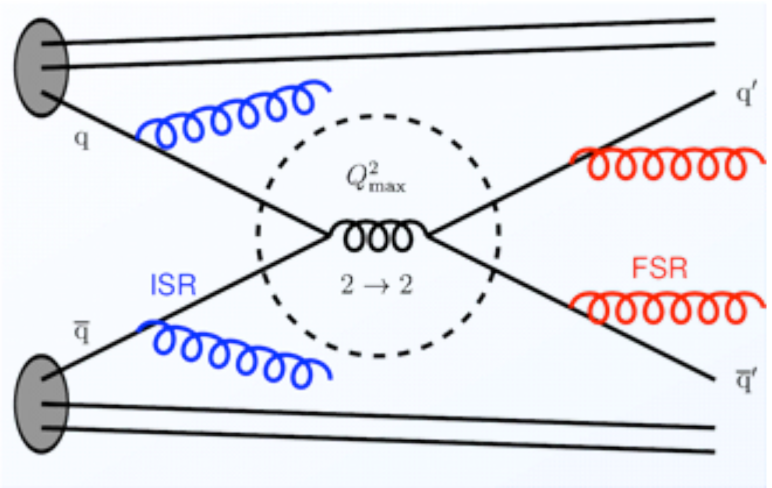
Introduction to QCD

1. Fundamentals of QCD
2. PDFs, Fixed-Order QCD, and Jet Algorithms
3. Parton Showers and Event Generators
4. QCD in the Infrared

Slides posted at:
www.cern.ch/skands/slides

Lecture Notes:
P. Skands, arXiv:1207.2389

General-Purpose Event Generators



Calculate Everything ≈ solve QCD → requires compromise!

Improve lowest-order perturbation theory,
by including the 'most significant' corrections
→ complete events (can evaluate any observable you want)

The Workhorses

PYTHIA : Successor to JETSET (begun in 1978). Originated in hadronization studies: Lund String.
HERWIG : Successor to EARWIG (begun in 1984). Originated in coherence studies: angular ordering.
SHERPA : Begun in 2000. Originated in "matching" of matrix elements to showers: CKKW-L.
+ MORE SPECIALIZED: ALPGEN, MADGRAPH, HELAC, ARIADNE, VINCIA, WHIZARD, (a)MC@NLO, POWHEG, HEJ, PHOJET, EPOS, QGSJET, SIBYLL, DPMJET, LDCMC, DIPSY, HIJING, CASCADE, GOSAM, BLACKHAT, …

(PYTHIA)

PYTHIA anno 1978

(then called JETSET)

LU TP 78-18
November, 1978

A Monte Carlo Program for Quark Jet Generation

T. Sjöstrand, B. Söderberg

A Monte Carlo computer program is presented, that simulates the fragmentation of a fast parton into a jet of mesons. It uses an iterative scaling scheme and is compatible with the jet model of Field and Feynman.

Note:
Field-Feynman was an early fragmentation model
Now superseded by the **String** (in PYTHIA) and **Cluster** (in HERWIG & SHERPA) models.

```
      SUBROUTINE JETGEN(N)
      COMMON /JET/ K(100,2), P(100,5)
      COMMON /PAR/ PUD, PS1, SIGMA, CX2, EBEG, WFIN, IFLBEG
      COMMON /DATA1/ MESO(9,2), CMIX(6,2), PMAS(19)
      IFLSGN=(10-IFLBEG)/5
      W=2.*EBEG
      I=0
      IPD=0
C 1 FLAVOUR AND PT FOR FIRST QUARK
      IFL1=IABS(IFLBEG)
      PT1=SIGMA*SQRT(-ALOG(RANF(0)))
      PHI1=6.2832*RANF(0)
      PX1=PT1*COS(PHI1)
      PY1=PT1*SIN(PHI1)
  100 I=I+1
C 2 FLAVOUR AND PT FOR NEXT ANTIQUARK
      IFL2=1+INT(RANF(0)/PUD)
      PT2=SIGMA*SQRT(-ALOG(RANF(0)))
      PHI2=6.2832*RANF(0)
      PX2=PT2*COS(PHI2)
      PY2=PT2*SIN(PHI2)
C 3 MESON FORMED, SPIN ADDED AND FLAVOUR MIXED
      K(I,1)=MESO(3*(IFL1-1)+IFL2,IFLSGN)
      ISPIN=INT(PS1+RANF(0))
      K(I,2)=1+9*ISPIN+K(I,1)
      IF(K(I,1).LE.6) GOTO 110
      TMIX=RANF(0)
      KM=K(I,1)-6+3*ISPIN
      K(I,2)=8+9*ISPIN+INT(TMIX+CMIX(KM,1))+INT(TMIX+CMIX(KM,2))
C 4 MESON MASS FROM TABLE, PT FROM CONSTITUENTS
  110 P(I,5)=PMAS(K(I,2))
      P(I,1)=PX1+PX2
      P(I,2)=PY1+PY2
      PMTS=P(I,1)**2+P(I,2)**2+P(I,5)**2
C 5 RANDOM CHOICE OF X=(E+PZ)MESON/(E+PZ)AVAILABLE GIVES E AND PZ
      X=RANF(0)
      IF(RANF(0).LT.CX2) X=1.-X**(1./3.)
      P(I,3)=(X*W-PMTS/(X*W))/2.
      P(I,4)=(X*W+PMTS/(X*W))/2.
C 6 IF UNSTABLE, DECAY CHAIN INTO STABLE PARTICLES
  120 IPD=IPD+1
      IF(K(IPD,2).GE.8) CALL DECAY(IPD,I)
      IF(IPD.LT.I.AND.I.LE.96) GOTO 120
C 7 FLAVOUR AND PT OF QUARK FORMED IN PAIR WITH ANTIQUARK ABOVE
      IFL1=IFL2
      PX1=-PX2
      PY1=-PY2
C 8 IF ENOUGH E+PZ LEFT, GO TO 2
      W=(1.-X)*W
      IF(W.GT.WFIN.AND.I.LE.95) GOTO 100
      N=I
      RETURN
      END
```

(PYTHIA)

PYTHIA anno 2013

(now called PYTHIA 8)

~ 100,000 lines of C++

What a modern MC generator has inside:

LU TP 07-28 (CPC 178 (2008) 852)
October, 2007

A Brief Introduction to PYTHIA 8.1

T. Sjöstrand, S. Mrenna, P. Skands

The Pythia program is a standard tool for the generation of high-energy collisions, comprising a coherent set of physics models for the evolution from a few-body hard process to a complex multihadronic final state. It contains a library of hard processes and models for initial- and final-state parton showers, multiple parton-parton interactions, beam remnants, string fragmentation and particle decays. It also has a set of utilities and interfaces to external programs. […]

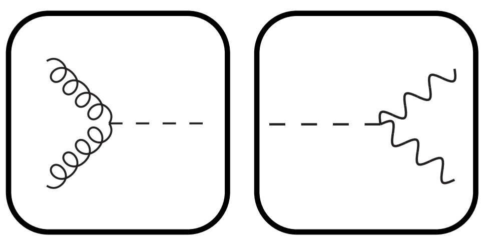
- Hard Processes (internal, interfaced, or via Les Houches events)
- BSM (internal or via interfaces)
- PDFs (internal or via interfaces)
- Showers (internal or inherited)
- Multiple parton interactions
- Beam Remnants
- String Fragmentation
- Decays (internal or via interfaces)
- Examples and Tutorial
- Online HTML / PHP Manual
- Utilities and interfaces to external programs

Divide and Conquer

Factorization → Split the problem into many (nested) pieces

+ Quantum mechanics → Probabilities → Random Numbers

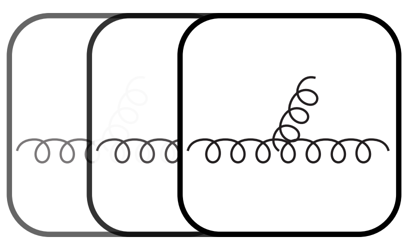
$$\mathcal{P}_{\text{event}} = \mathcal{P}_{\text{hard}} \otimes \mathcal{P}_{\text{dec}} \otimes \mathcal{P}_{\text{ISR}} \otimes \mathcal{P}_{\text{FSR}} \otimes \mathcal{P}_{\text{MPI}} \otimes \mathcal{P}_{\text{Had}} \otimes \ldots$$



Hard Process & Decays:

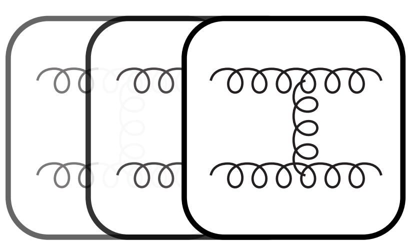
Use (N)LO matrix elements

→ Sets "hard" resolution scale for process: Q_{MAX}



Initial- & Final-State Radiation (ISR & FSR):

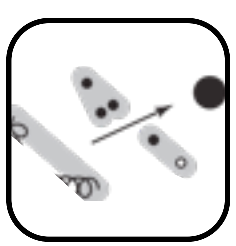
Altarelli-Parisi equations → differential evolution, dP/dQ^2, as function of resolution scale; run from Q_{MAX} to ~ 1 GeV (This Lecture)



MPI (Multi-Parton Interactions)

Additional (soft) parton-parton interactions: LO matrix elements

→ Additional (soft) "Underlying-Event" activity

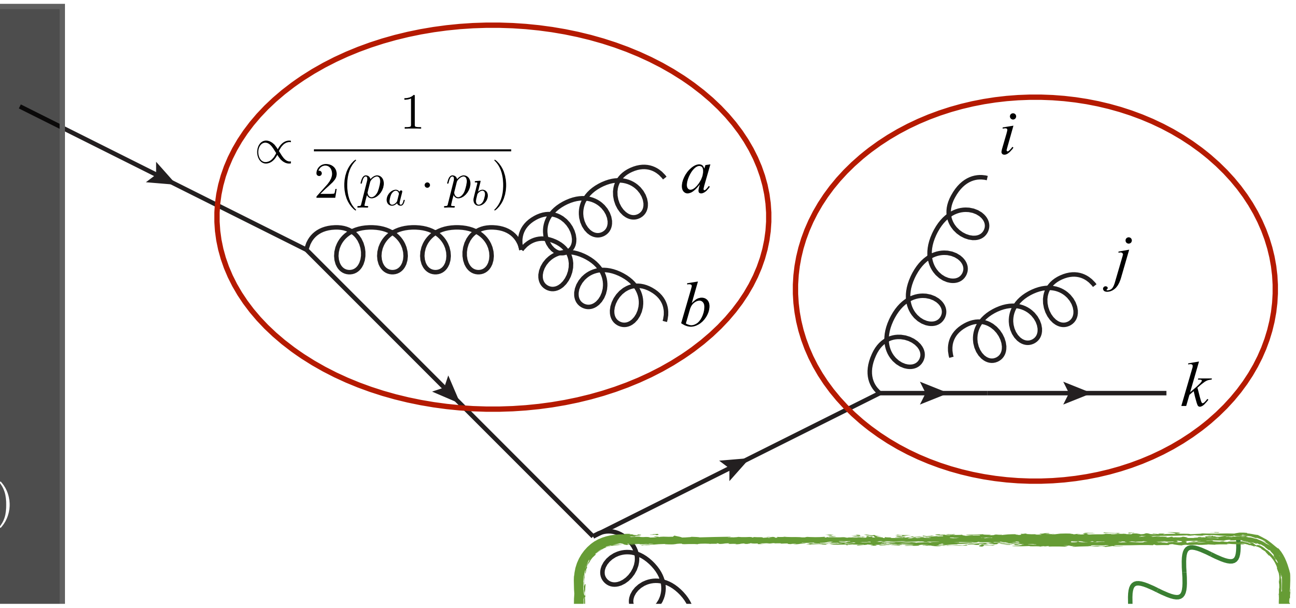


Hadronization

Non-perturbative model of color-singlet parton systems → hadrons

Recall : Jets ≈ Fractals (First Lecture)

- **Most bremsstrahlung** is driven by divergent propagators → simple structure
- **Amplitudes factorize in singular limits** (→ universal "conformal" or "fractal" structure)



Partons ab → "collinear":

P(z) = DGLAP splitting kernels, with z = energy fraction = $E_a/(E_a+E_b)$

$$|\mathcal{M}_{F+1}(\ldots,a,b,\ldots)|^2 \overset{a||b}{\to} g_s^2 \mathcal{C} \frac{P(z)}{2(p_a \cdot p_b)} |\mathcal{M}_F(\ldots,a+b,\ldots)|^2$$

Gluon j → "soft":

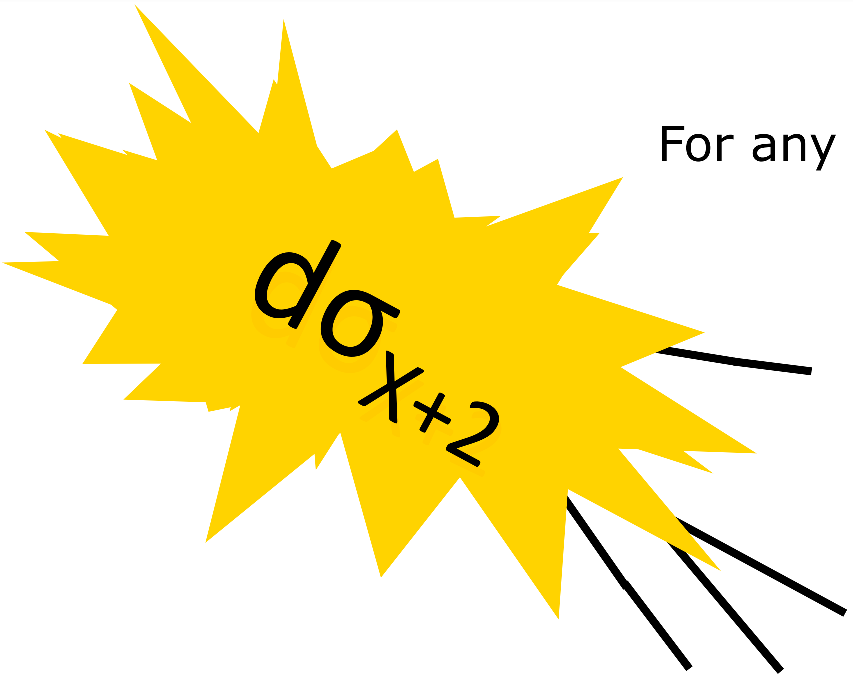
Coherence → Parton j really emitted by (i,k) "colour antenna"

$$|\mathcal{M}_{F+1}(\ldots,i,j,k\ldots)|^2 \overset{j_g\to 0}{\to} g_s^2 \mathcal{C} \frac{(p_i \cdot p_k)}{(p_i \cdot p_j)(p_j \cdot p_k)} |\mathcal{M}_F(\ldots,i,k,\ldots)|^2$$

+ scaling violation: $g_s^2 \to 4\pi\alpha_s(Q^2)$

Can apply this many times
→ nested factorizations

Bremsstrahlung



For any basic process $d\sigma_X$ = ✓ (calculated process by process)

$$d\sigma_{X+1} \sim N_C 2 g_s^2 \frac{ds_{i1}}{s_{i1}} \frac{ds_{1j}}{s_{1j}} d\sigma_X \quad ✓$$

$$d\sigma_{X+2} \sim N_C 2 g_s^2 \frac{ds_{i2}}{s_{i2}} \frac{ds_{2j}}{s_{2j}} d\sigma_{X+1} \quad ✓$$

$$d\sigma_{X+3} \sim N_C 2 g_s^2 \frac{ds_{i3}}{s_{i3}} \frac{ds_{3j}}{s_{3j}} d\sigma_{X+2} \quad \ldots$$

Factorization in Soft and Collinear Limits

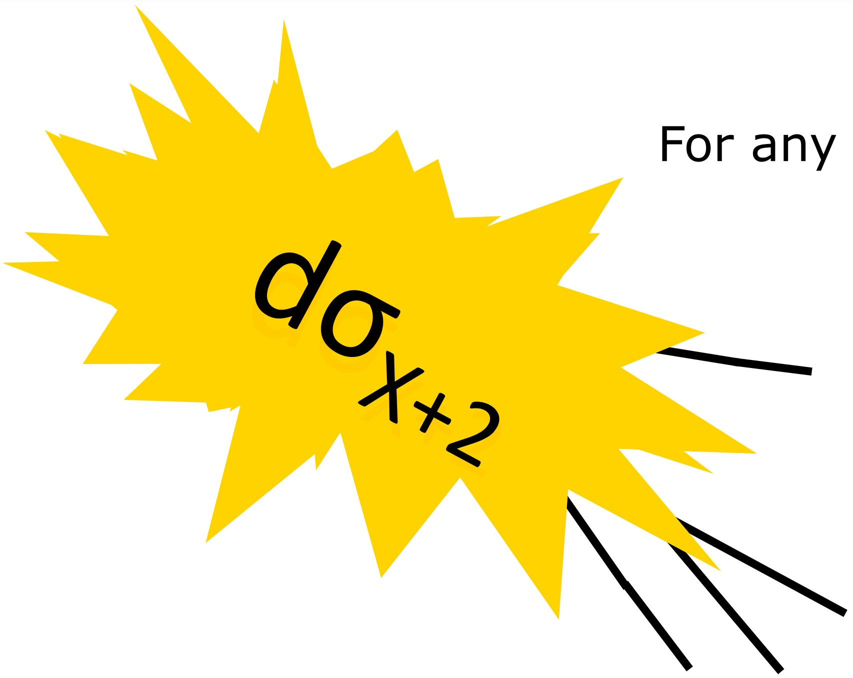
P(z) : "DGLAP Splitting Functions"

$$|M(\ldots,p_i,p_j\ldots)|^2 \overset{i||j}{\to} g_s^2 \mathcal{C} \frac{P(z)}{s_{ij}} |M(\ldots,p_i+p_j,\ldots)|^2$$

$$|M(\ldots,p_i,p_j,p_k\ldots)|^2 \overset{j_g\to 0}{\to} g_s^2 \mathcal{C} \frac{2s_{ik}}{s_{ij}s_{jk}} |M(\ldots,p_i,p_k,\ldots)|^2$$

"Soft Eikonal" : generalizes to Dipole/Antenna Functions (more later)

Bremsstrahlung



For any basic process $d\sigma_X = $ ✓ (calculated process by process)

$$d\sigma_{X+1} \sim N_C 2 g_s^2 \frac{ds_{i1}}{s_{i1}} \frac{ds_{1j}}{s_{1j}} d\sigma_X \quad ✓$$

$$d\sigma_{X+2} \sim N_C 2 g_s^2 \frac{ds_{i2}}{s_{i2}} \frac{ds_{2j}}{s_{2j}} d\sigma_{X+1} \quad ✓$$

$$d\sigma_{X+3} \sim N_C 2 g_s^2 \frac{ds_{i3}}{s_{i3}} \frac{ds_{3j}}{s_{3j}} d\sigma_{X+2} \quad \ldots$$

Singularities: mandated by gauge theory
Non-singular terms: process-dependent

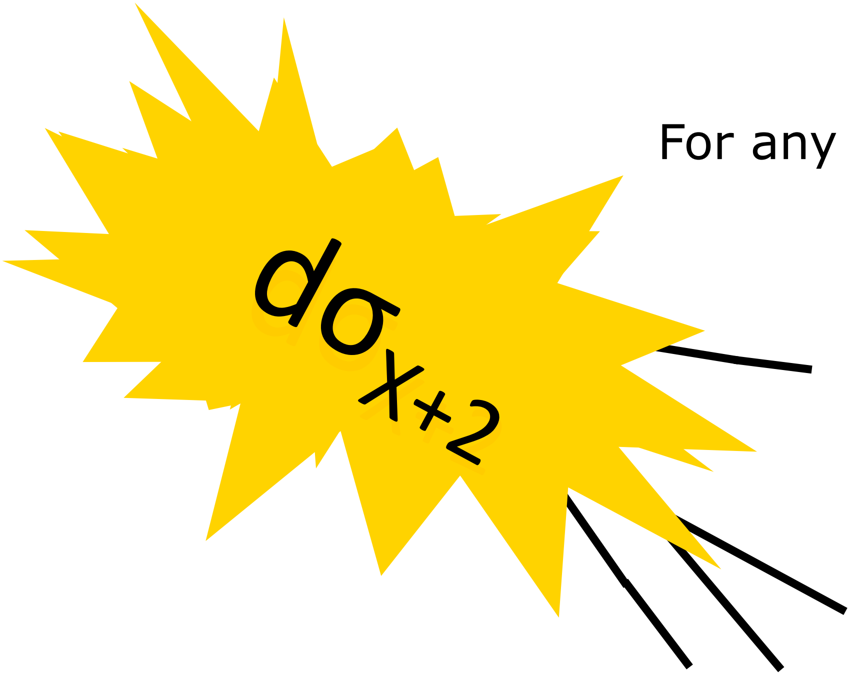
SOFT COLLINEAR

$$\frac{|\mathcal{M}(Z^0 \to q_i g_j \bar{q}_k)|^2}{|\mathcal{M}(Z^0 \to q_I \bar{q}_K)|^2} = g_s^2\, 2C_F \left[\frac{2 s_{ik}}{s_{ij} s_{jk}} + \frac{1}{s_{IK}} \left(\frac{s_{ij}}{s_{jk}} + \frac{s_{jk}}{s_{ij}} \right) \right]$$

$$\frac{|\mathcal{M}(H^0 \to q_i g_j \bar{q}_k)|^2}{|\mathcal{M}(H^0 \to q_I \bar{q}_K)|^2} = g_s^2\, 2C_F \left[\frac{2 s_{ik}}{s_{ij} s_{jk}} + \frac{1}{s_{IK}} \left(\frac{s_{ij}}{s_{jk}} + \frac{s_{jk}}{s_{ij}} + 2 \right) \right]$$

SOFT COLLINEAR +F

Bremsstrahlung



For any basic process $d\sigma_X$ = ✓ (calculated process by process)

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$$d\sigma_{X+2} \sim N_C 2 g_s^2 \frac{ds_{i2}}{s_{i2}} \frac{ds_{2j}}{s_{2j}} d\sigma_{X+1} \quad ✓$$

$$d\sigma_{X+3} \sim N_C 2 g_s^2 \frac{ds_{i3}}{s_{i3}} \frac{ds_{3j}}{s_{3j}} d\sigma_{X+2} \quad \ldots$$

Iterated factorization

Gives us a universal approximation to ∞-order tree-level cross sections.

