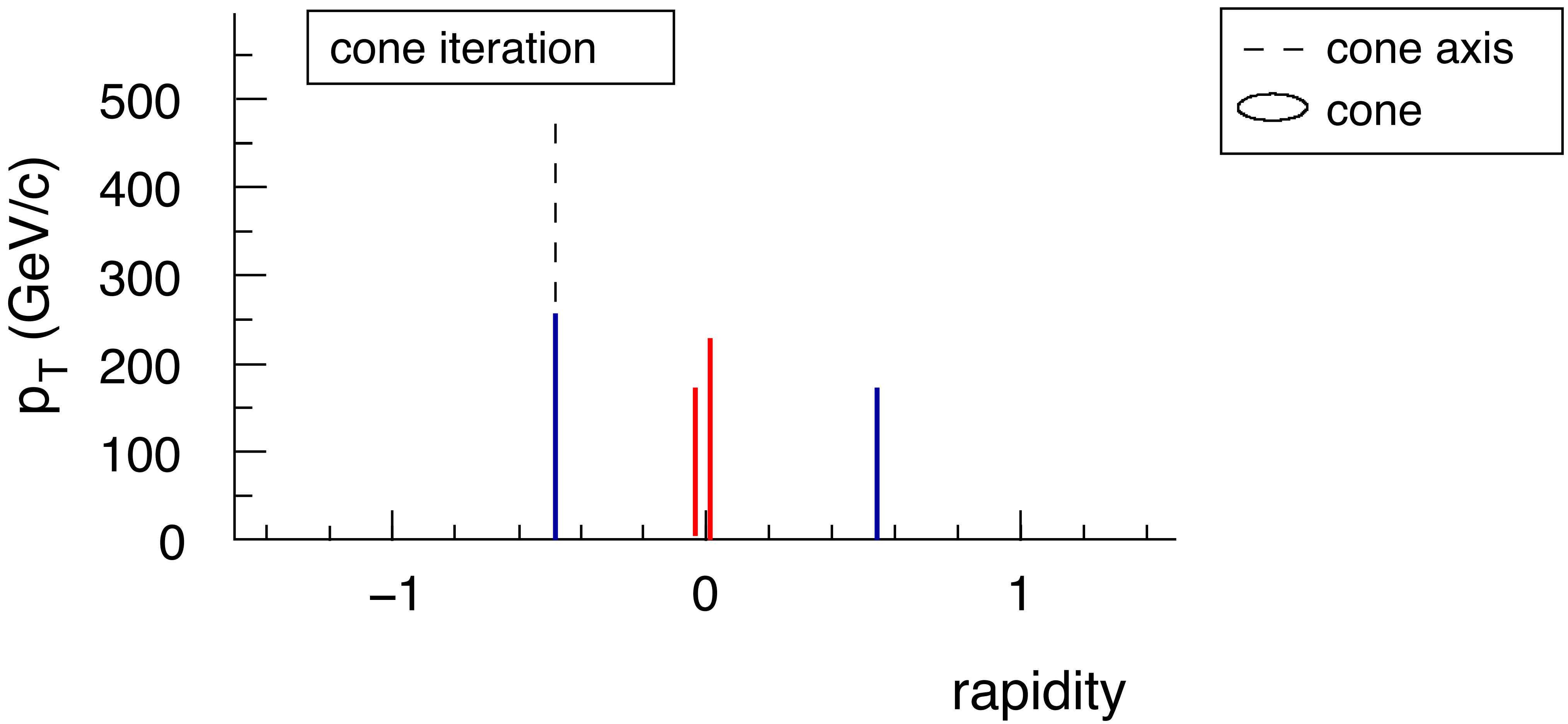

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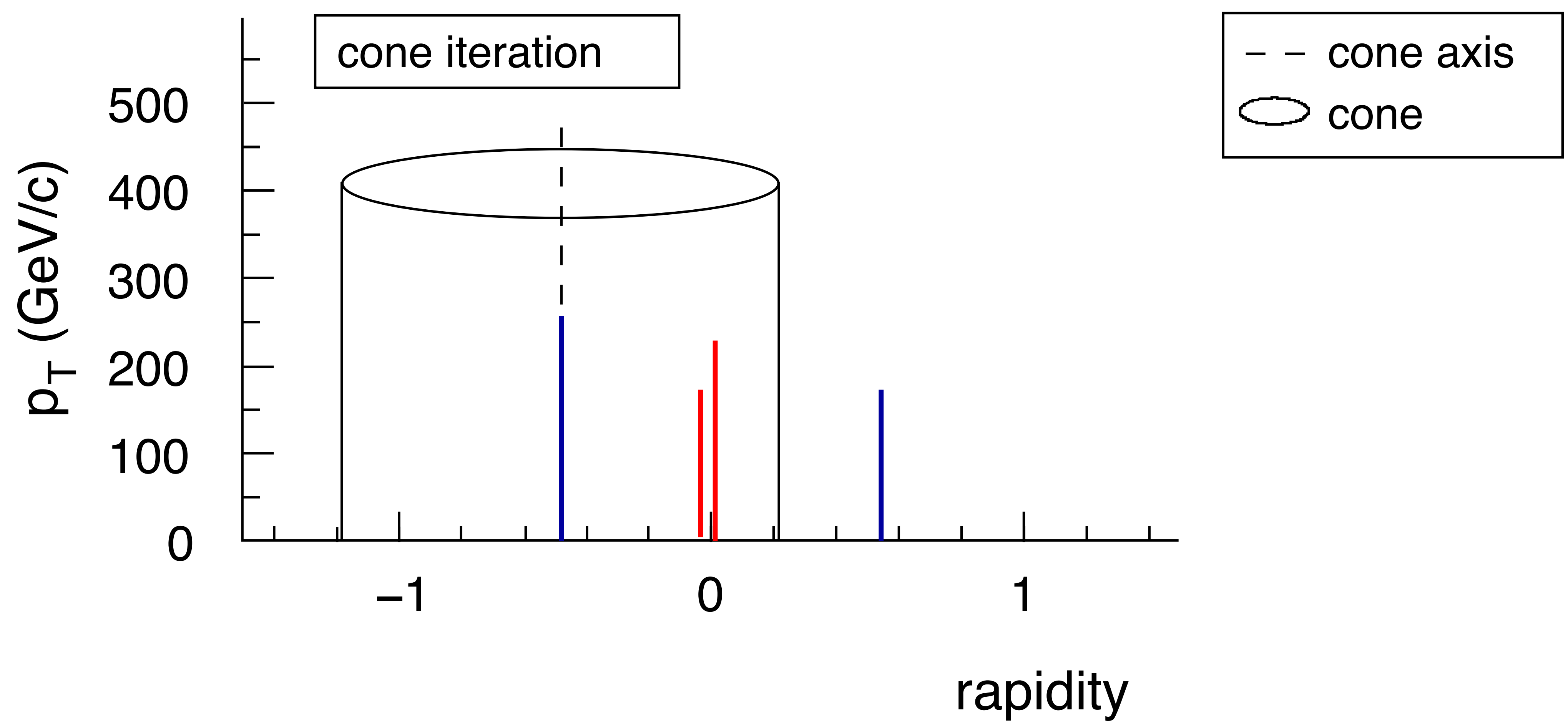