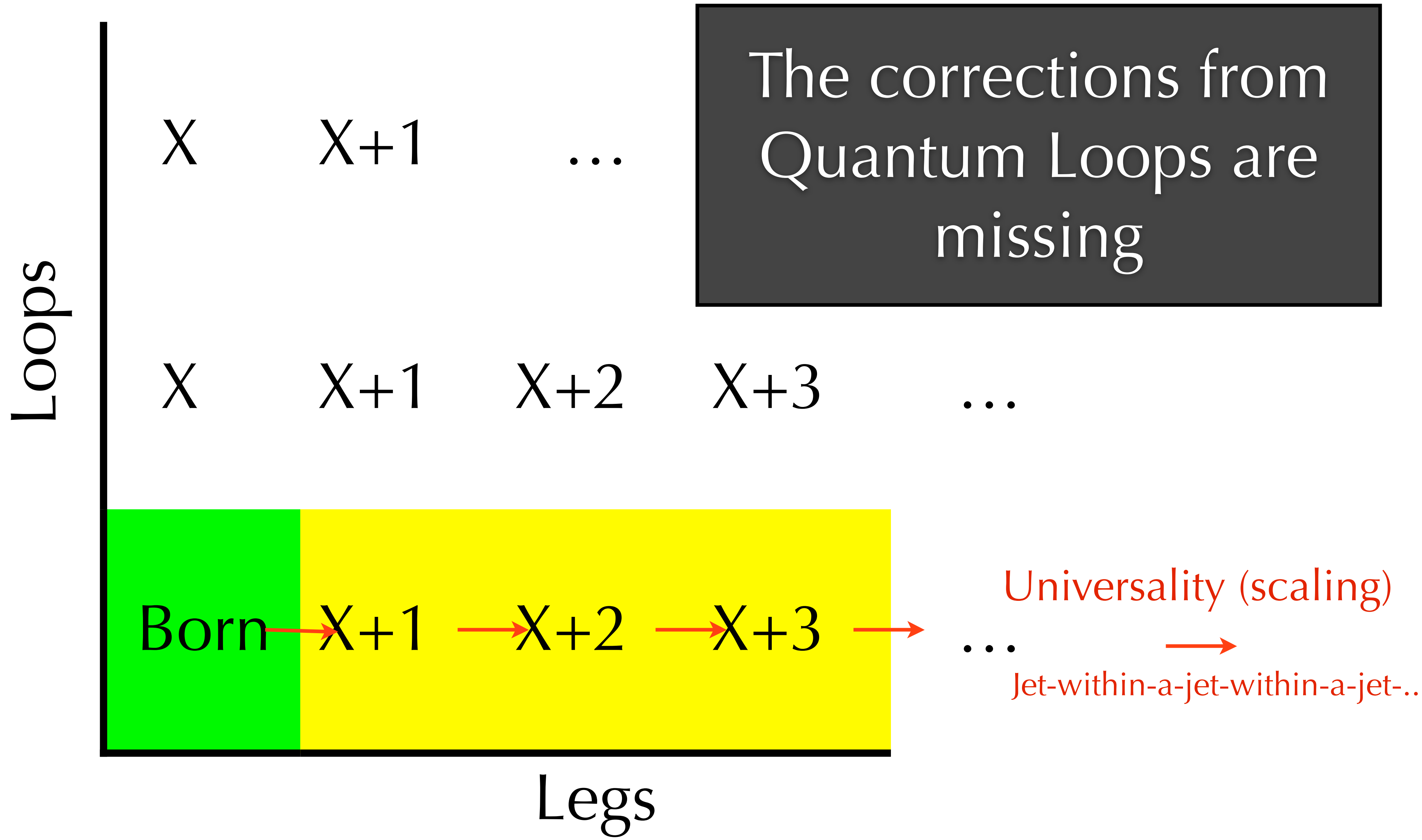
Exact in singular (strongly ordered) limit.

Finite terms (non-universal) → Uncertainties for non-singular (hard) radiation

But something is not right … Total σ would be infinite …

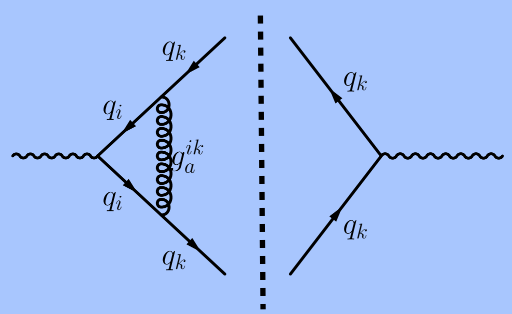
Loops and Legs

Coefficients of the Perturbative Series



Unitarity → Evolution (Resummation)

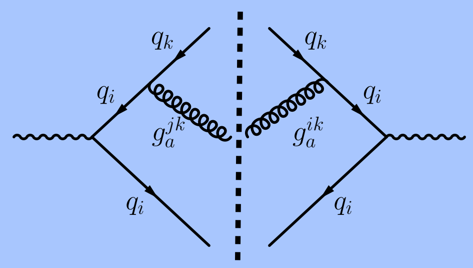
Unitarity: sum(probability) = 1



Kinoshita-Lee-Nauenberg (Lecture 2):

(sum over degenerate quantum states = finite)

Loop = - Int(Tree) + F



Parton Showers neglect F → Leading-Logarithmic (LL) Approximation

Imposed by Event *evolution*:

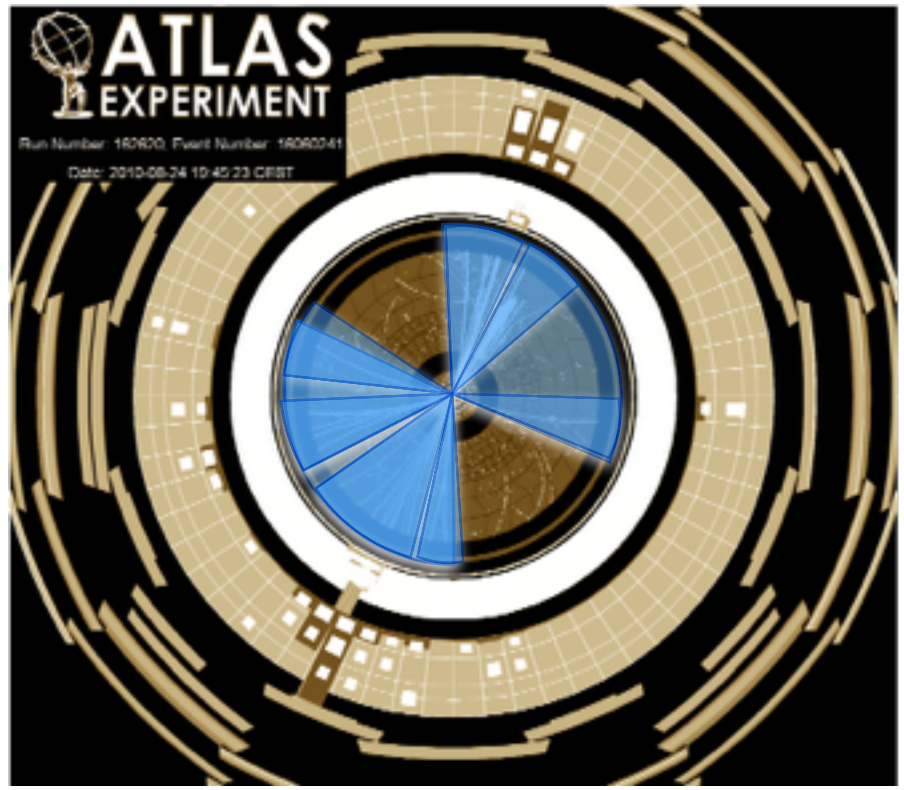
When (X) branches to (X+1): **Gain** one (X+1). **Loose** one (X).

→ *evolution equation with kernel* $\frac{d\sigma_{X+1}}{d\sigma_X}$

Evolve in some measure of *resolution* ~ hardness, 1/time … ~ fractal scale

→ includes both real (tree) and virtual (loop) corrections

Unitarity → Evolution (Resummation)



- ► Interpretation: the structure evolves! (example: X = 2-jets)
  - Take a jet algorithm, with resolution measure “Q”, apply it to your events
  - At a very crude resolution, you find that everything is 2-jets

Evolution Equations

What we need is a differential equation

Boundary condition: a few partons defined at a high scale (Q_F)

Then evolves (or "runs") that parton system down to a low scale (the hadronization cutoff ~ 1 GeV) → It's an evolution equation in Q_F

Close analogue: nuclear decay

Evolve an unstable nucleus. Check if it decays + follow chains of decays.

Decay constant

$$\frac{\mathrm{d}P(t)}{\mathrm{d}t} = c_N$$

Probability to remain undecayed in the time interval $[t_1,t_2]$

$$\Delta(t_1,t_2) = \exp\left(-\int_{t_1}^{t_2} c_N \,\mathrm{d}t\right) = \exp\left(-c_N\,\Delta t\right)$$

$$= 1 - c_N\Delta t + \mathcal{O}(c_N^2)$$

Decay probability per unit time

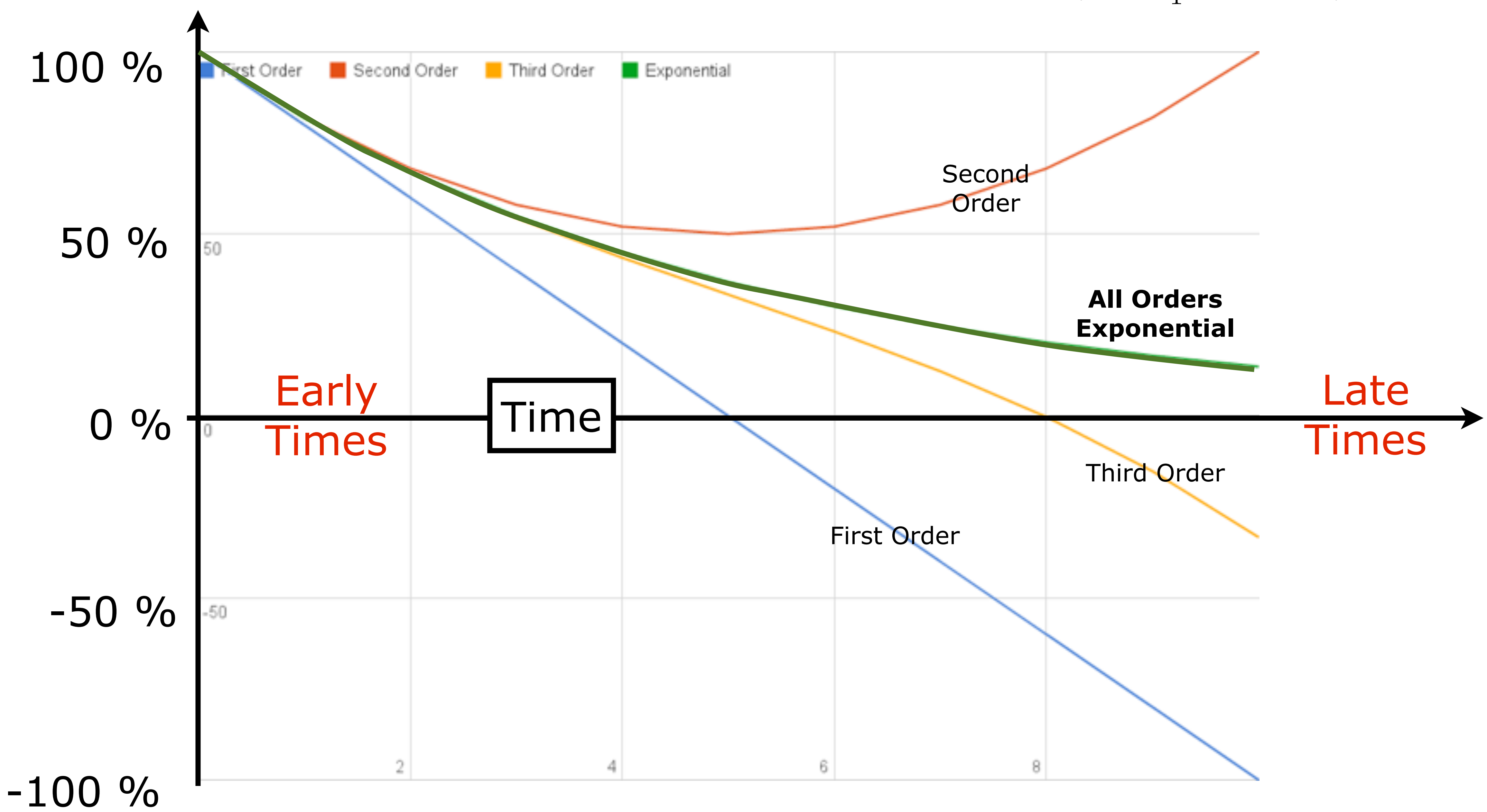
$$\frac{\mathrm{d}P_{\mathrm{res}}(t)}{\mathrm{d}t} = \frac{-\mathrm{d}\Delta}{\mathrm{d}t} = c_N\,\Delta(t_1,t)$$

(requires that the nucleus did not already decay)

$\Delta(t_1,t_2)$: "Sudakov Factor"

Nuclear Decay

Nuclei remaining undecayed after time t $= \Delta(t_1, t_2) = \exp\left(-\int_{t_1}^{t_2} \mathrm{d}t \, \frac{\mathrm{d}\mathcal{P}}{\mathrm{d}t}\right)$



The Sudakov Factor

In nuclear decay, the Sudakov factor counts:

How many nuclei remain undecayed after a time t

Probability to remain undecayed in the time interval $[t_1,t_2]$

$$\Delta(t_1,t_2) = \exp\left(-\int_{t_1}^{t_2} c_N \, \mathrm{d}t\right) = \exp\left(-c_N \, \Delta t\right)$$

The Sudakov factor for a parton system counts:

The probability that the parton system doesn't evolve (branch) when we run the factorization scale (~1/time) from a high to a low scale

Evolution probability per unit "time"

$$\frac{\mathrm{d}P_{\mathrm{res}}(t)}{\mathrm{d}t} = \frac{-\mathrm{d}\Delta}{\mathrm{d}t} = c_N \, \Delta(t_1,t)$$

(replace t by shower evolution scale)

(replace c_N by proper shower evolution kernels)

What's the evolution kernel?

cf. conformal (fractal) QCD, Lecture 1
(and PDF evolution, Lecture 2)

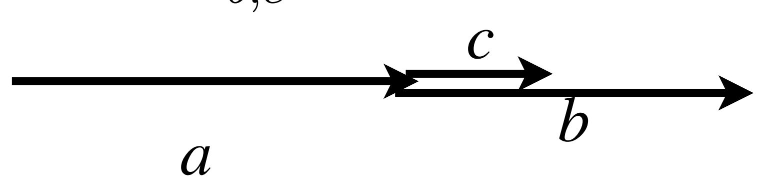
DGLAP splitting functions

Can be derived from *collinear limit* of MEs $(p_b+p_c)^2 \to 0$

+ evolution equation from invariance with respect to $Q_F \to$ RGE

DGLAP
(E.g., PYTHIA)

$$\mathrm{d}\mathcal{P}_a = \sum_{b,c} \frac{\alpha_{abc}}{2\pi} P_{a\to bc}(z)\, \mathrm{d}t\, \mathrm{d}z \ .$$



$p_b = z\, p_a$

$p_c = (1\text{-}z)\, p_a$

$$\begin{aligned}
P_{\mathrm{q}\to\mathrm{qg}}(z) &= C_F \frac{1+z^2}{1-z} \ , \\
P_{\mathrm{g}\to\mathrm{gg}}(z) &= N_C \frac{(1-z(1-z))^2}{z(1-z)} \ , \\
P_{\mathrm{g}\to\mathrm{q\bar{q}}}(z) &= T_R \left(z^2+(1-z)^2\right) \ , \\
P_{\mathrm{q}\to\mathrm{q}\gamma}(z) &= e_{\mathrm{q}}^2 \frac{1+z^2}{1-z} \ , \\
P_{\ell\to\ell\gamma}(z) &= e_{\ell}^2 \frac{1+z^2}{1-z} \ ,
\end{aligned}$$

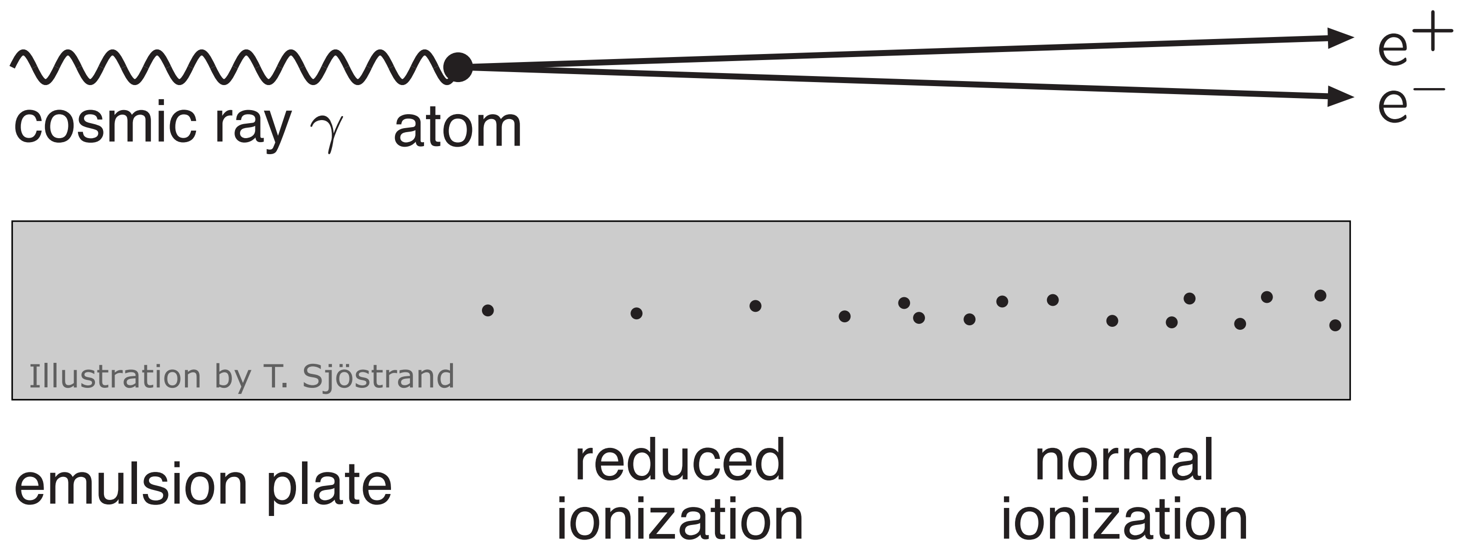
$$\mathrm{d}t = \frac{\mathrm{d}Q^2}{Q^2} = \mathrm{d}\ln Q^2$$

… with Q^2 some measure of "hardness"
= event/jet resolution
measuring parton virtualities / formation time / …

Note: there exist now also alternatives to AP kernels (with same collinear limits!): dipoles, antennae, …

Coherence

QED: Chudakov effect (mid-fifties)



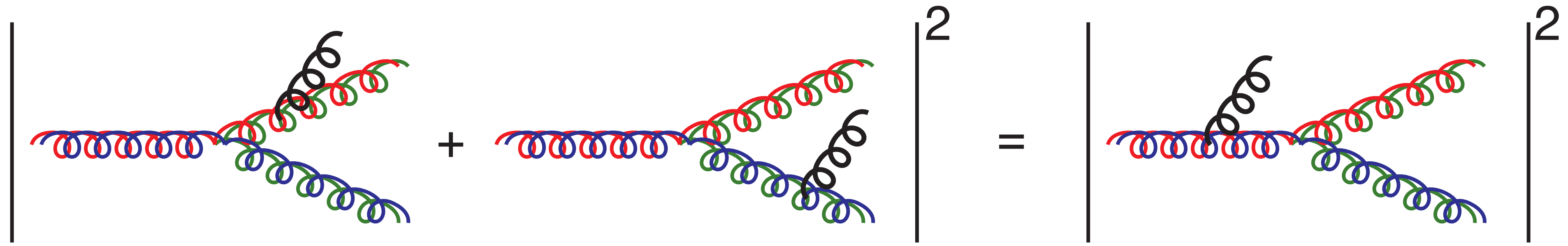
Approximations to Coherence:

Angular Ordering (HERWIG)

Angular Vetos (PYTHIA)

Coherent Dipoles/Antennae (ARIADNE, Catani-Seymour, VINCIA)

QCD: colour coherence for **soft** gluon emission



→ an example of an interference effect that can be treated probabilistically

More interference effects can be included by matching to full matrix elements

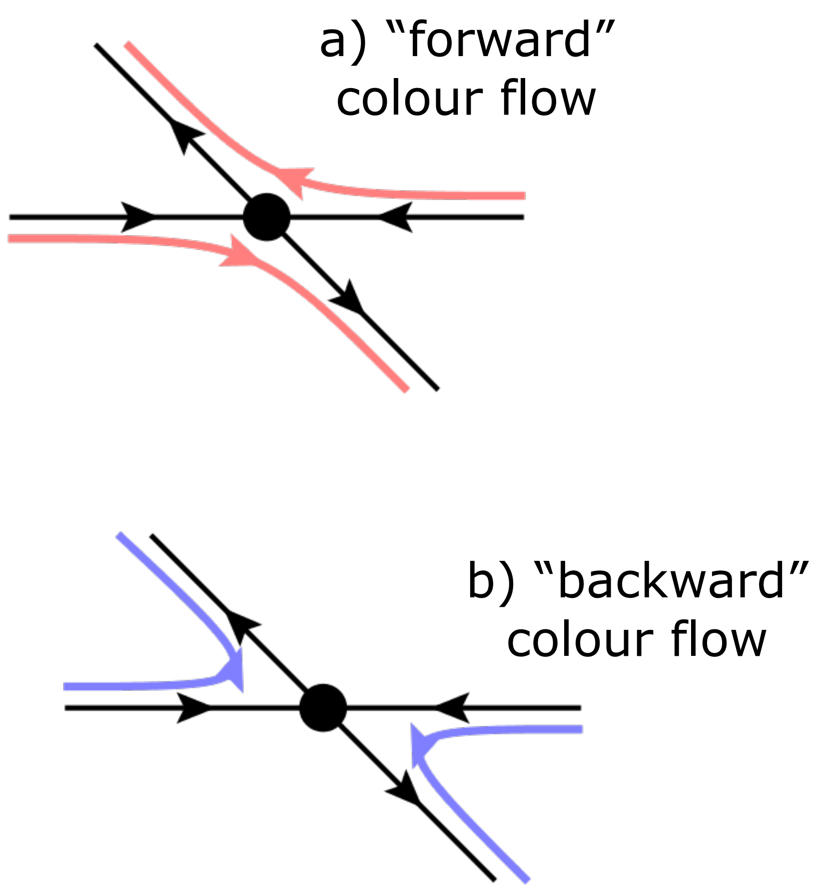
Coherence at Work

Example taken from: Ritzmann, Kosower, PS, PLB718 (2013) 1345

Example: quark-quark scattering in hadron collisions

Consider one specific phase-space point (eg scattering at 45°)

2 possible colour flows: a and b



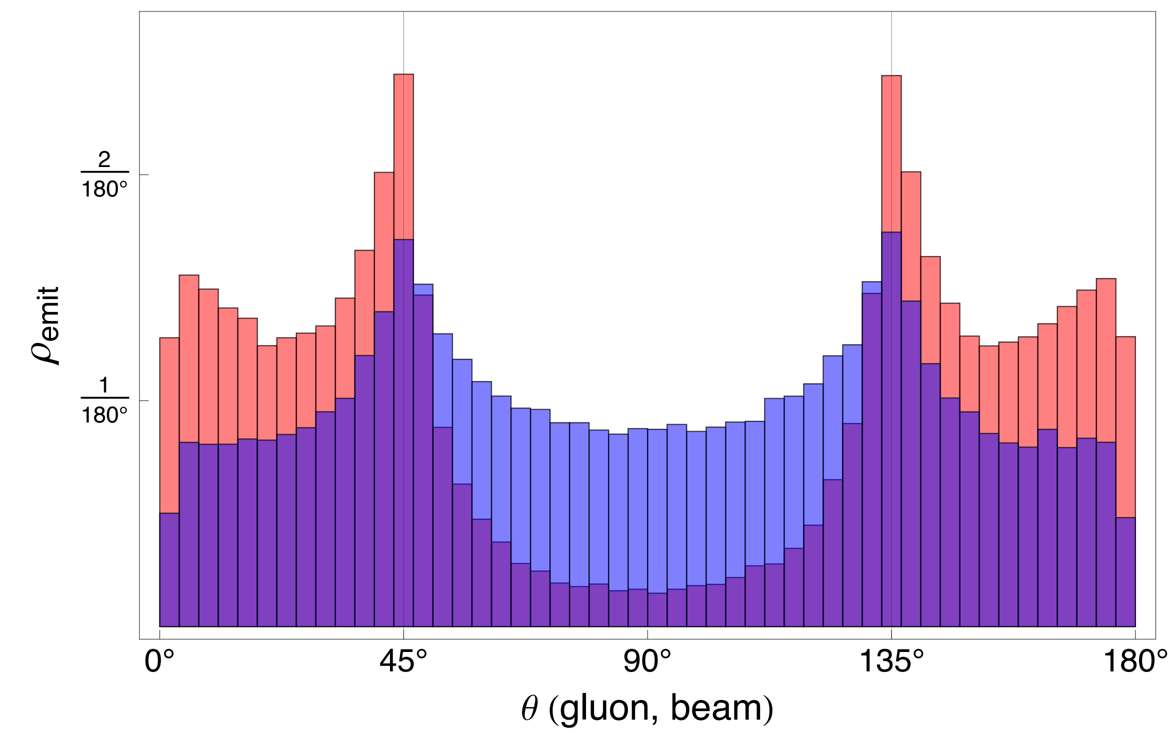
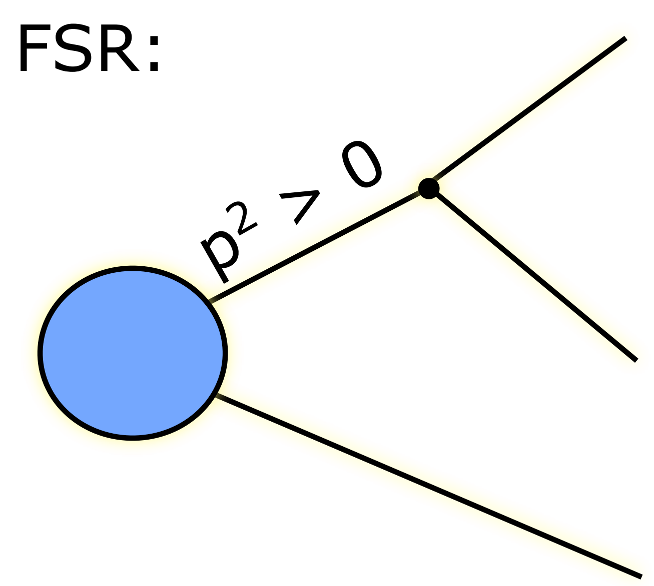
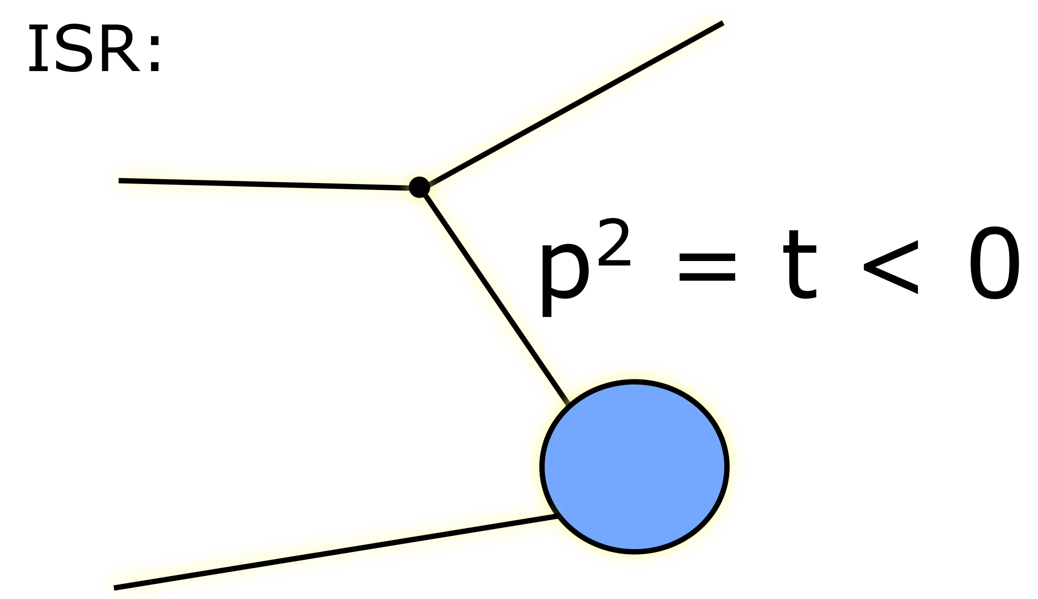


Figure 4: Angular distribution of the first gluon emission in $qq \to qq$ scattering at 45°, for the two different color flows. The light (red) histogram shows the emission density for the forward flow, and the dark (blue) histogram shows the emission density for the backward flow.

Another good recent example is the SM contribution to the Tevatron top-quark forward-backward asymmetry from coherent showers, see: PS, Webber, Winter, JHEP 1207 (2012) 151

Initial-State vs Final-State Evolution





Virtualities are
Timelike: $p^2>0$

Start at $Q^2 = Q_F^2$
"Forwards evolution"

Virtualities are
Spacelike: $p^2<0$

Start at $Q^2 = Q_F^2$
Constrained backwards evolution
towards boundary condition = proton

Separation meaningful for collinear radiation, but not for soft …

(Initial-State Evolution)

DGLAP for Parton Density

$$\frac{\mathrm{d}f_b(x,t)}{\mathrm{d}t} = \sum_{a,c} \int \frac{\mathrm{d}x'}{x'} f_a(x',t) \frac{\alpha_{abc}}{2\pi} P_{a\to bc}\left(\frac{x}{x'}\right)$$

→ Sudakov for ISR

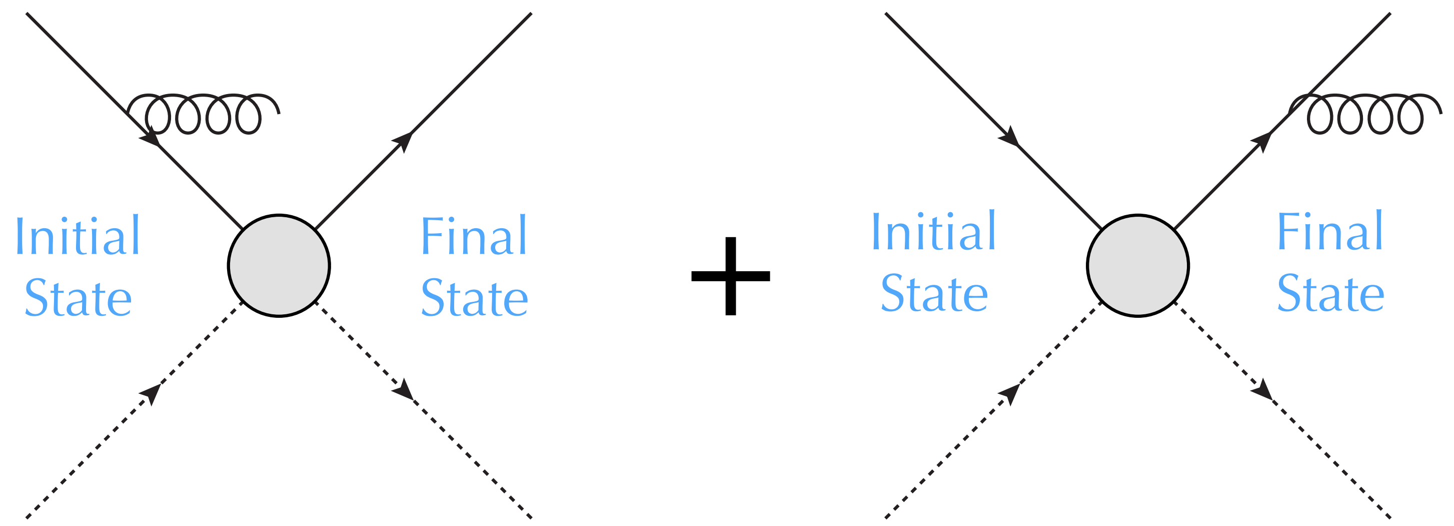
Contains a ratio of PDFs

$$\begin{aligned}\Delta(x, t_{\max}, t) &= \exp\left\{-\int_t^{t_{\max}} \mathrm{d}t' \sum_{a,c} \int \frac{\mathrm{d}x'}{x'} \frac{f_a(x',t')}{f_b(x,t')} \frac{\alpha_{abc}(t')}{2\pi} P_{a\to bc}\left(\frac{x}{x'}\right)\right\} \\ &= \exp\left\{-\int_t^{t_{\max}} \mathrm{d}t' \sum_{a,c} \int \mathrm{d}z \frac{\alpha_{abc}(t')}{2\pi} P_{a\to bc}(z) \frac{x' f_a(x',t')}{x f_b(x,t')}\right\},\end{aligned}$$

Initial-Final Interference

A tricky aspect for many parton showers. Illustrates that quantum ≠ classical !

Who emitted that gluon?



Real QFT = sum over amplitudes, then square → interference (IF coherence)

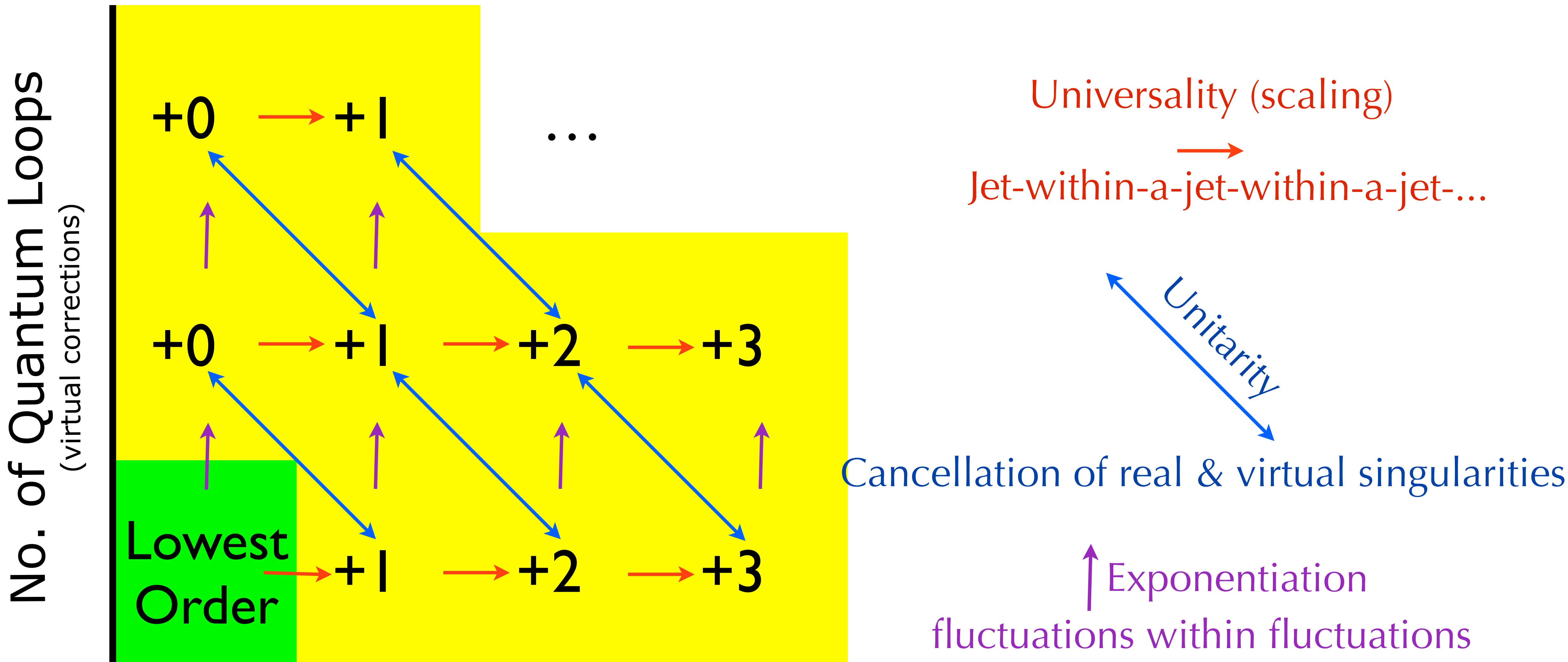
Respected by dipole/antenna languages (and by angular ordering), but not by conventional DGLAP (→ all PDFs are "wrong")

Separation meaningful for collinear radiation, but not for soft …

Bootstrapped Perturbation Theory

Start from an **arbitrary lowest-order** process (green = QFT amplitude squared)

Parton showers generate the (LL) bremsstrahlung terms of the rest of the perturbative series (approximate infinite-order resummation)



Note! LL ≠ full QCD! (→ matching)

(real corrections)

The Shower Operator

H = Hard process

Born $\left.\frac{d\sigma_H}{d\mathcal{O}}\right|_{\text{Born}} = \int d\Phi_H \; |M_H^{(0)}|^2 \, \delta(\mathcal{O} - \mathcal{O}(\{p\}_H))$

{p} : partons

But instead of evaluating O directly on the Born final state, first insert a showering operator

Born + shower $\left.\frac{d\sigma_H}{d\mathcal{O}}\right|_{\mathcal{S}} = \int d\Phi_H \; |M_H^{(0)}|^2 \, \mathcal{S}(\{p\}_H, \mathcal{O})$

{p} : partons

S : showering operator

Unitarity: to first order, S does nothing

$$\mathcal{S}(\{p\}_H, \mathcal{O}) = \delta\left(\mathcal{O} - \mathcal{O}(\{p\}_H)\right) \; + \; \mathcal{O}(\alpha_s)$$

The Shower Operator

To ALL Orders

(Markov Chain)

$$S(\{p\}_X, \mathcal{O}) = \Delta(t_{\rm start}, t_{\rm had})\delta(\mathcal{O} - \mathcal{O}(\{p\}_X))$$

"Nothing Happens" → "Evaluate Observable"

$$- \int_{t_{\rm start}}^{t_{\rm had}} {\rm d}t \frac{{\rm d}\Delta(t_{\rm start}, t)}{{\rm d}t} S(\{p\}_{X+1}, \mathcal{O})$$

"Something Happens" → "Continue Shower"

All-orders Probability that nothing happens

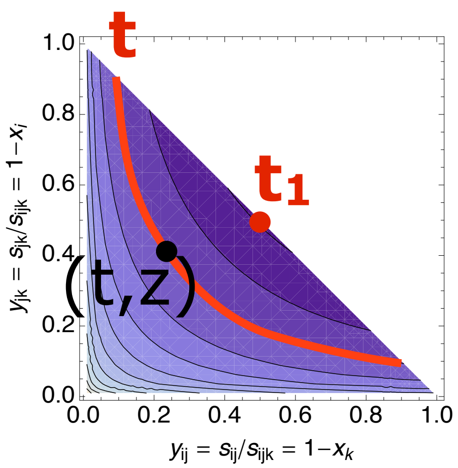
$$\Delta(t_1, t_2) = \exp\left(- \int_{t_1}^{t_2} {\rm d}t \, \frac{{\rm d}\mathcal{P}}{{\rm d}t} \right)$$

(Exponentiation)
Analogous to nuclear decay
$N(t) \approx N(0)\exp(-ct)$

A Shower Algorithm

Note: on this slide, I use results from the theory of Random numbers, interesting in itself but would need more time to give details

1. Generate Random Number, R ∈ [0,1]



Solve equation $R = \Delta(t_1, t)$ for t (with starting scale t_1)

Analytically for simple splitting kernels,
else numerically (or by trial+veto)
→ t scale for next branching

2. Generate another Random Number, R_z ∈ [0,1]

To find second (linearly independent) phase-space invariant

Solve equation $R_z = \frac{I_z(z,t)}{I_z(z_{\max}(t),t)}$ for z (at scale t)

With the "primitive function" $I_z(z,t) = \int_{z_{\min}(t)}^{z} \mathrm{d}z \left. \frac{\mathrm{d}\Delta(t')}{\mathrm{d}t'} \right|_{t'=t}$

3. Generate a third Random Number, R_φ ∈ [0,1]

Solve equation $R_\varphi = \varphi/2\pi$ for φ → Can now do 3D branching

Perturbative Ambiguities

The final states generated by a shower algorithm will depend on

1. The choice of perturbative evolution variable(s) $t^{[i]}$. ← Ordering & Evolution-scale choices
2. The choice of phase-space mapping $\mathrm{d}\Phi^{[i]}_{n+1}/\mathrm{d}\Phi_n$. ← Recoils, kinematics
3. The choice of radiation functions a_i, as a function of the phase-space variables. ← Non-singular terms, Reparametrizations, Subleading Colour
4. The choice of renormalization scale function μ_R.
5. Choices of starting and ending scales. ← Phase-space limits / suppressions for hard radiation and choice of hadronization scale

→ gives us additional handles for uncertainty estimates, beyond just μ_R
(+ ambiguities can be reduced by including more pQCD → matching!)

Jack of All Orders, Master of None?