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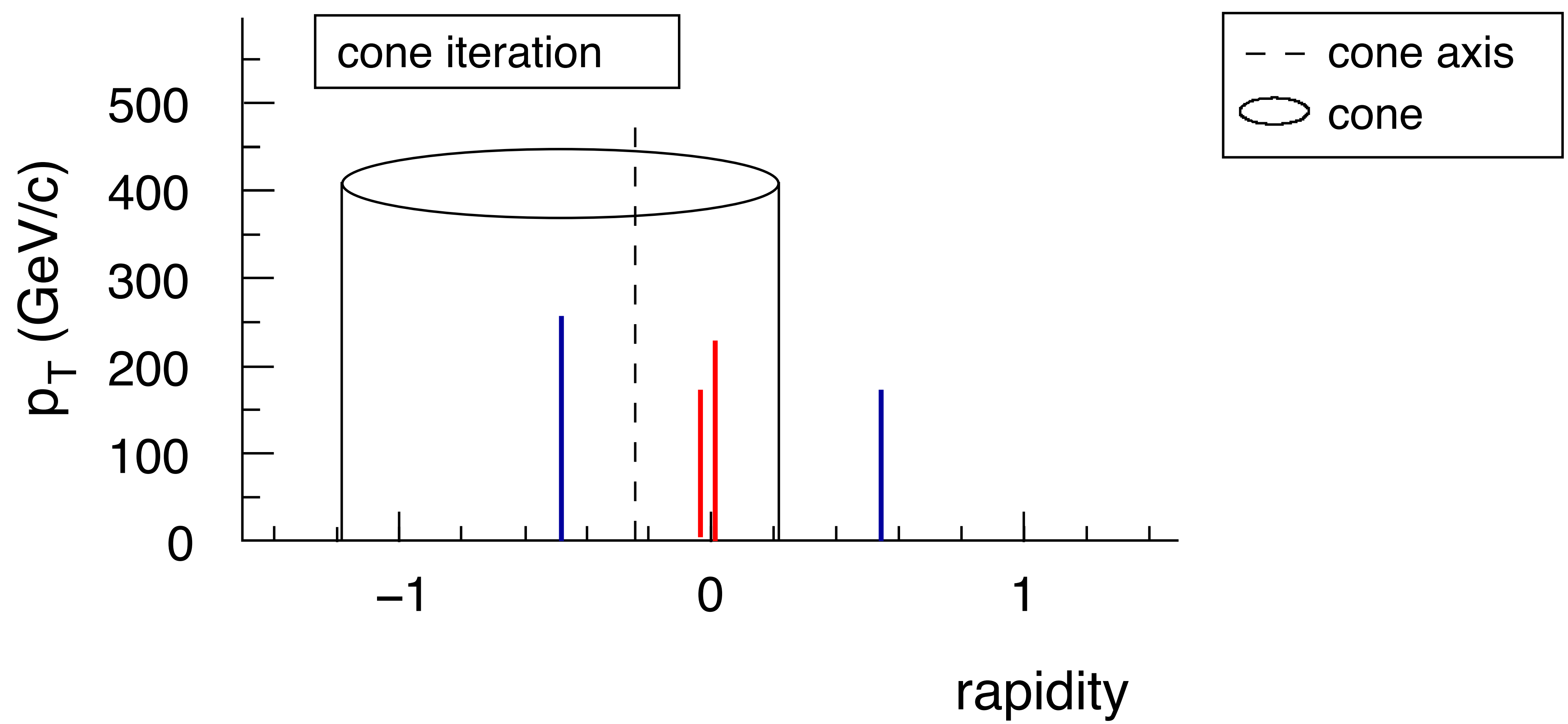