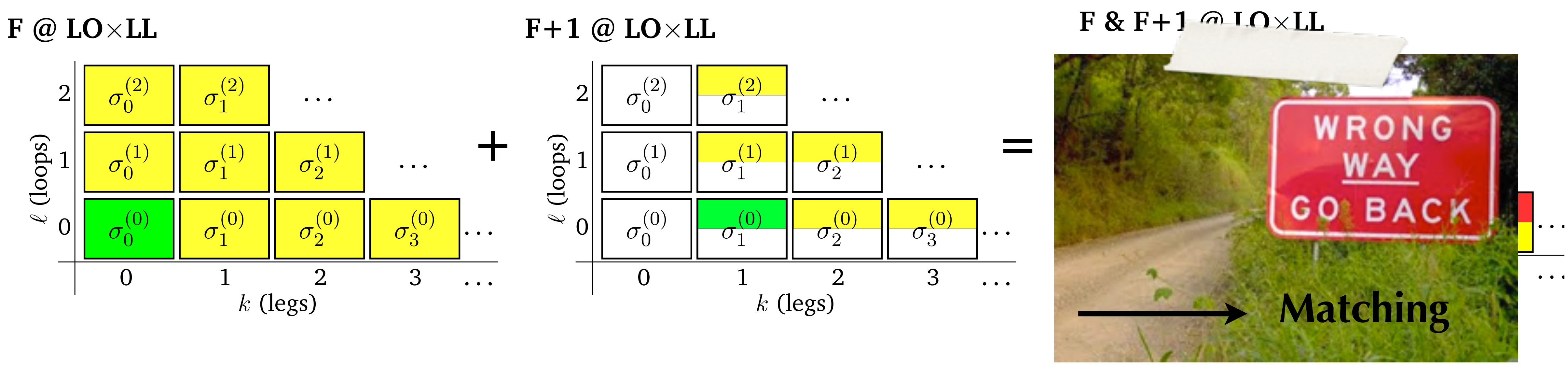
Nice to have all-orders solution

But it is only exact in the singular (soft & collinear) limits

→ gets the bulk of bremsstrahlung corrections right, but fails equally spectacularly: for hard wide-angle radiation: **visible, extra jets**

… which is exactly where fixed-order calculations work!

So combine them!



Summary: Parton Showers

Aim: generate events in as much detail as mother nature

→ Make stochastic choices ~ as in Nature (Q.M.) → Random numbers

Factor complete event probability into separate universal pieces, treated independently and/or sequentially (Markov-Chain MC)

Improve Born-level theory by including 'most significant' corrections

Resonance decays (e.g., $t \to bW^+$, $W \to qq'$, $H^0 \to \gamma^0\gamma^0$, $Z^0 \to \mu^+\mu^-$, …)

Bremsstrahlung (FSR and ISR, exact in collinear and soft* limits)

Hard radiation (matching)

Hadronization (strings/clusters, discussed tomorrow)

Additional Soft Physics: multiple parton-parton interactions, Bose-Einstein correlations, colour reconnections, hadron decays, …

Coherence*

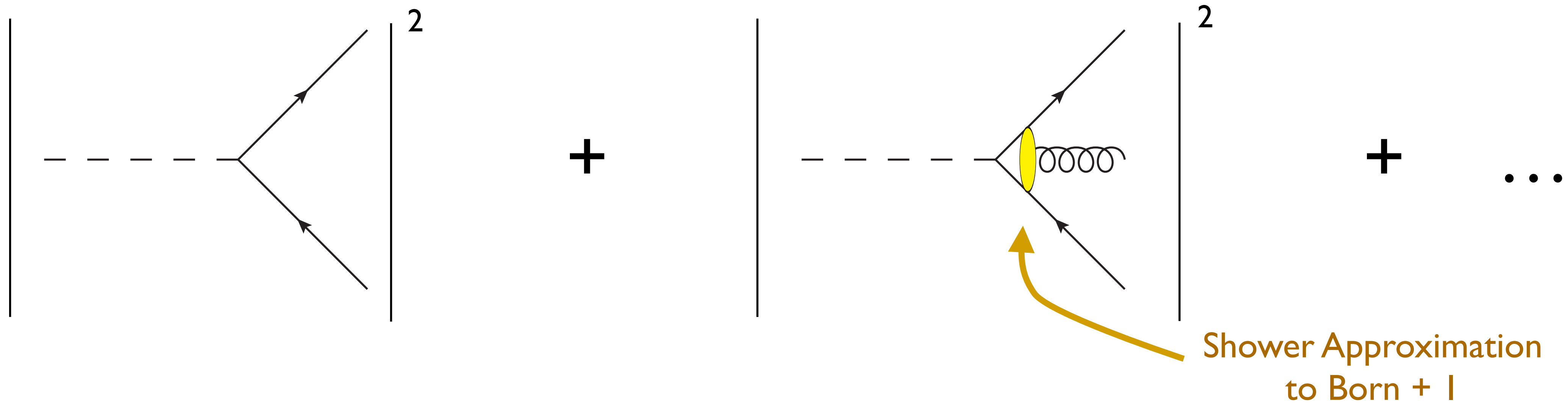
Soft radiation → Angular ordering or Coherent Dipoles/Antennae

Matching

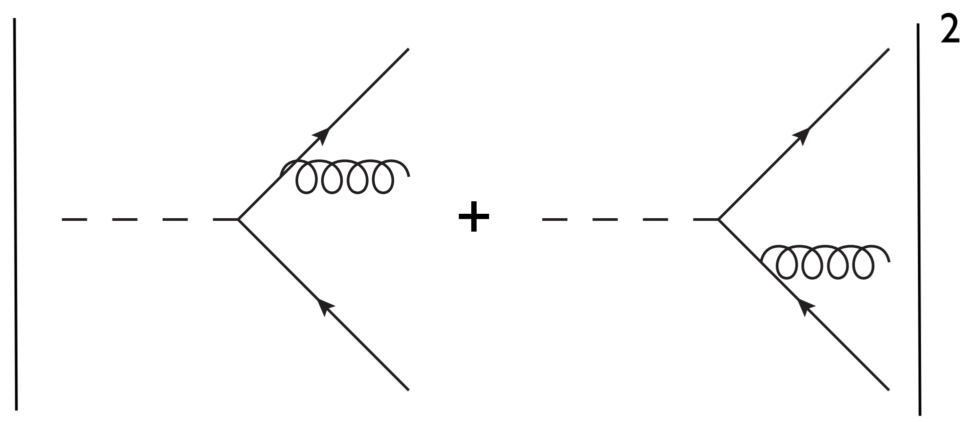
Image Credits: istockphoto

Example: $H^0 \to b\bar{b}$

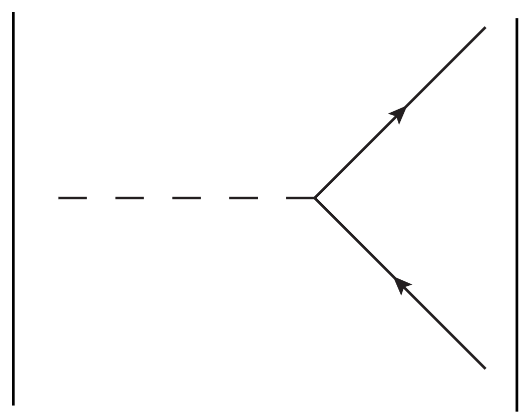
Born + Shower



Born + 1 @ LO

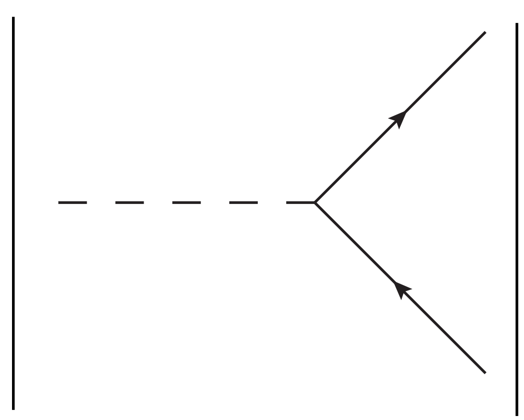


Born + Shower



$$\left| \cdots \right|^2 \left(1 + g_s^2\, 2C_F \left[\frac{2s_{ik}}{s_{ij}s_{jk}} + \frac{1}{s_{IK}} \left(\frac{s_{ij}}{s_{jk}} + \frac{s_{jk}}{s_{ij}} \right) \right] + \ldots \right)$$

Born + 1 @ LO

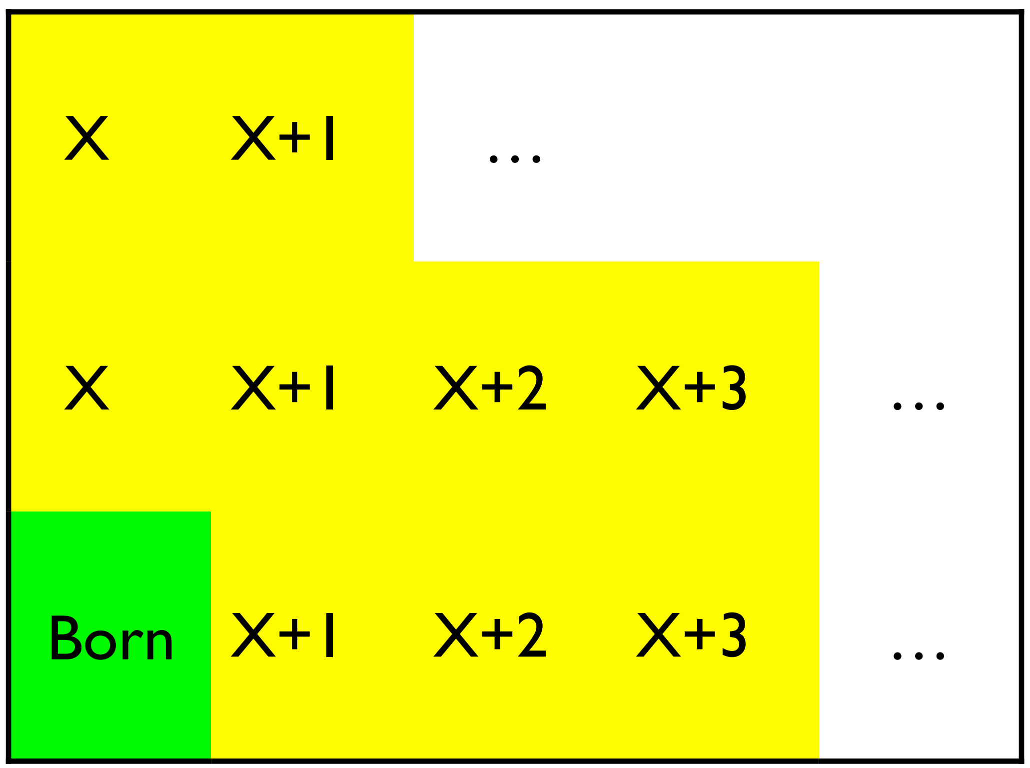


$$\left| \cdots \right|^2 \left(g_s^2\, 2C_F \left[\frac{2s_{ik}}{s_{ij}s_{jk}} + \frac{1}{s_{IK}} \left(\frac{s_{ij}}{s_{jk}} + \frac{s_{jk}}{s_{ij}} + 2 \right) \right] \right)$$

Total Overkill to add these two. All we really need is just that **+2** …

Adding Calculations

Born × Shower

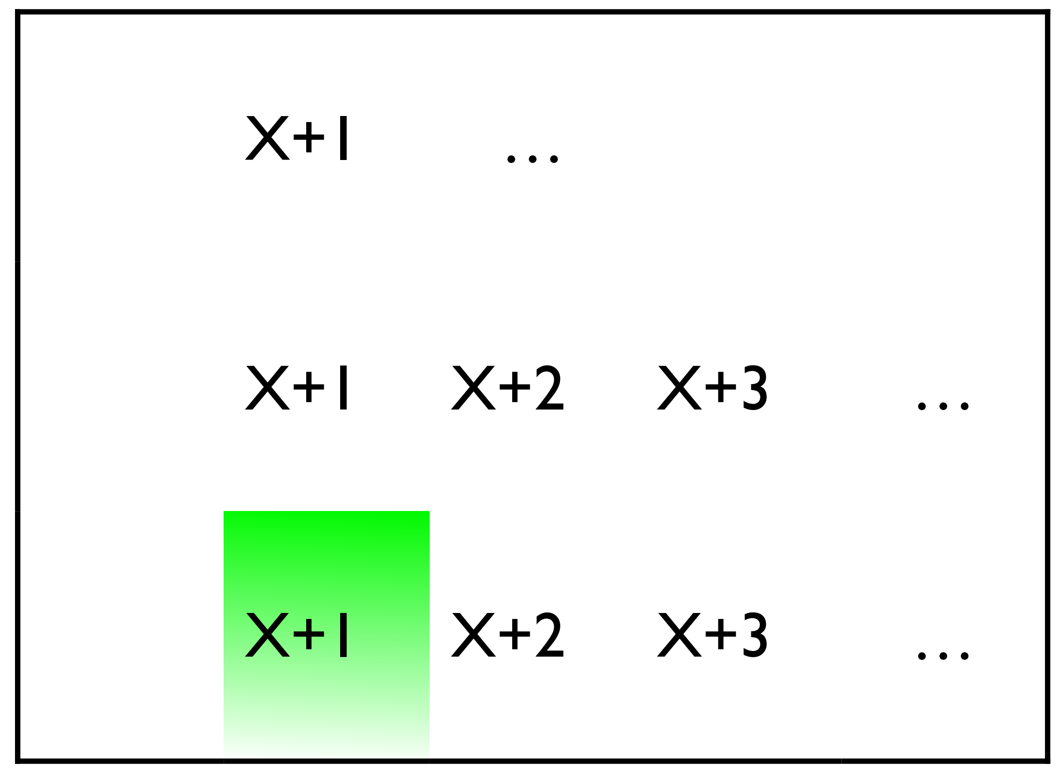


… Fixed-Order Matrix Element

… Shower Approximation

X+1 @ LO

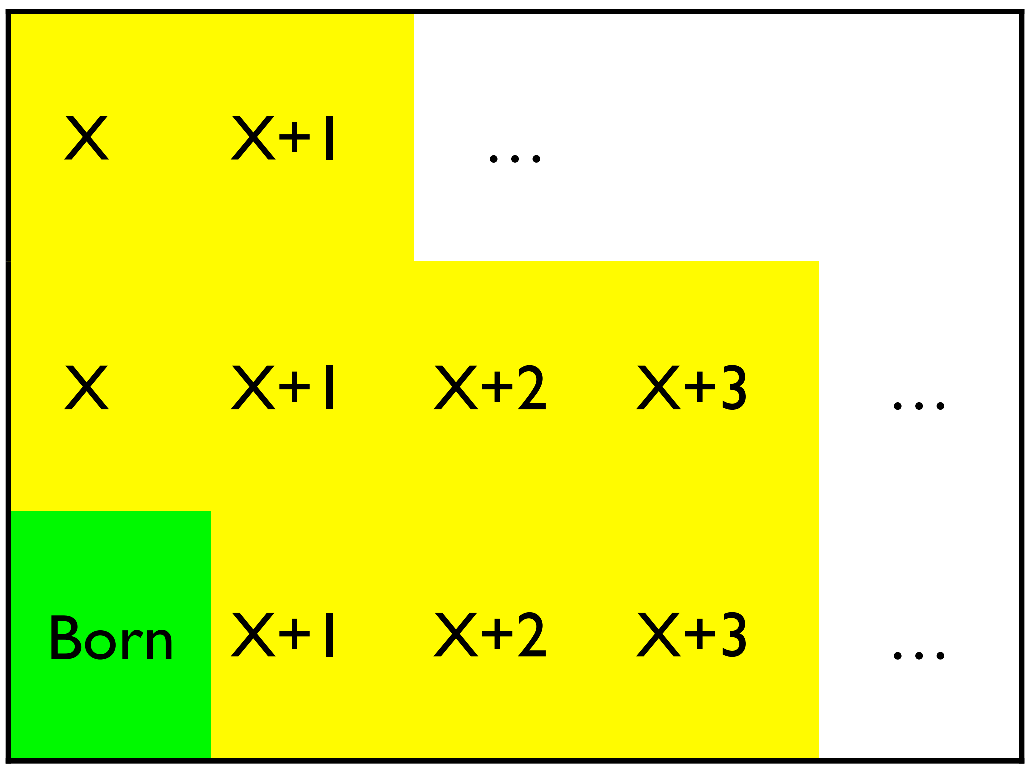
(with p_T cutoff, see previous lectures)



… Fixed-Order ME above p_T cut & nothing below

Adding Calculations

Born × Shower

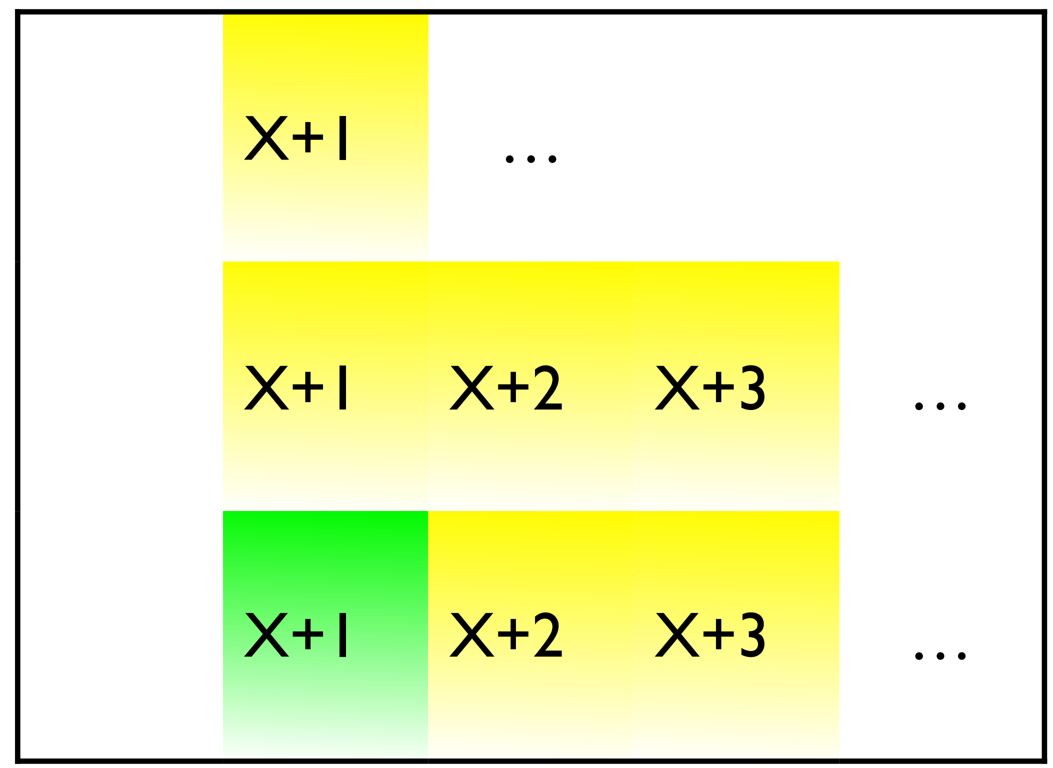


… Fixed-Order Matrix Element

… Shower Approximation

X+1 @ LO × Shower

(with p_T cutoff, see previous lectures)



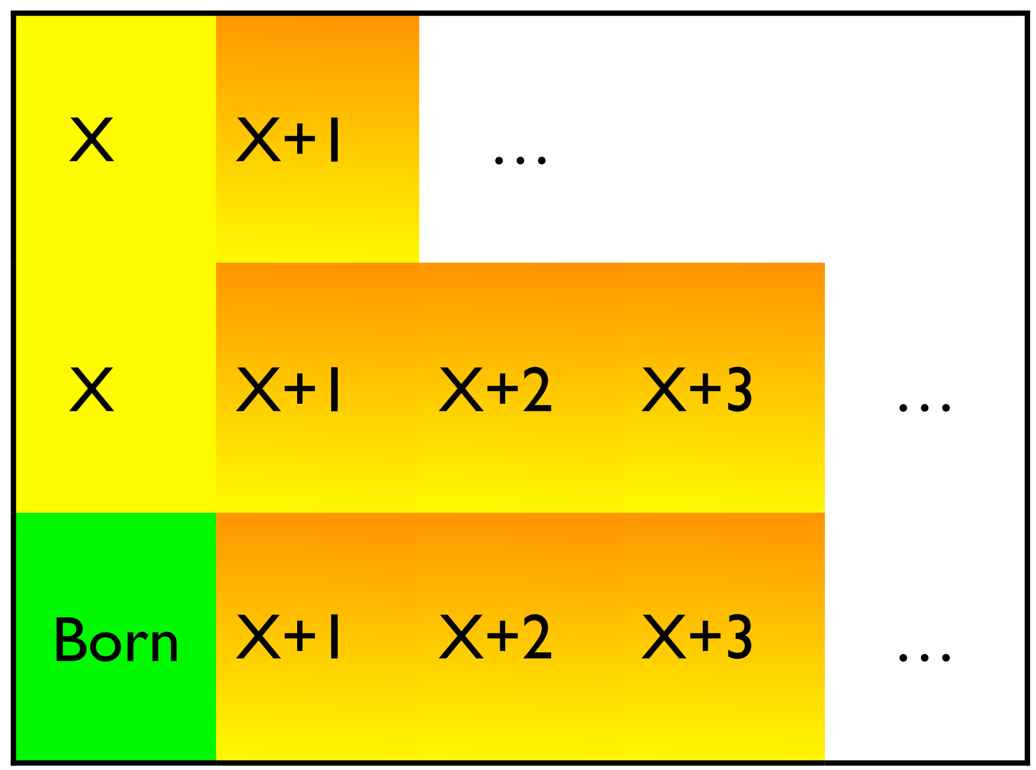
… Fixed-Order ME above p_T cut & nothing below