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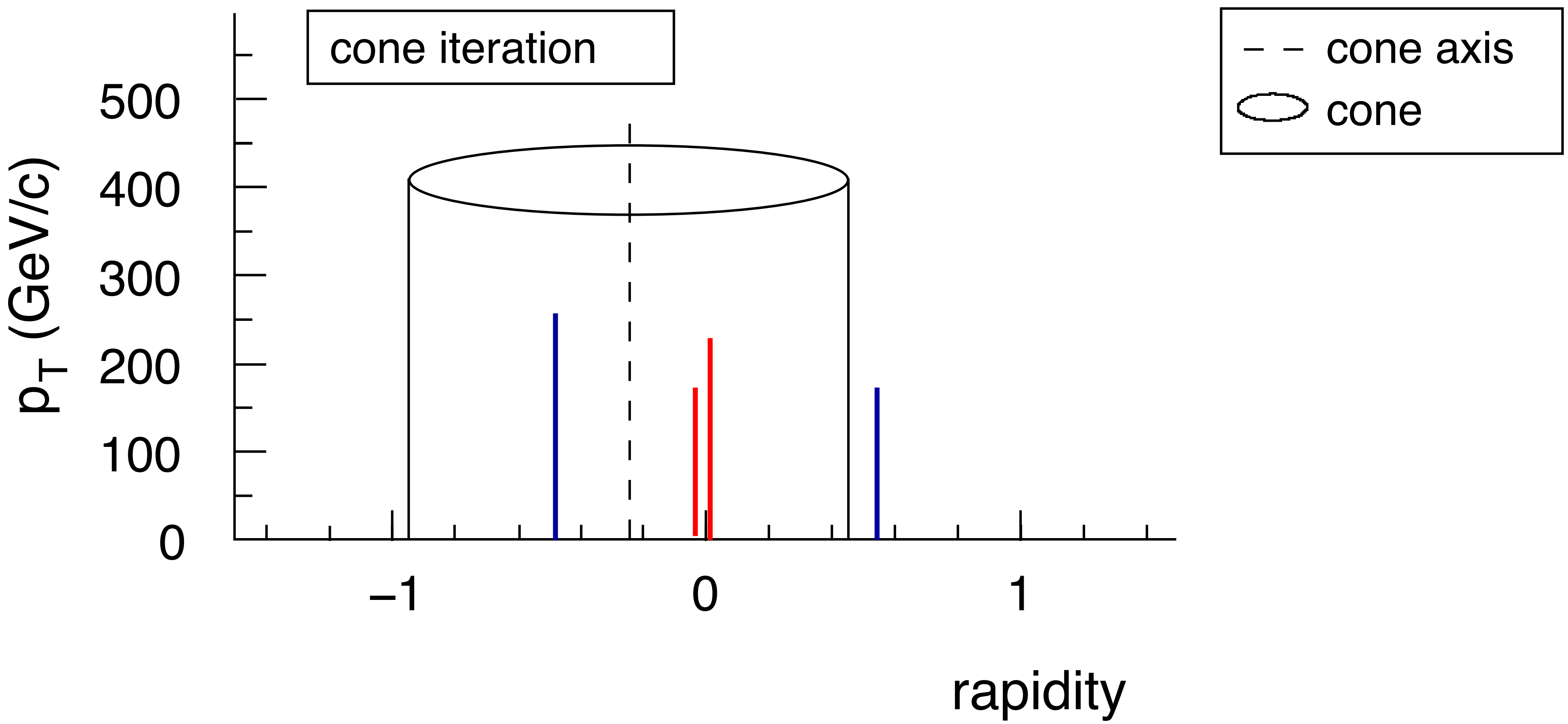

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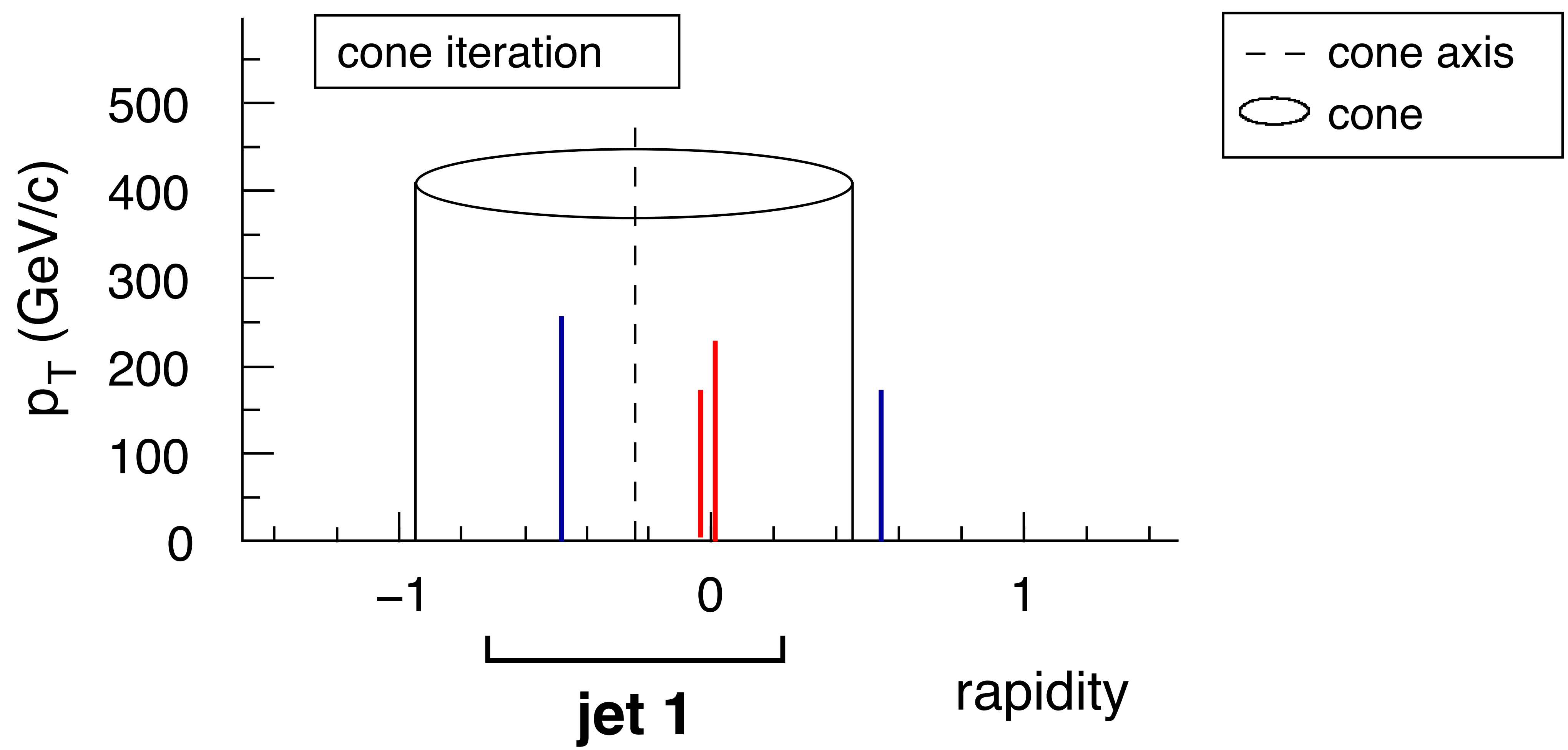