… Shower approximation above p_T cut & nothing below

→ Double Counting

Born × Shower + (X+1) × shower

Double Counting of terms present in both expansions



Worse than useless

… Fixed-Order Matrix Element

… Shower Approximation

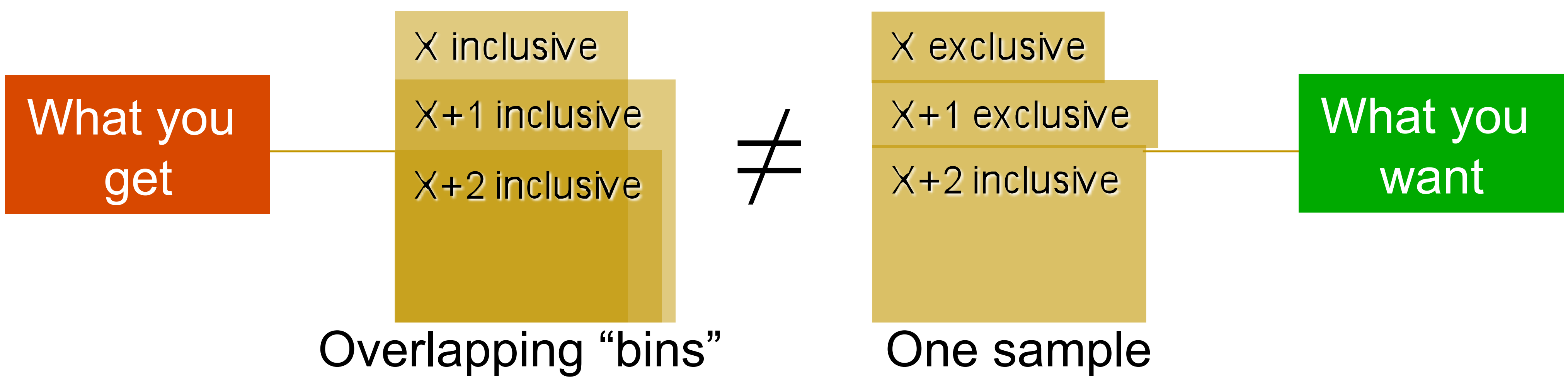
… Double counting above p_T cut & shower approximation below

Interpretation

- A (Complete Idiot's) Solution – Combine
  1. $[X]_{ME}$ + showering
  2. $[X + 1\ \text{jet}]_{ME}$ + showering
  3. …

Run generator for X (+ shower)
Run generator for X+1 (+ shower)
Run generator for … (+ shower)
Combine everything into one sample

- Doesn't work
  - [X] + shower is inclusive
  - [X+1] + shower is also inclusive

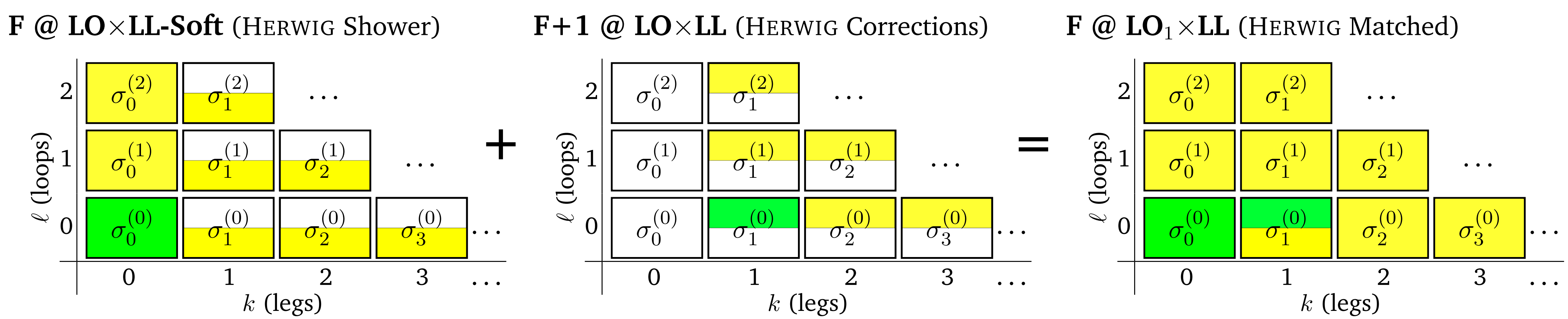


Matching 1: Slicing

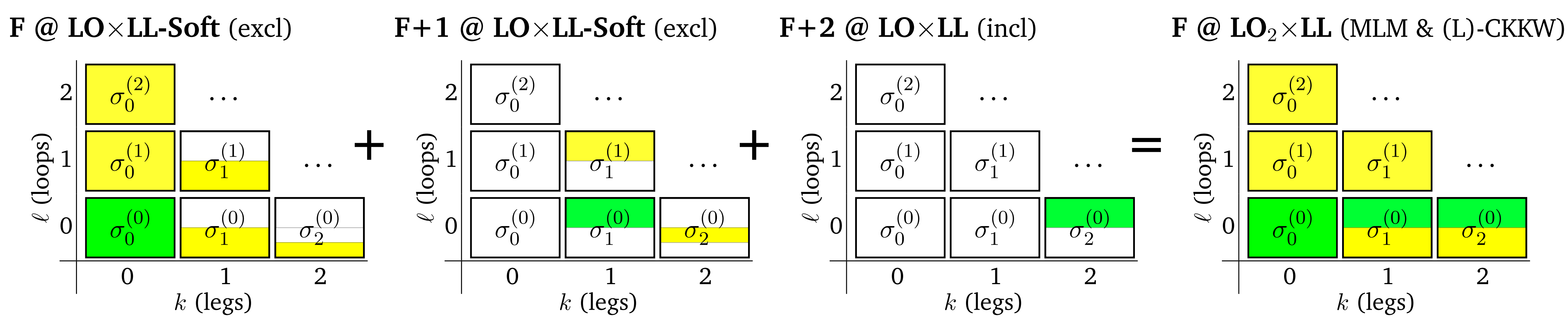
Examples: MLM, CKKW, CKKW-L

First emission: "the HERWIG correction"

Use the fact that the angular-ordered HERWIG parton shower has a "dead zone" for hard wide-angle radiation (Seymour, 1995)



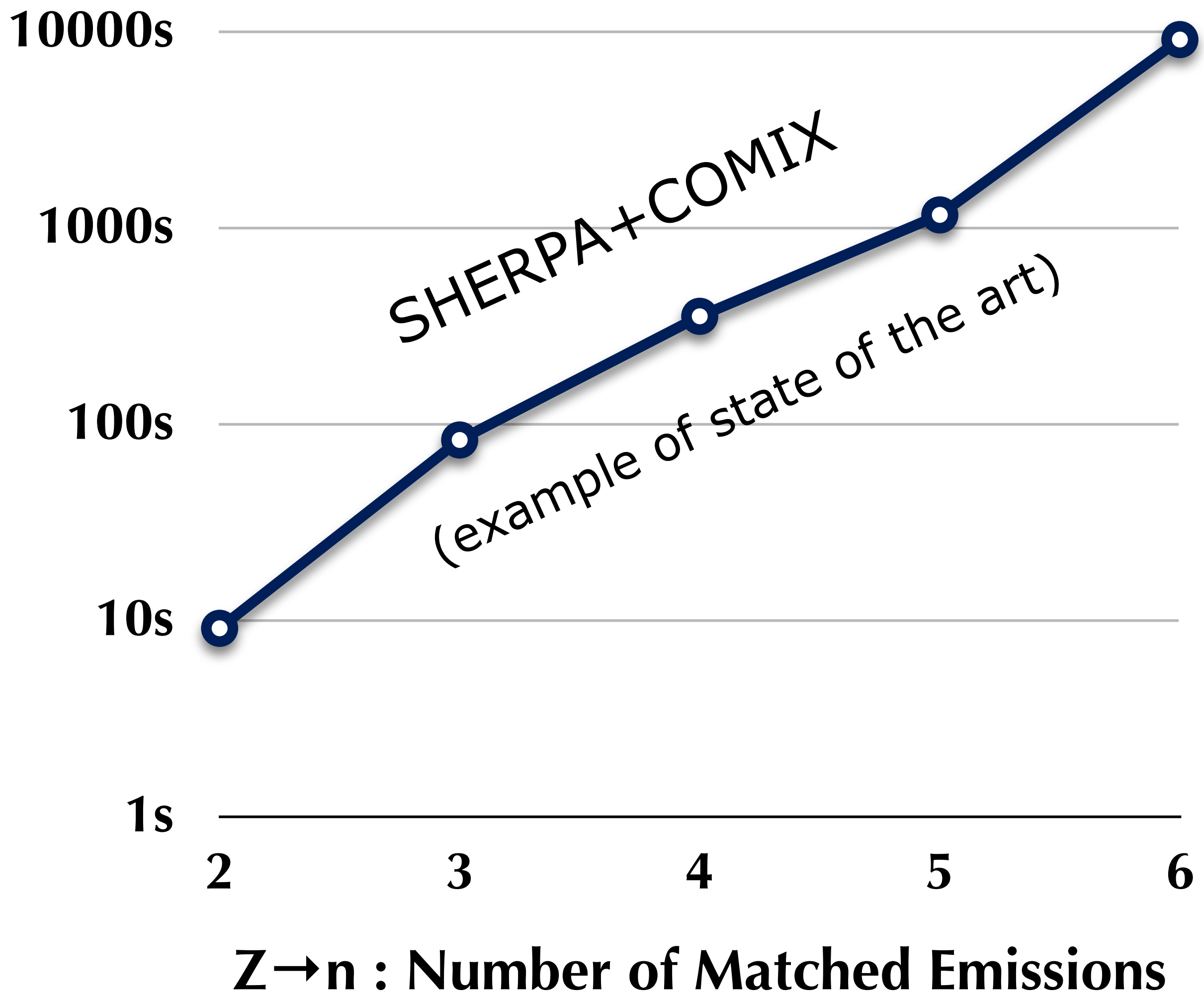
Many emissions: the MLM & CKKW-L prescriptions



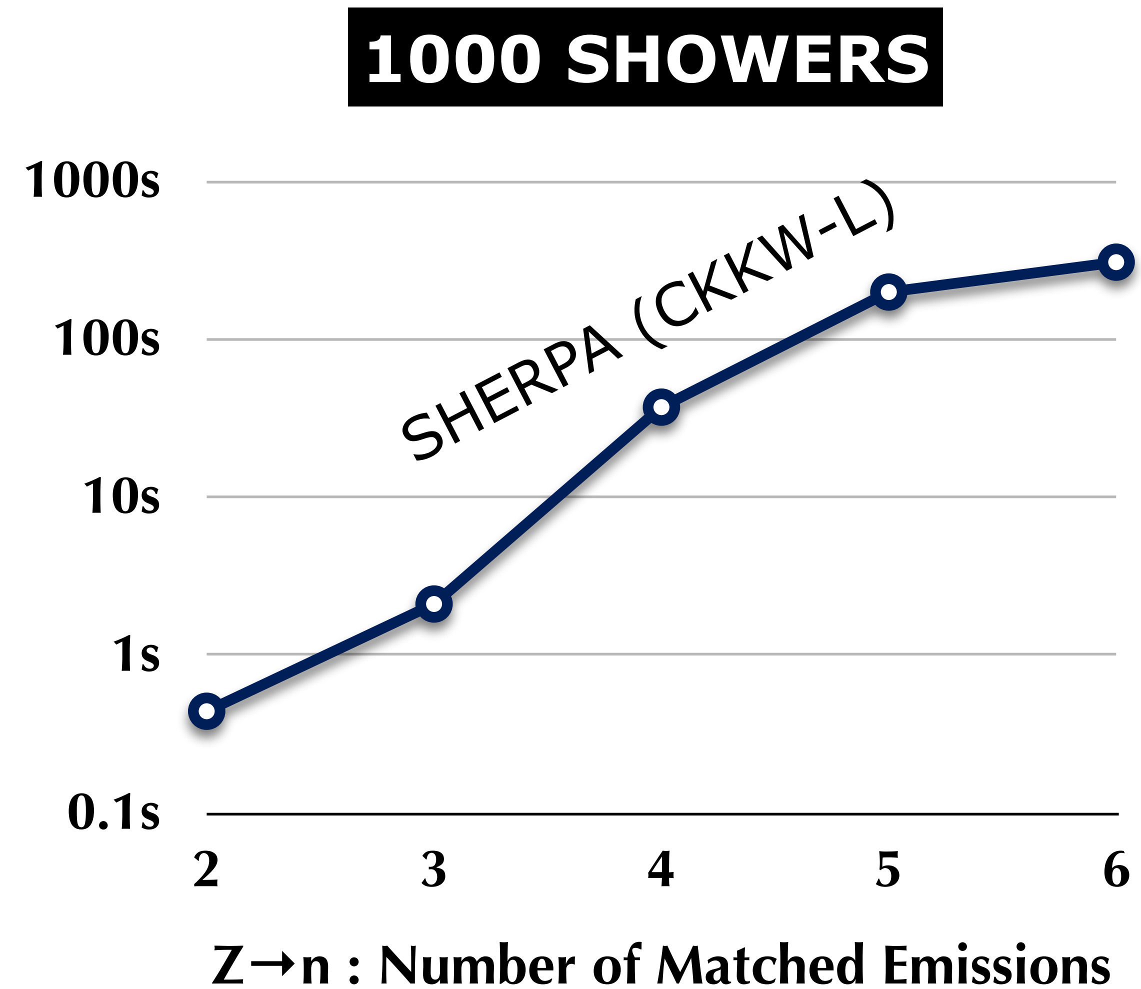
(CKKW & Lönnblad, 2001) (Mangano, 2002) (+many more recent; see Alwall et al., EPJC53(2008)473)

Slicing: The Cost

1. Initialization time
(to pre-compute cross sections and warm up phase-space grids)



2. Time to generate 1000 events
(Z → partons, fully showered & matched. No hadronization.)



Z→udscb ; Hadronization OFF ; ISR OFF ; udsc MASSLESS ; b MASSIVE ; E_{CM} = 91.2 GeV ; Q_{match} = 5 GeV
SHERPA 1.4.0 (+COMIX) ; PYTHIA 8.1.65 ; VINCIA 1.0.29 (+MADGRAPH 4.4.26) ;
gcc/gfortran v 4.7.1 -O2 ; single 3.06 GHz core (4GB RAM)

The Gain

Example: W + Jets

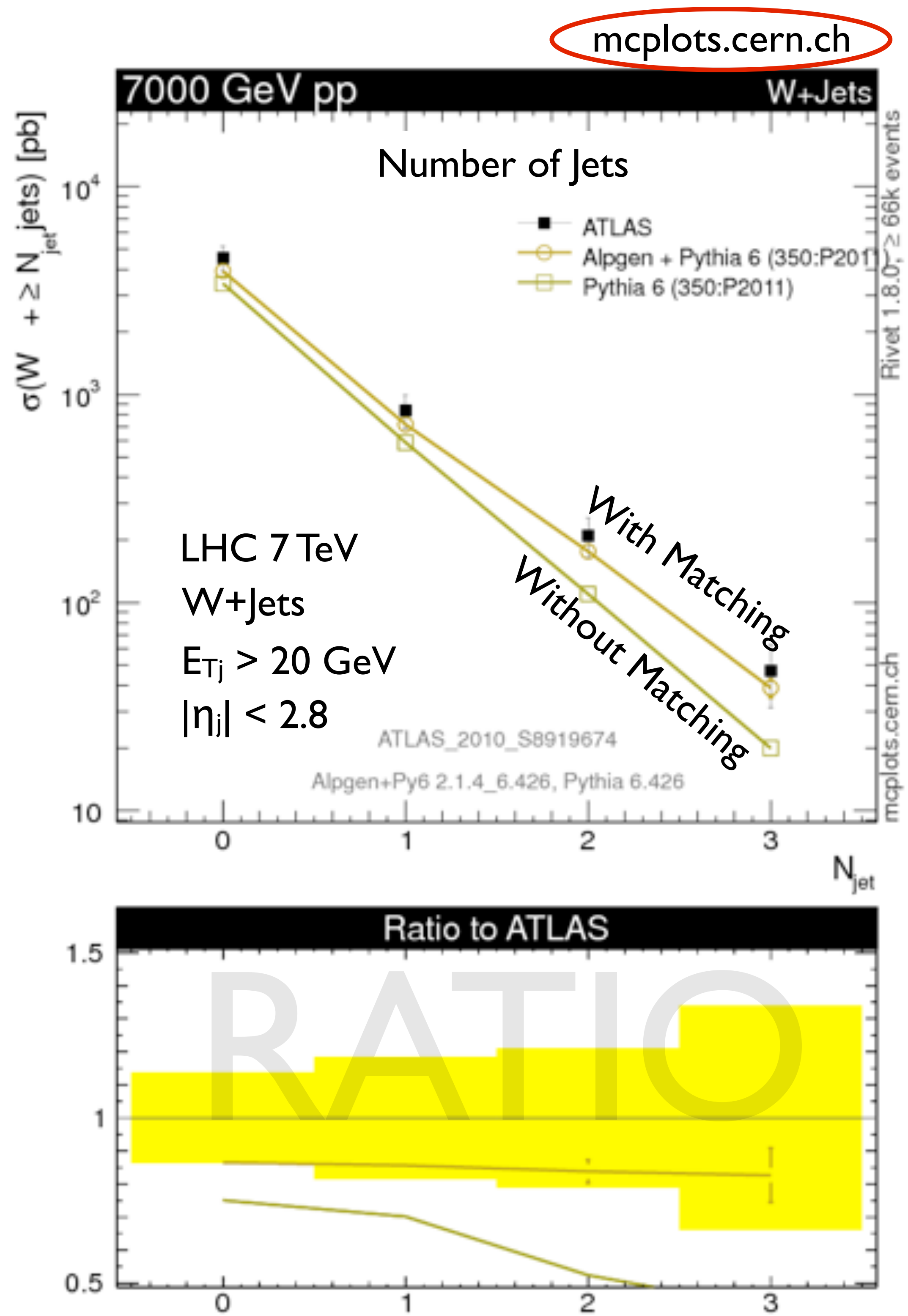
Number of jets in pp→W+X at the LHC

From 0 (W inclusive) to W+3 jets

PYTHIA includes matching up to W+1 jet + shower

With ALPGEN, also the LO matrix elements for 2 and 3 jets are included

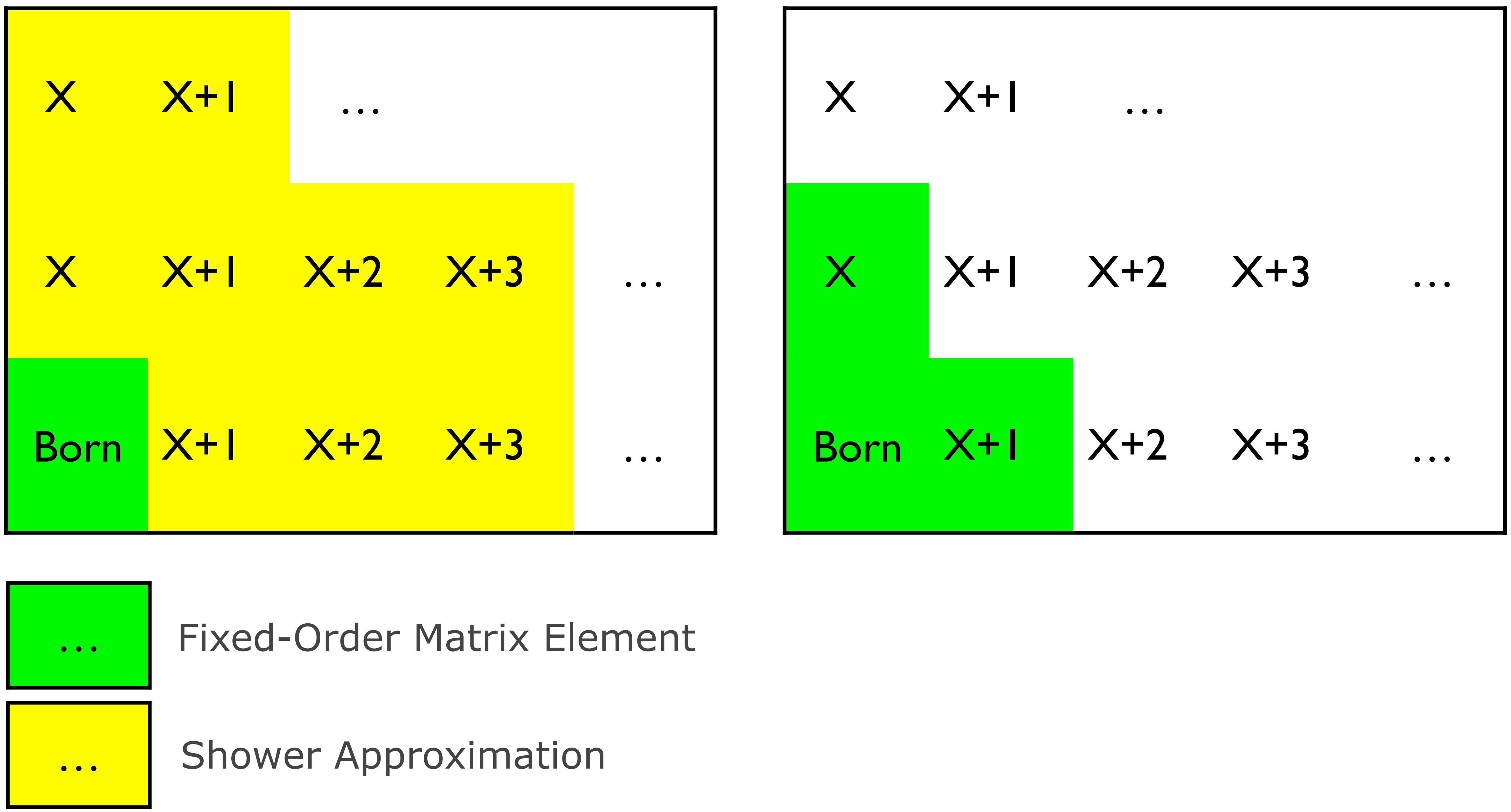
(but Normalization still only LO)



Matching 2: Subtraction

Examples: MC@NLO, aMC@NLO

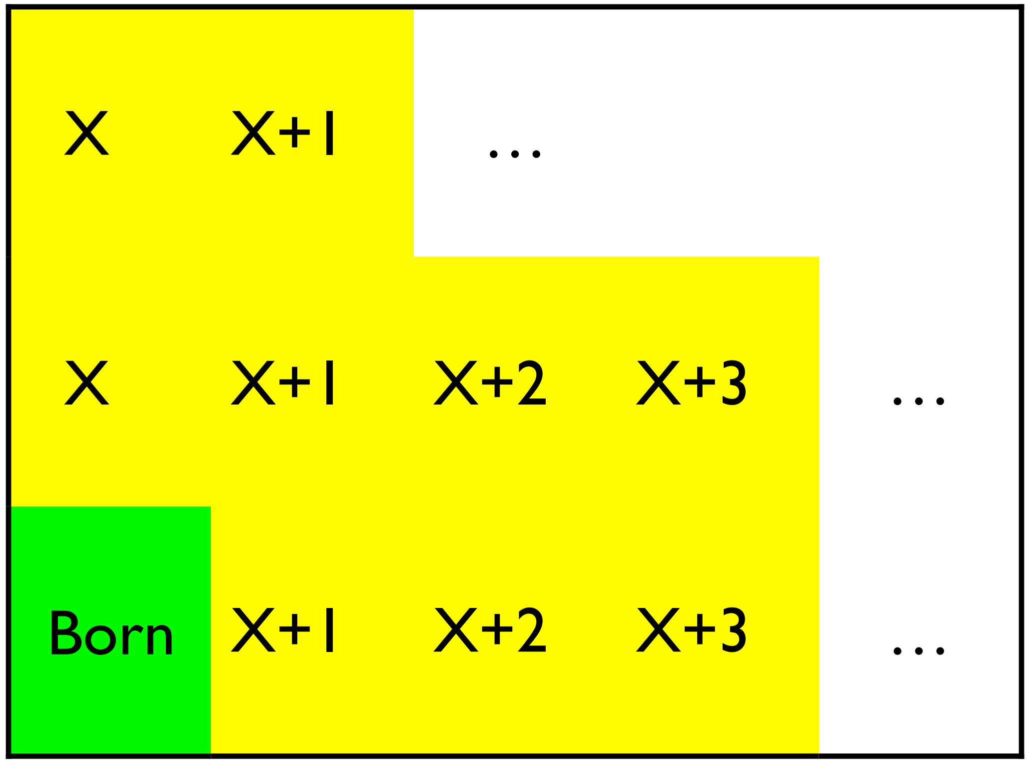
LO × Shower   NLO



Matching 2: Subtraction

Examples: MC@NLO, aMC@NLO

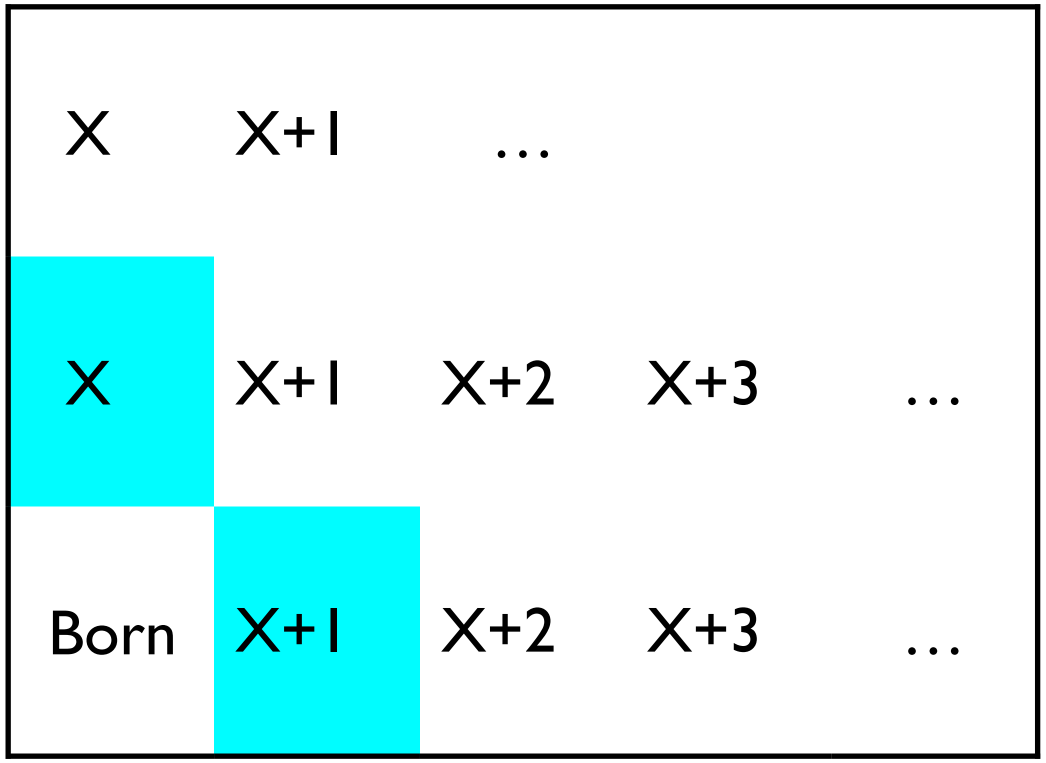
LO × Shower



… Fixed-Order Matrix Element

… Shower Approximation

NLO - Shower$_{NLO}$



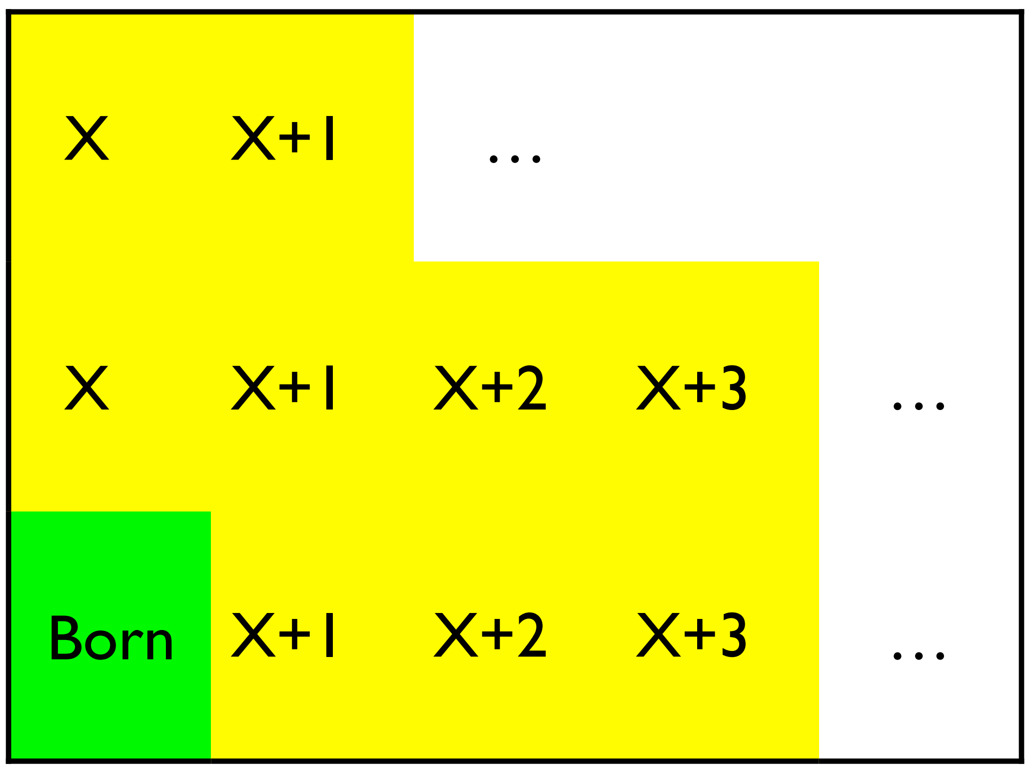
Expand shower approximation to NLO analytically, then subtract:

… Fixed-Order ME minus Shower Approximation (NOTE: can be < 0!)

Matching 2: Subtraction

Examples: MC@NLO, aMC@NLO

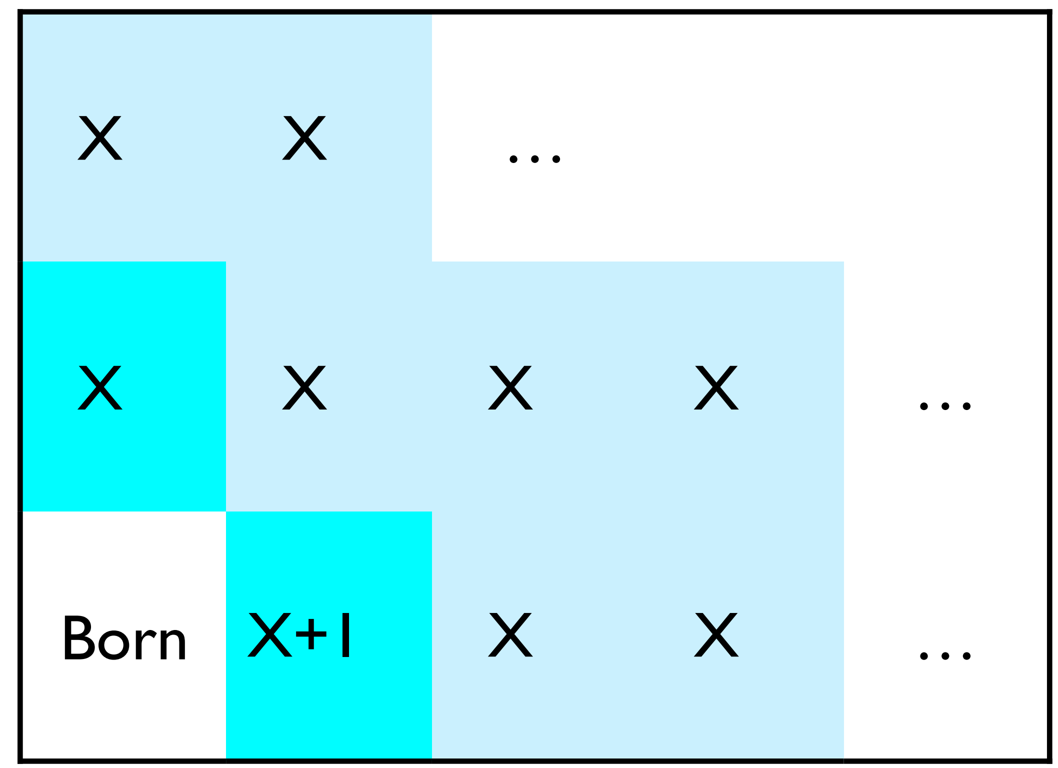
LO × Shower



… Fixed-Order Matrix Element

… Shower Approximation

(NLO - Shower$_{NLO}$) × Shower



… Fixed-Order ME minus Shower Approximation (NOTE: can be < 0!)

… Subleading corrections generated by shower off subtracted ME

Matching 2: Subtraction

Examples: MC@NLO, aMC@NLO

Combine → MC@NLO

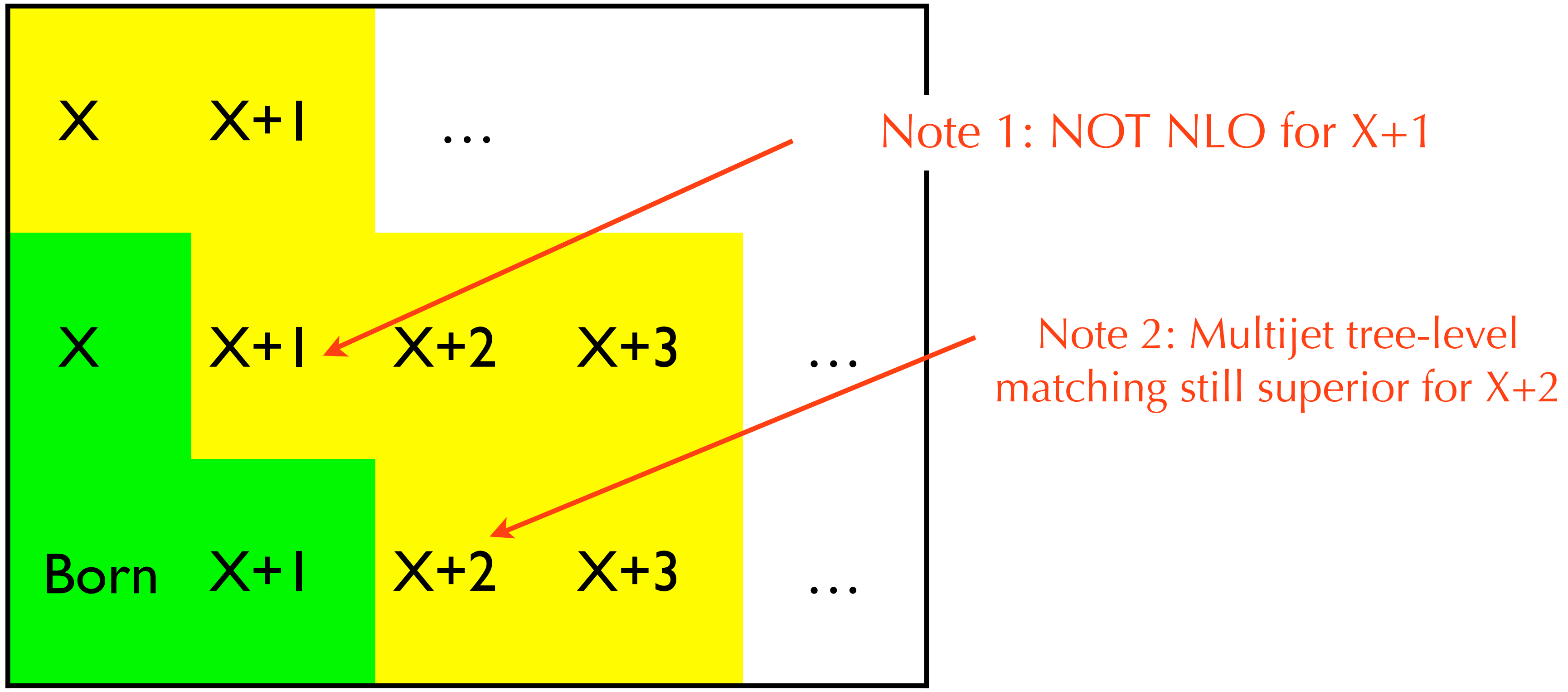
Frixione, Webber, JHEP 0206 (2002) 029

Consistent NLO + parton shower (though correction events can have w<0)

Recently, has been almost fully automated in aMC@NLO

Frederix, Frixione, Hirschi, Maltoni, Pittau, Torrielli, JHEP 1202 (2012) 048

NLO: for X inclusive
LO for X+1
LL: for everything else



NB: w < 0 are a problem because they kill efficiency:
Extreme example: 1000 positive-weight - 999 negative-weight events → statistical precision of 1 event, for 2000 generated (for comparison, normal MC@NLO has ~ 10% neg-weights)

Matching 3: ME Corrections

Standard Paradigm:

Have ME for X, X+1,…, X+n;

Want to combine and add showers → "The Soft Stuff"

Works pretty well at low multiplicities

Still, only corrected for "hard" scales; Soft still pure LL.

At high multiplicities:

Efficiency problems: slowdown from need to compute and generate phase space from $d\sigma_{X+n}$, and from unweighting (efficiency also reduced by negative weights, if present)

Scale hierarchies: smaller single-scale phase-space region

Powers of alphaS pile up

Better Starting Point: a QCD fractal?

(shameless VINCIA promo)

(plug-in to PYTHIA 8 for ME-improved final-state showers, uses helicity matrix elements from MadGraph)

Interleaved Paradigm:

Have shower; want to improve it using ME for X, X+1, …, X+n.

Interpret all-orders shower structure as a "trial distribution"

Quasi-scale-invariant: intrinsically multi-scale (resums logs)

Unitary: automatically unweighted (& IR divergences → multiplicities)

More precise expressions imprinted via veto algorithm: ME corrections at LO, NLO, … → soft *and* hard corrections

No additional phase-space generator or σ_{X+n} calculations → **fast**

+ Can get Automated Theory Uncertainties

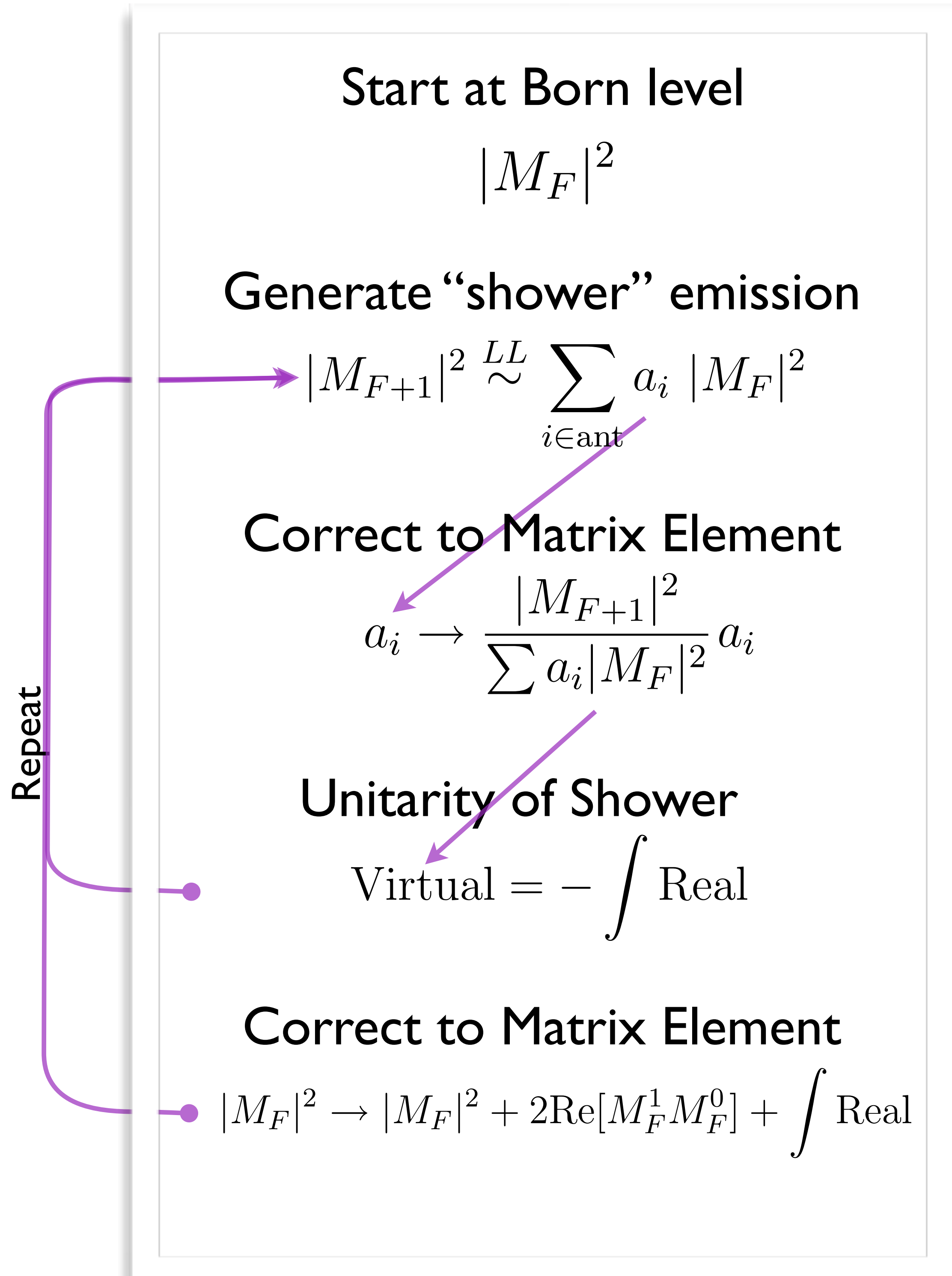
For each event: vector of output weights (central value = 1)

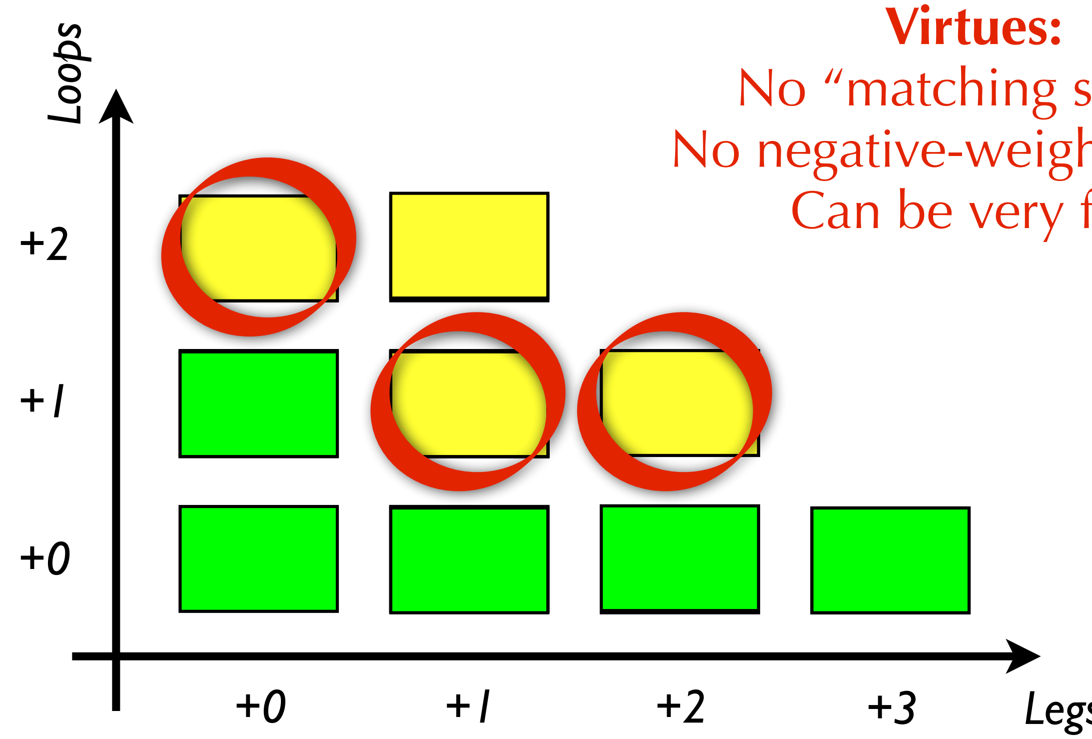
+ Uncertainty variations. Faster than N separate samples; only one sample to analyse, pass through detector simulations, etc.