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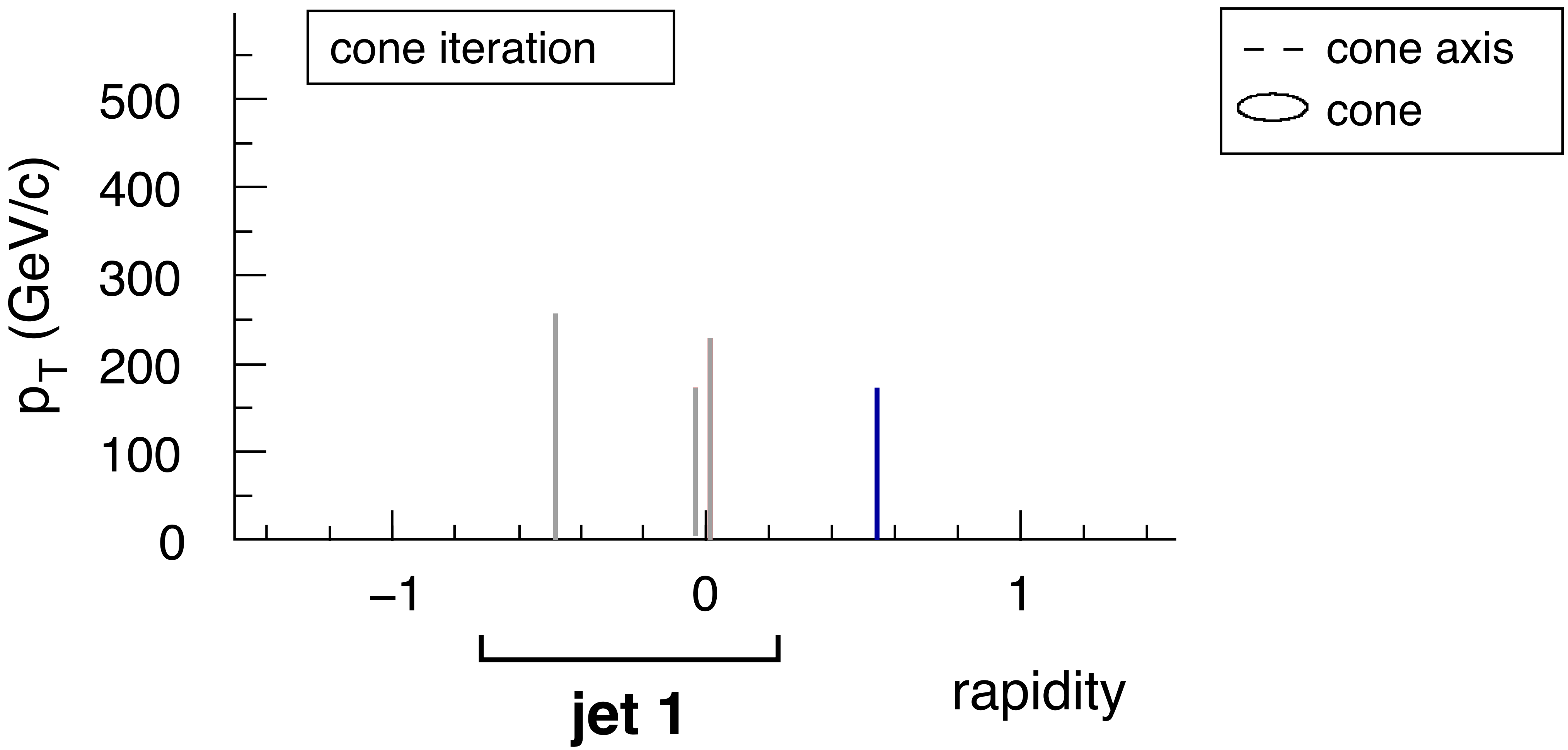

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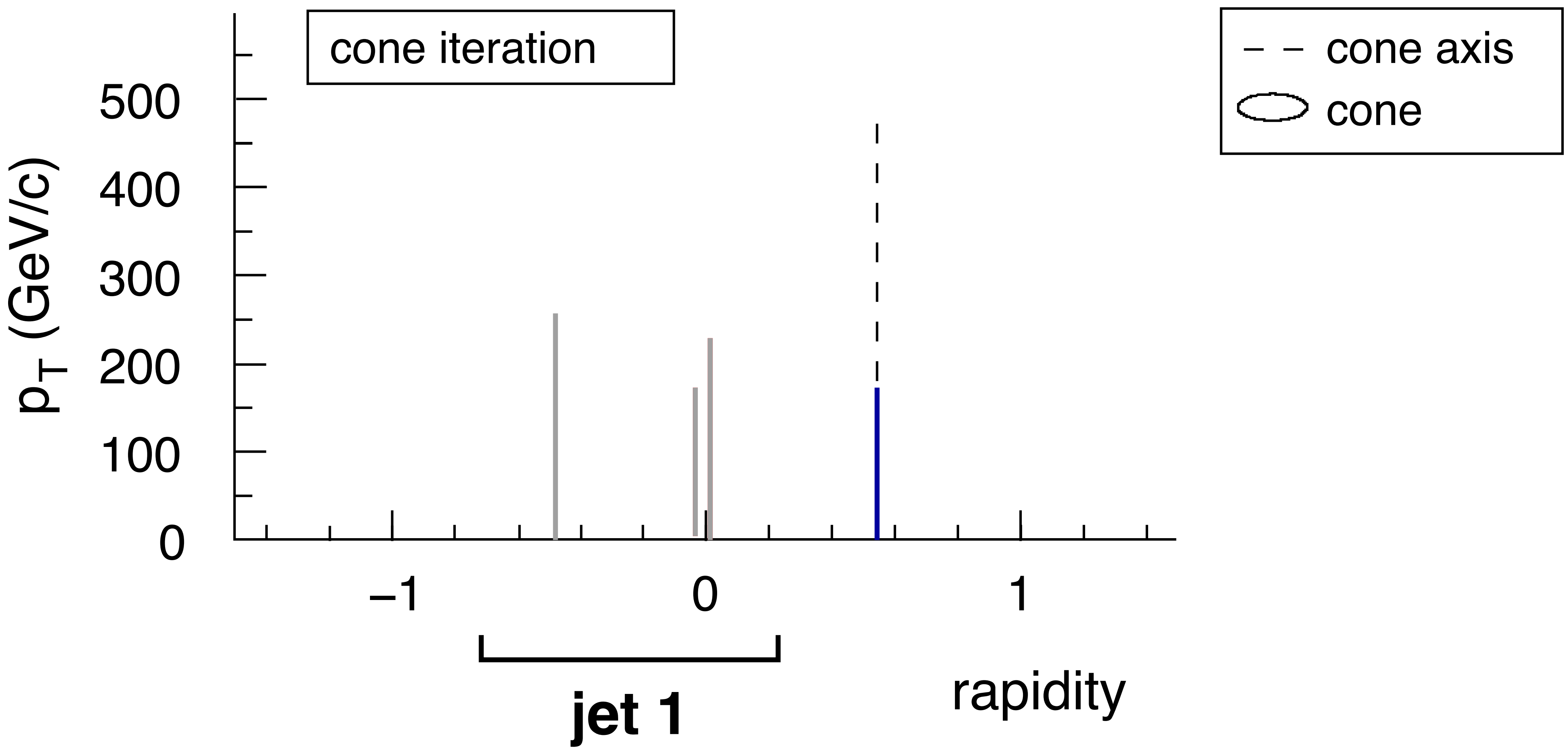

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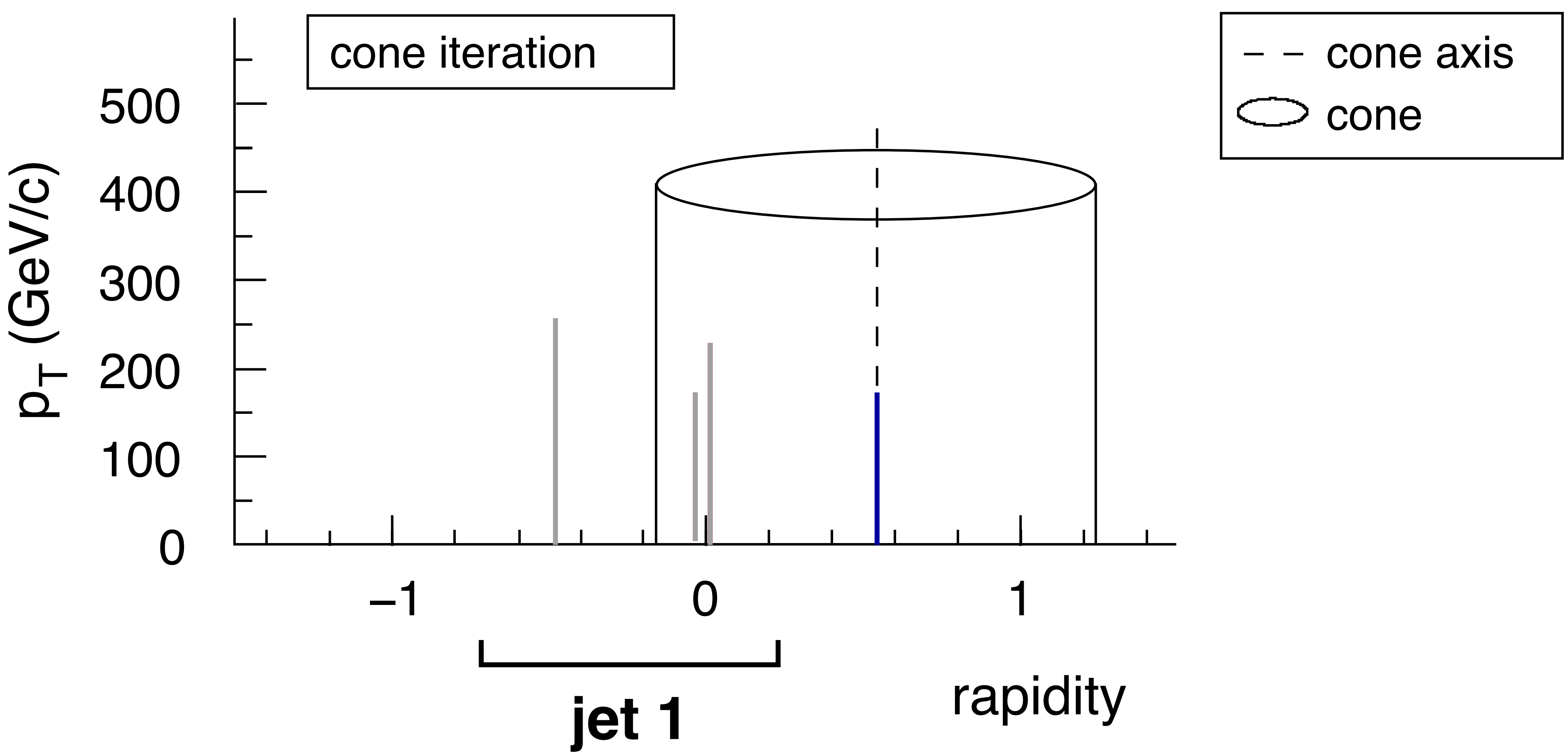