LO: Giele, Kosower, Skands, PRD84(2011)054003 **NLO:** Hartgring, Laenen, Skands, arXiv:1303.4974

Matching 3: ME Corrections

Examples: PYTHIA, POWHEG, VINCIA





Virtues:
No "matching scale"
No negative-weight events
Can be very fast

First Order

PYTHIA: LO_1 corrections to most SM and BSM decay processes, and for pp → Z/W/H (Sjöstrand 1987)

POWHEG (& POWHEG BOX): LO_1 + NLO_0 corrections for generic processes (Frixione, Nason, Oleari, 2007)

Multileg NLO:

VINCIA: $LO_{1,2,3,4}$ + $NLO_{0,1}$ (shower plugin to PYTHIA 8; formalism for pp soon to appear) (see previous slide)

MiNLO-merged POWHEG: $LO_{1,2}$ + $NLO_{0,1}$ for pp → Z/W/H

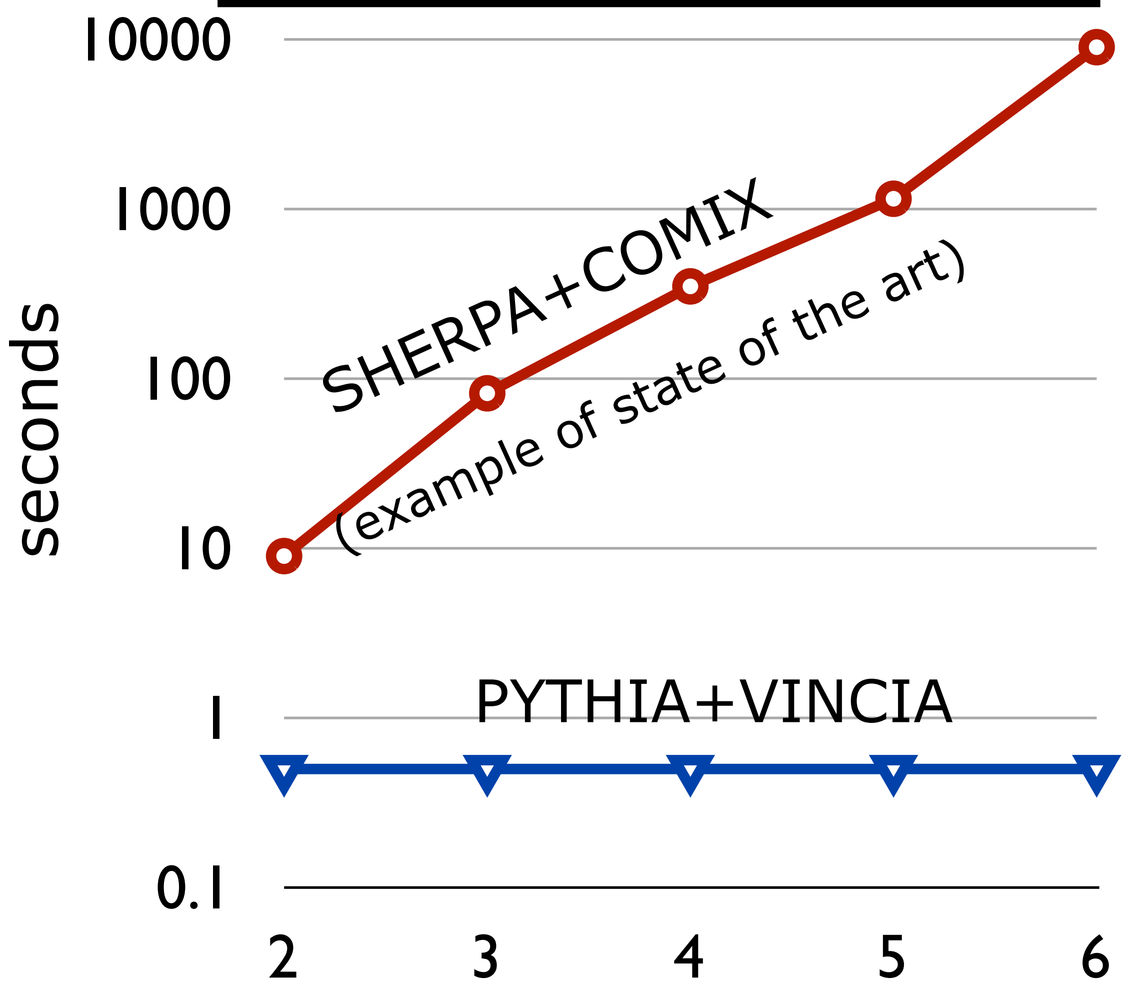
UNLOPS: for generic processes (in PYTHIA 8, based on POWHEG input) (Lönnblad & Prestel, 2013)

Illustrations from: PS, TASI Lectures, arXiv:1207.2389

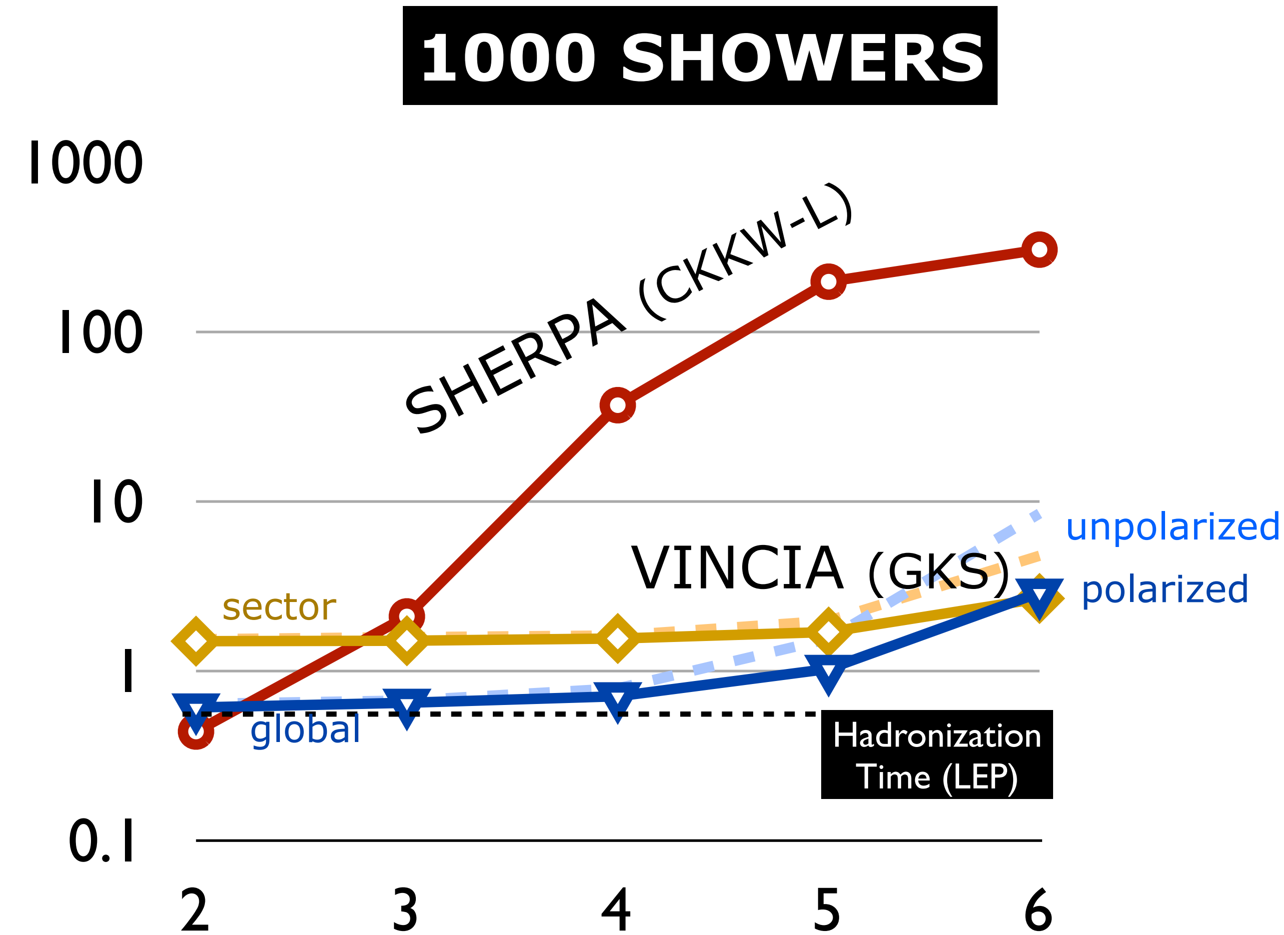
Speed

Larkoski, Lopez-Villarejo, Skands, PRD 87 (2013) 054033

1. Initialization time
(to pre-compute cross sections and warm up phase-space grids)



2. Time to generate 1000 events
(Z → partons, fully showered & matched. No hadronization.)



Z→udscb ; Hadronization OFF ; ISR OFF ; udsc MASSLESS ; b MASSIVE ; E_{CM} = 91.2 GeV ; Q_{match} = 5 GeV
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gcc/gfortran v 4.7.1 -O2 ; single 3.06 GHz core (4GB RAM)

Summary: Two ways to compute Quantum Corrections

Fixed Order: consider a specific physical process

Explicit solutions (to given perturbative order)

Standard-Model: typically NLO or NNLO
Beyond-SM: typically LO or NLO

LO: Leading Order (Born)
NLO = Next-to-LO, …

Limited generality

Event generators: consider all possible physical processes (within perturbative QFT)

Approximate solutions

LL: Leading Log + some NLL = Next-to-LL, …

Process-dependence = subleading correction (→ matching)

Maximum generality

Emphasis is on universalities; physics
Common property of all processes is, eg, the limits in which they factorize!

Increasingly, the gold standard is calculations that combine the best of both worlds!
These are, however, subtle, and the structure of the perturbative series remains intriguing

Simple Monte Carlo Example: Number of AEPSHEP students who will get hit by a car this week

Complicated Function:

Time-dependent

Traffic density during day, week-days vs week-ends

(i.e., non-trivial time evolution of system)

No two students are the same

Need to compute probability for each and sum

(simulates having several distinct types of “evolvers”)

Multiple outcomes:

Hit → keep walking, or go to hospital?

Multiple hits = Product of single hits, or more complicated?

Monte Carlo Approach

Approximate Traffic

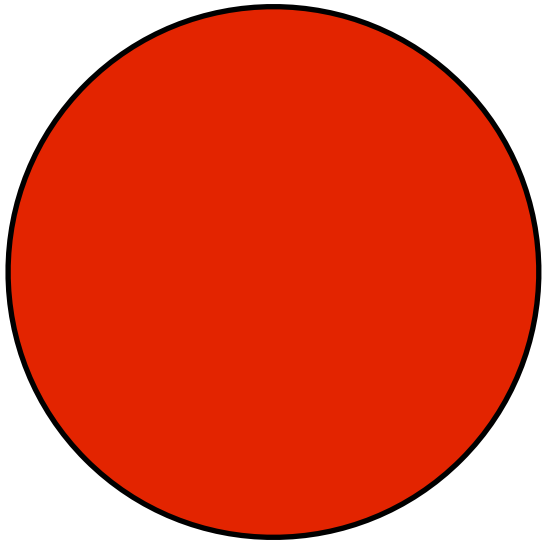
Simple overestimate:

highest recorded density
of most careless drivers,
driving at highest recorded speed
…

Approximate Student

by most completely reckless and accident-prone student
(wandering the streets lost in thought after these lectures …)

This extreme guess will be the equivalent of our simple overestimate from yesterday:



Hit Generator

Off we go…

Throw random accidents according to:

$$R = \int_{t_0}^{t_e} \mathrm{d}t \int_x \mathrm{d}x \sum_{i=1}^{n_{\mathrm{stud}}} \alpha_i(x,t)\, \rho_i(x,t)\, \rho_c(x,t)$$

Sum over students

Student-Car hit rate

Density of Student i

Density of Cars

Too Difficult

t_e : time of accident

$$R = (t_e - t_0)\Delta x \quad \alpha_{\max}\; n_{\mathrm{stud}}\; \rho_{c\max}$$

Hit rate for most accident-prone student

Rush-hour density of cars

Simple Overestimate

(Also generate trial x_e, e.g., uniformly in circle around Puri)

(Also generate trial *i;* a random student gets hit)

Hit Generator

Accept trial hit (i,x,t) with probability

$$\text{Prob(accept)} = \frac{\alpha_i(x,t)\ \rho_i(x,t)\ \rho_c(x,t)}{\alpha_{\text{max}}\ n_{\text{stud}}\ \rho_{\text{cmax}}}$$

Using the following:

ρ_c : actual density of cars at location x at time t

ρ_i : actual density of student i at location x at time t

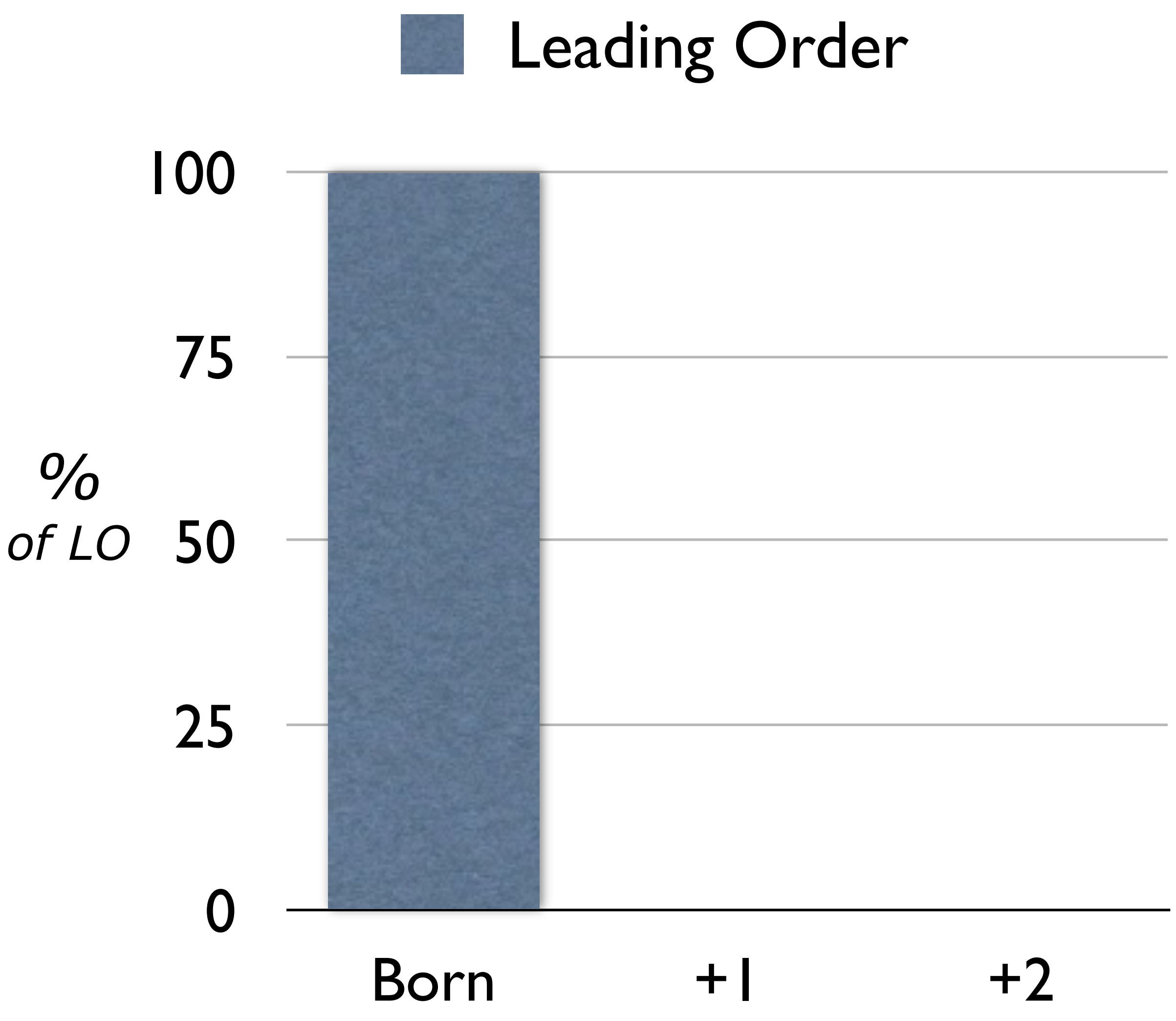
α_i : The actual "hit rate" (OK, not really known, but can make one up)

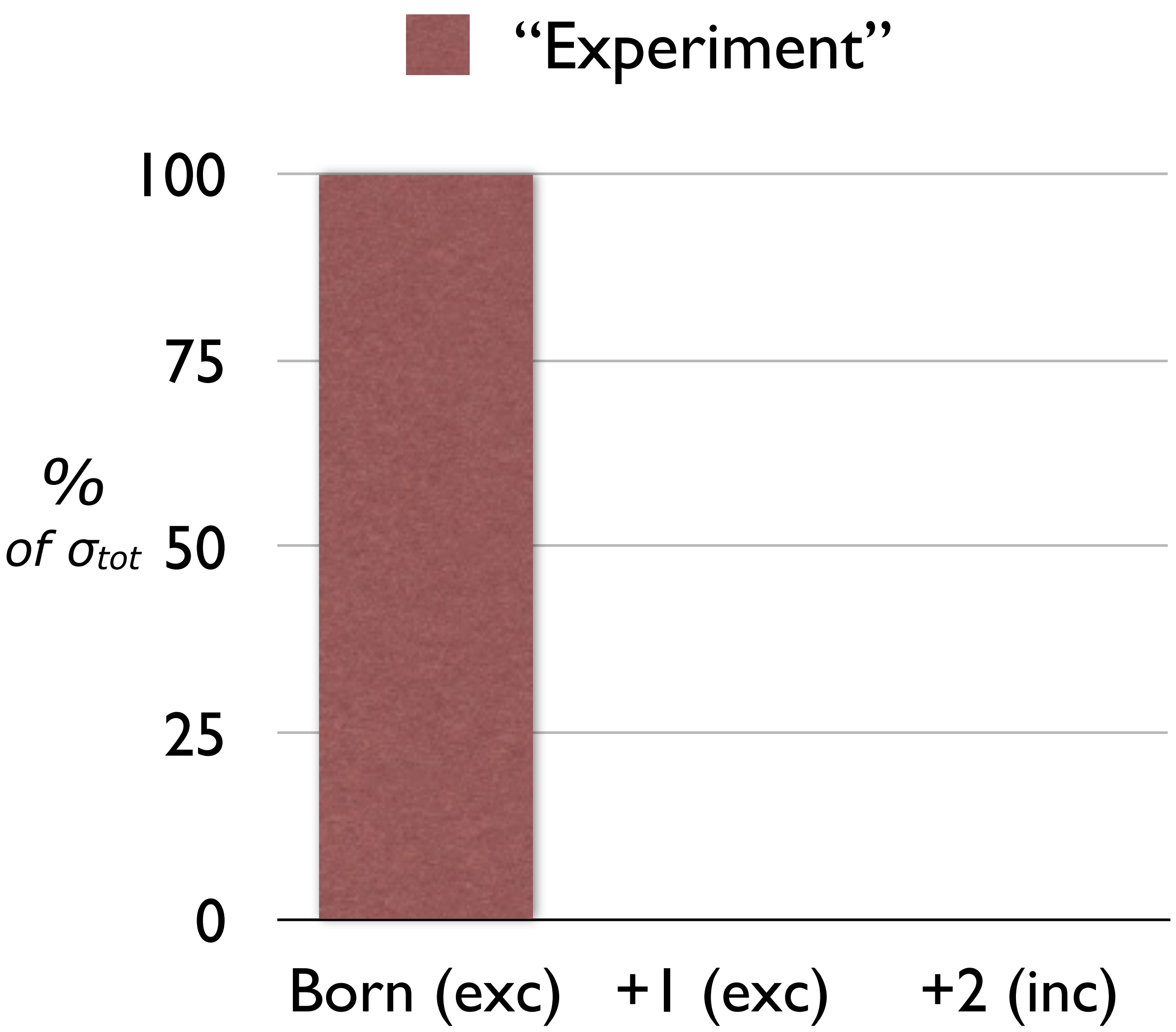
→ True number = number of accepted hits

(note: we didn't really treat multiple hits … → Markov Chain)