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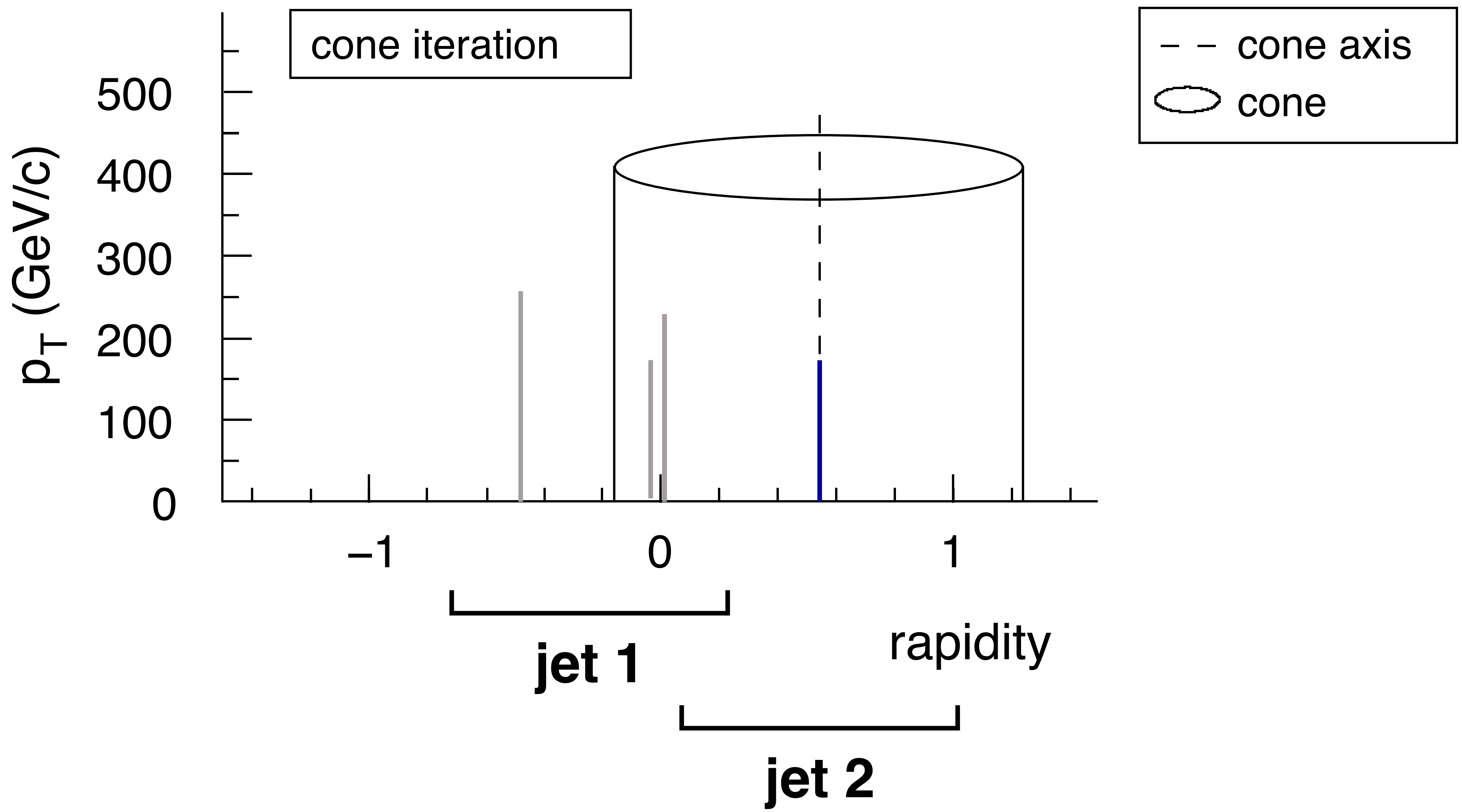

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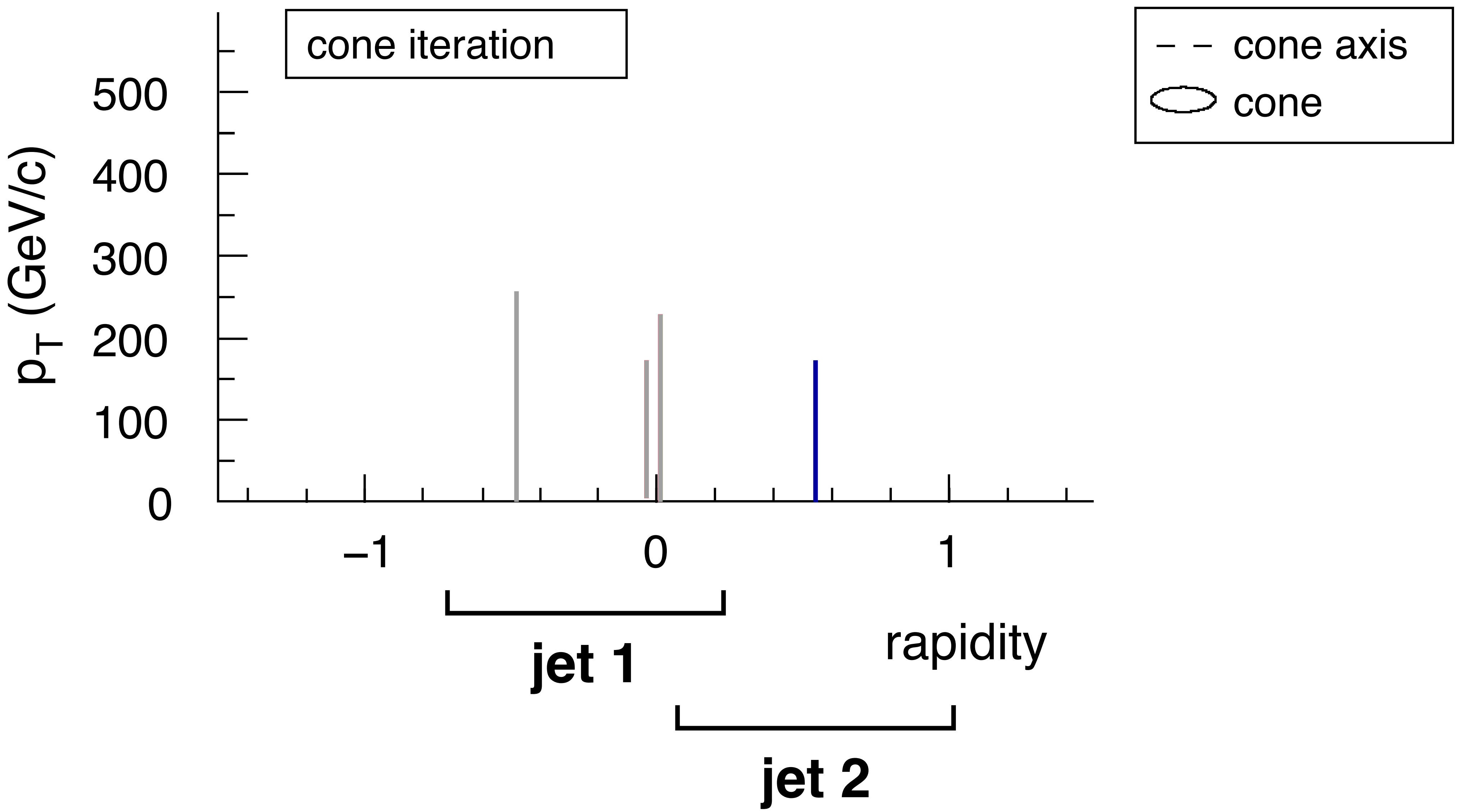

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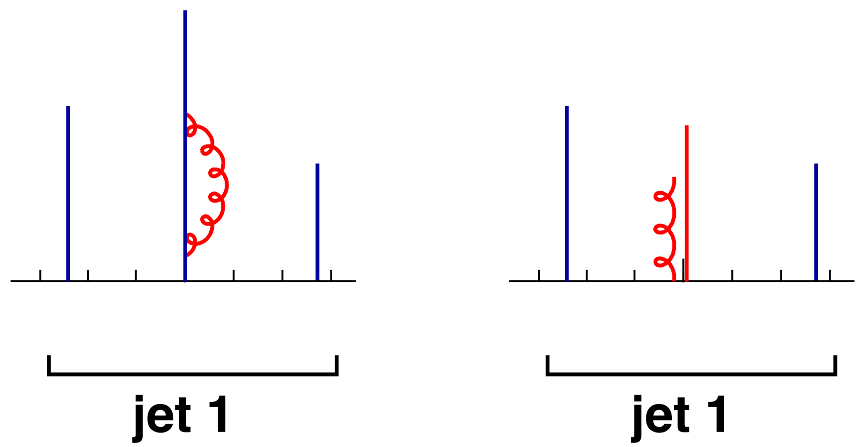
Consequences of Collinear Unsafety

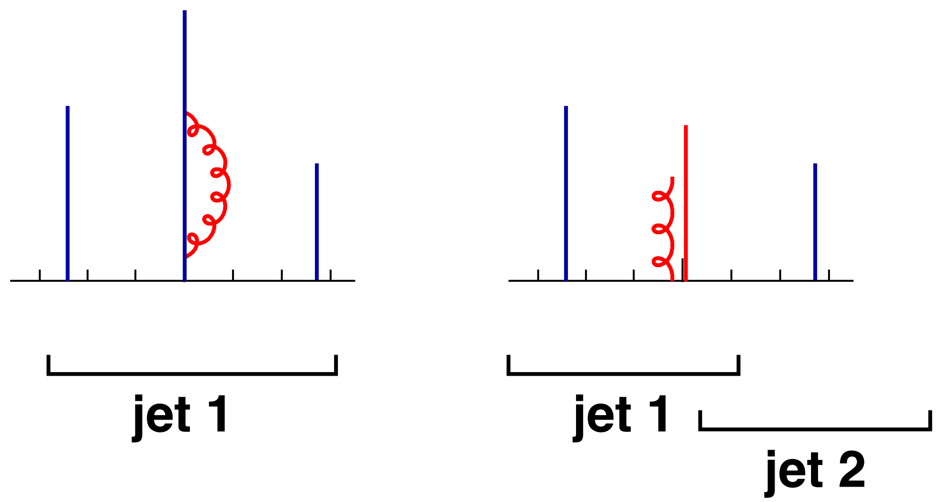
Collinear Safe



α_s^n x $(-\infty)$ α_s^n x $(+\infty)$

Infinities cancel

Collinear Unsafe



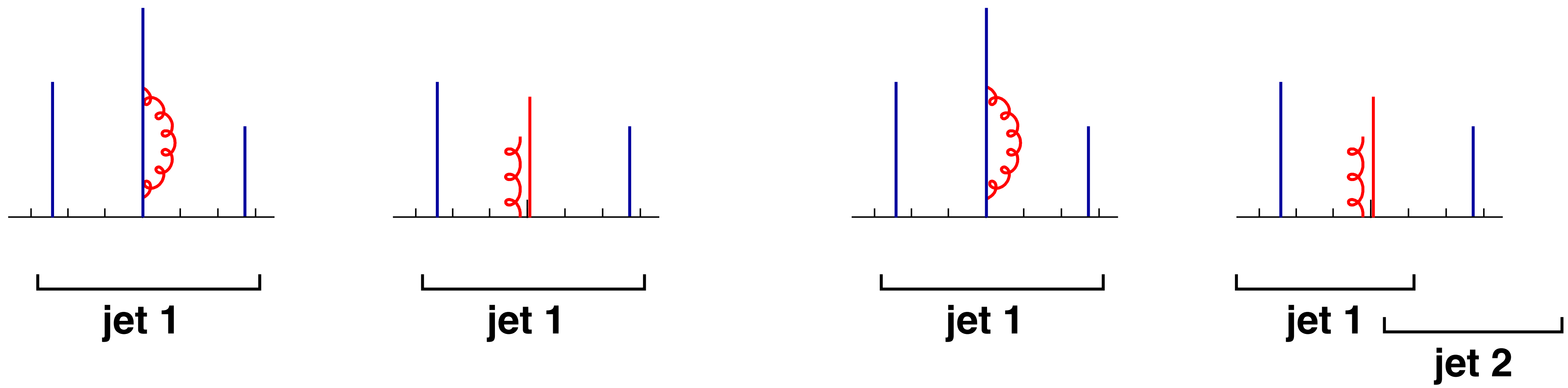
α_s^n x $(-\infty)$ α_s^n x $(+\infty)$

Infinities do not cancel

Invalidates perturbation theory

Consequences of Collinear Unsafety

Collinear Safe | **Collinear Unsafe**



$\alpha_s^n \times (-\infty)$ $\alpha_s^n \times (+\infty)$ $\alpha_s^n \times (-\infty)$ $\alpha_s^n \times (+\infty)$

Infinities cancel

Infinities do not cancel

Invalidates perturbation theory

Real life does not have infinities, but pert. infinity leaves a real-life trace

$$\alpha_s^2 + \alpha_s^3 + \alpha_s^4 \times \infty \to \alpha_s^2 + \alpha_s^3 + \alpha_s^4 \times \ln p_t/\Lambda \to \alpha_s^2 + \underbrace{\alpha_s^3 + \alpha_s^3}_{\text{BOTH WASTED}}$$

Stereo Vision

Use IR Safe algorithms

http://www.fastjet.fr/

To study short-distance physics

These days, ≈ as fast as IR unsafe algos and widely implemented (e.g., FASTJET), including

"Cone-like": SiSCone, Anti-k_T, …

"Recombination-like": k_T, Cambridge/Aachen, Anti-k_T…

Then use IR Sensitive observables

E.g., number of tracks, identified particles, …

To explicitly check hadronization and other IR models

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More about IR in lecture on soft QCD …

Introduction to QCD

1. **Fundamentals of QCD**
2. **Jets and Fixed-Order QCD**
3. **Monte Carlo Generators and Parton Showers**
4. **Matching at LO and NLO**
5. **QCD in the Infrared**

Note: slides posted at:
www.cern.ch/skands/slides

Supplementary Slides

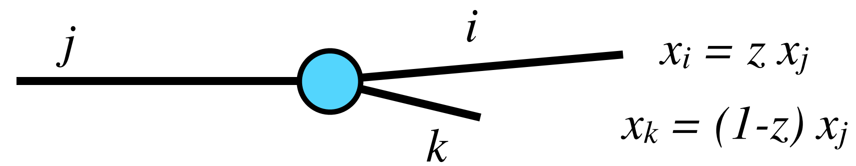
Evolution in Q^2 by DGLAP

(Dokshitzer-Gribov-Lipatov-Altarelli-Parisi)

Require cross section independent of μ_F (at calculated order) → RGE

$$\frac{df_i(x_i,\mu_F^2)}{d\ln\mu_F^2} = \sum_j \int_{x_i}^{1} \frac{dx_j}{x_j} f_j(x_j,\mu_F^2) \frac{\alpha_s}{2\pi} P_{j\to ik}\left(\frac{x_i}{x_j}\right)$$

Altarelli-Parisi
Splitting Kernels



$$\begin{aligned}
P_{\mathrm{q}\to\mathrm{qg}}(z) &= C_F \frac{1+z^2}{1-z} \ , \\
P_{\mathrm{g}\to\mathrm{gg}}(z) &= N_C \frac{(1-z(1-z))^2}{z(1-z)} \ , \\
P_{\mathrm{g}\to\mathrm{q\bar{q}}}(z) &= T_R\,(z^2+(1-z)^2) \ , \\
P_{\mathrm{q}\to\mathrm{q}\gamma}(z) &= e_{\mathrm{q}}^2 \frac{1+z^2}{1-z} \ , \\
P_{\ell\to\ell\gamma}(z) &= e_\ell^2 \frac{1+z^2}{1-z} \ ,
\end{aligned}$$

Note: to be used directly in above equation, these splitting kernels should be defined with the "plus prescription"

For example:

$$\frac{1+z^2}{1-z} \to \left(\frac{1+z^2}{1-z}\right)_+$$

Reason for this covered in next two slides

PDF DGLAP : Details

We Wrote:

$$\frac{dq(x,\mu^2)}{d\ln\mu^2} = \frac{\alpha_s}{2\pi}\underbrace{\int_x^1 dz\, P_{qq}(z)\,\frac{q(x/z,\mu^2)}{z}}_{P_{qq}\otimes q},$$

More properly, it's a gain-loss equation (same equation, rewritten):

$$\frac{dq(x,\mu^2)}{d\ln\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 dz\, p_{qq}(z)\,\frac{q(x/z,\mu^2)}{z} - \frac{\alpha_s}{2\pi}\int_0^1 dz\, p_{qq}(z)\, q(x,\mu^2)$$

p_{qq} is real $q \leftarrow q$ **splitting kernel:** $p_{qq}(z) = C_F\frac{1+z^2}{1-z}$

First term: some partons flow from higher y=x/z to x (POSITIVE)
Second term: some partons at x flow to lower y=zx (NEGATIVE)

How can they be the same equation?

PDF DGLAP : Details

Awkward to write real and virtual parts separately. Use more compact notation:

$$\frac{dq(x,\mu^2)}{d\ln\mu^2} = \frac{\alpha_s}{2\pi}\underbrace{\int_x^1 dz\, P_{qq}(z)\, \frac{q(x/z,\mu^2)}{z}}_{P_{qq}\otimes q}, \qquad P_{qq} = C_F\left(\frac{1+z^2}{1-z}\right)_+$$

This involves the *plus prescription:*

$$\int_0^1 dz\,[g(z)]_+\, f(z) = \int_0^1 dz\, g(z)\, f(z) - \int_0^1 dz\, g(z)\, f(1)$$

$z = 1$ divergences of $g(z)$ cancelled if $f(z)$ sufficiently smooth at $z = 1$