Evolution

$$Q \sim Q_X$$



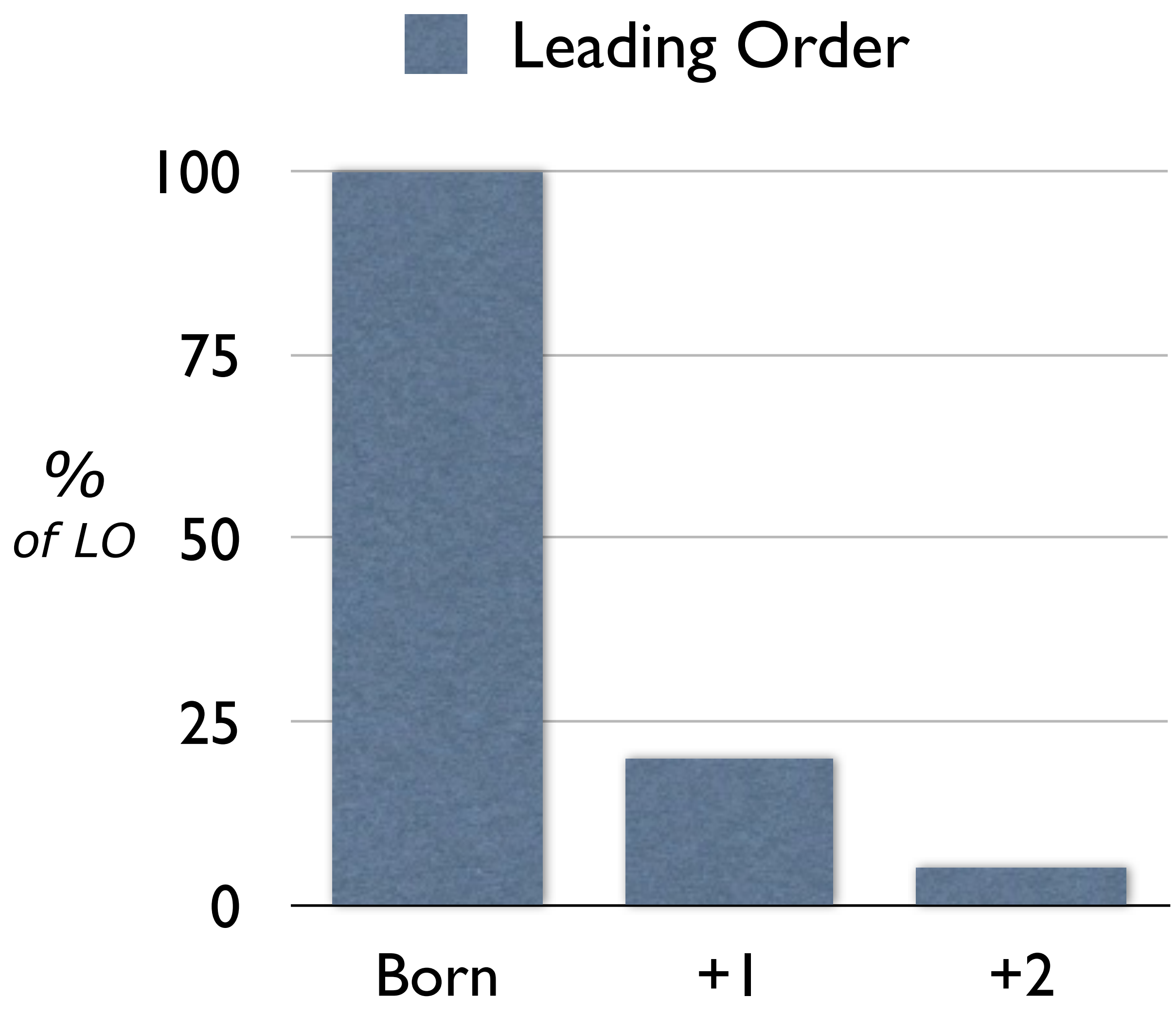


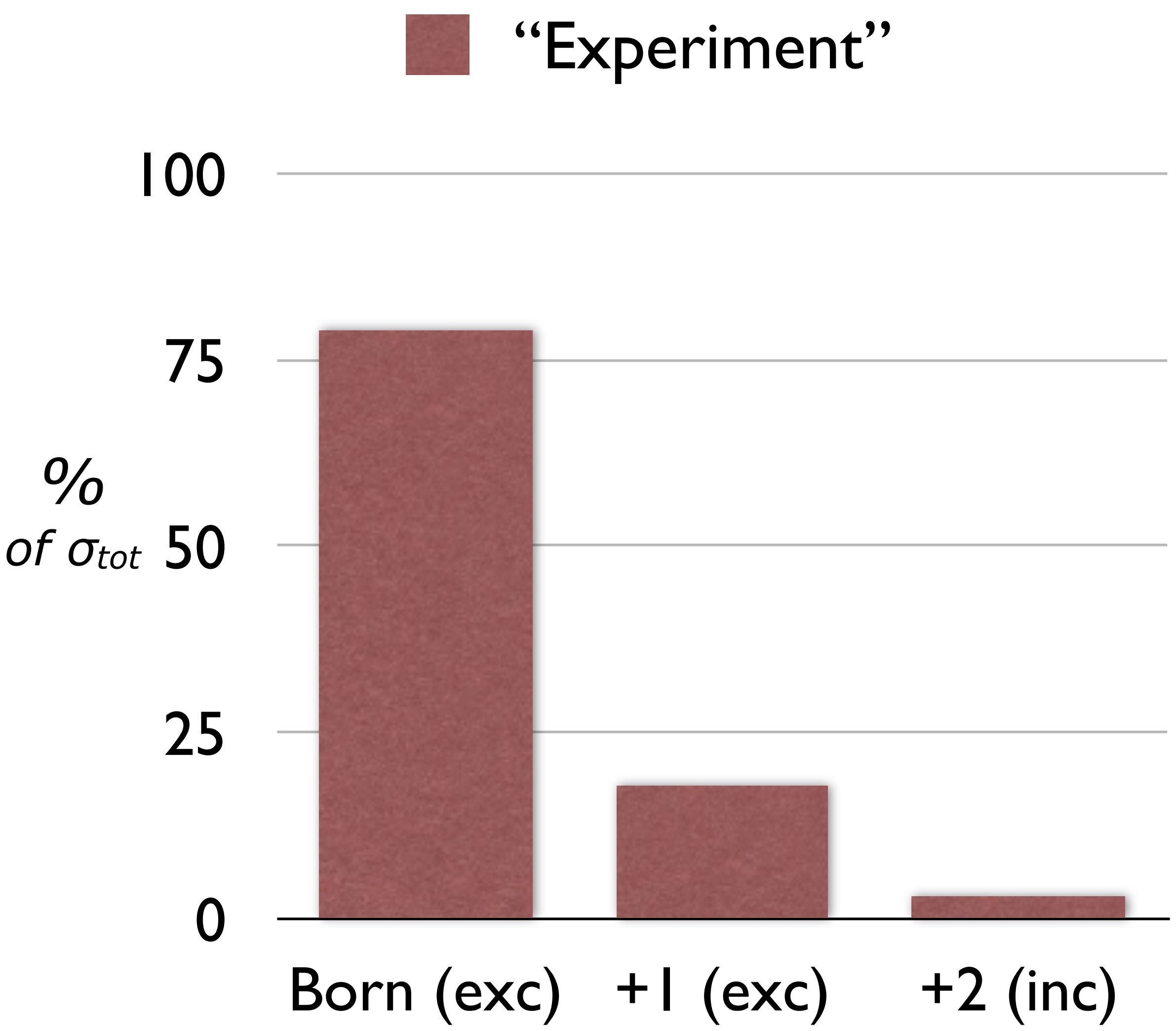
Exclusive = n and only n jets

Inclusive = n or more jets

Evolution

$$Q \sim \frac{Q_X}{\text{``A few''}}$$



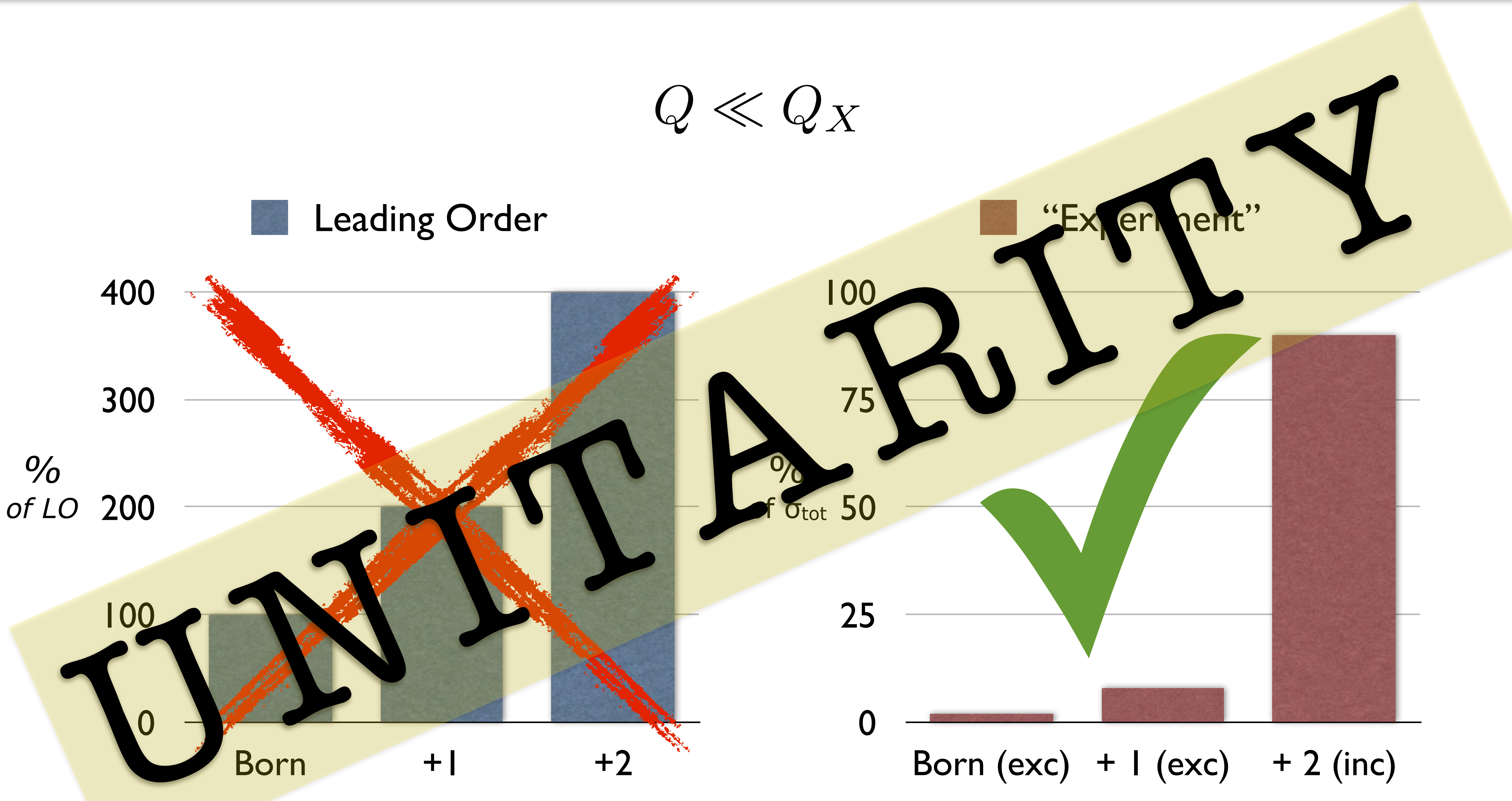


Exclusive = n and only n jets

Inclusive = n or more jets

Evolution

$$Q \ll Q_X$$



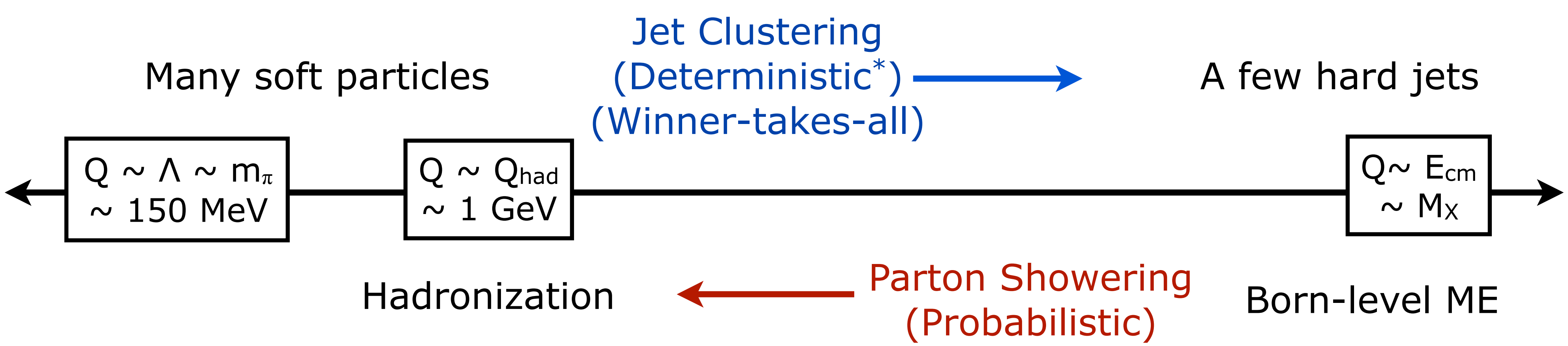
Cross Section Diverges

Cross Section Remains = Total (IR safe)
Number of Partons Diverges (IR unsafe)

Jets vs **Parton Showers**

Jet clustering algorithms

Map event from low E-resolution scale (i.e., with many partons/hadrons, most of which are soft) to a higher E-resolution scale (with fewer, hard, IR-safe, jets)



Parton shower algorithms

Map a few hard partons to many softer ones

Probabilistic → closer to nature.
Not uniquely invertible by any jet algorithm*

(* See "Qjets" for a probabilistic jet algorithm, arXiv:1201.1914)
(* See "Sector Showers" for a deterministic shower, arXiv:1109.3608)

Slicing: Some Subtleties

Choice of slicing scale (=matching scale)

Fixed order must still be reliable when regulated with this scale

→ matching scale should never be chosen more than ~ one order of magnitude below hard scale.

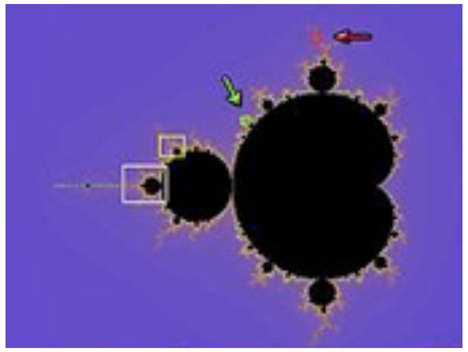
Precision still "only" Leading Order

Choice of Renormalization Scale

We already saw this can be very important (and tricky) in multi-scale problems.

Caution advised (see also supplementary slides & lecture notes)

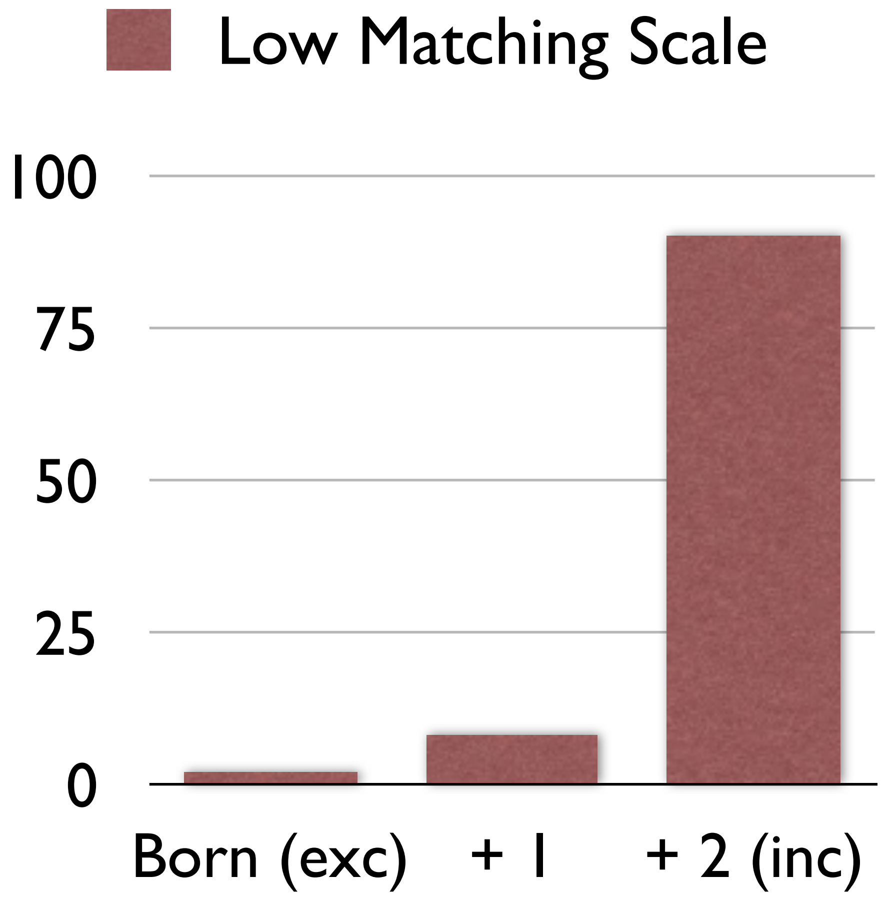
Choice of Matching Scale



Reminder: in perturbative region, QCD is approximately ***scale invariant***

→ A scale of 20 GeV for a W boson becomes 40 GeV for something weighing $2M_W$, etc … (+ adjust for C_A/C_F if g-initiated)

→ The matching scale should be written as a **ratio** (Bjorken scaling)
Using a too low matching scale → everything just becomes highest ME →

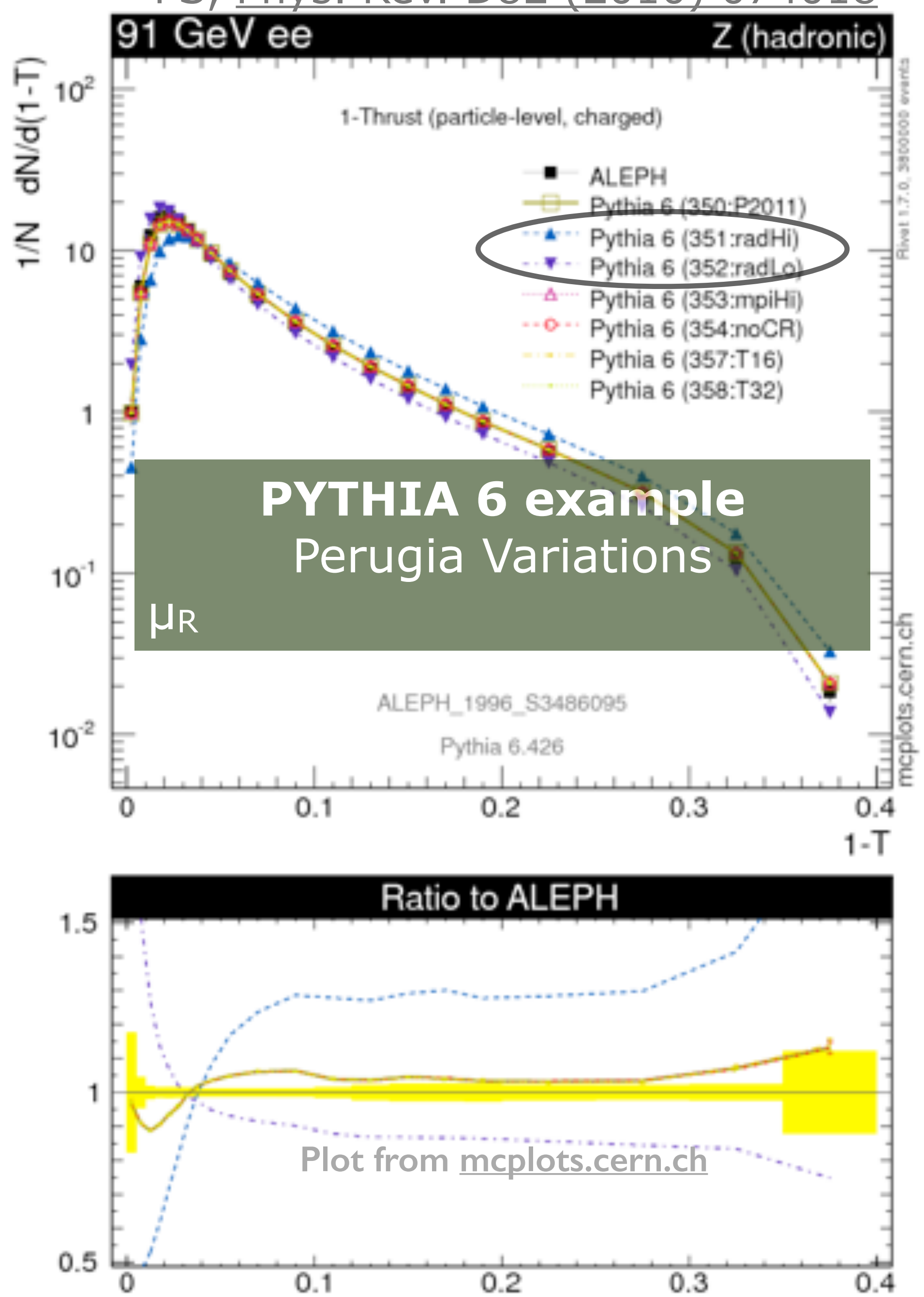


Caveat emptor: showers generally do not include helicity correlations

Uncertainty Estimates

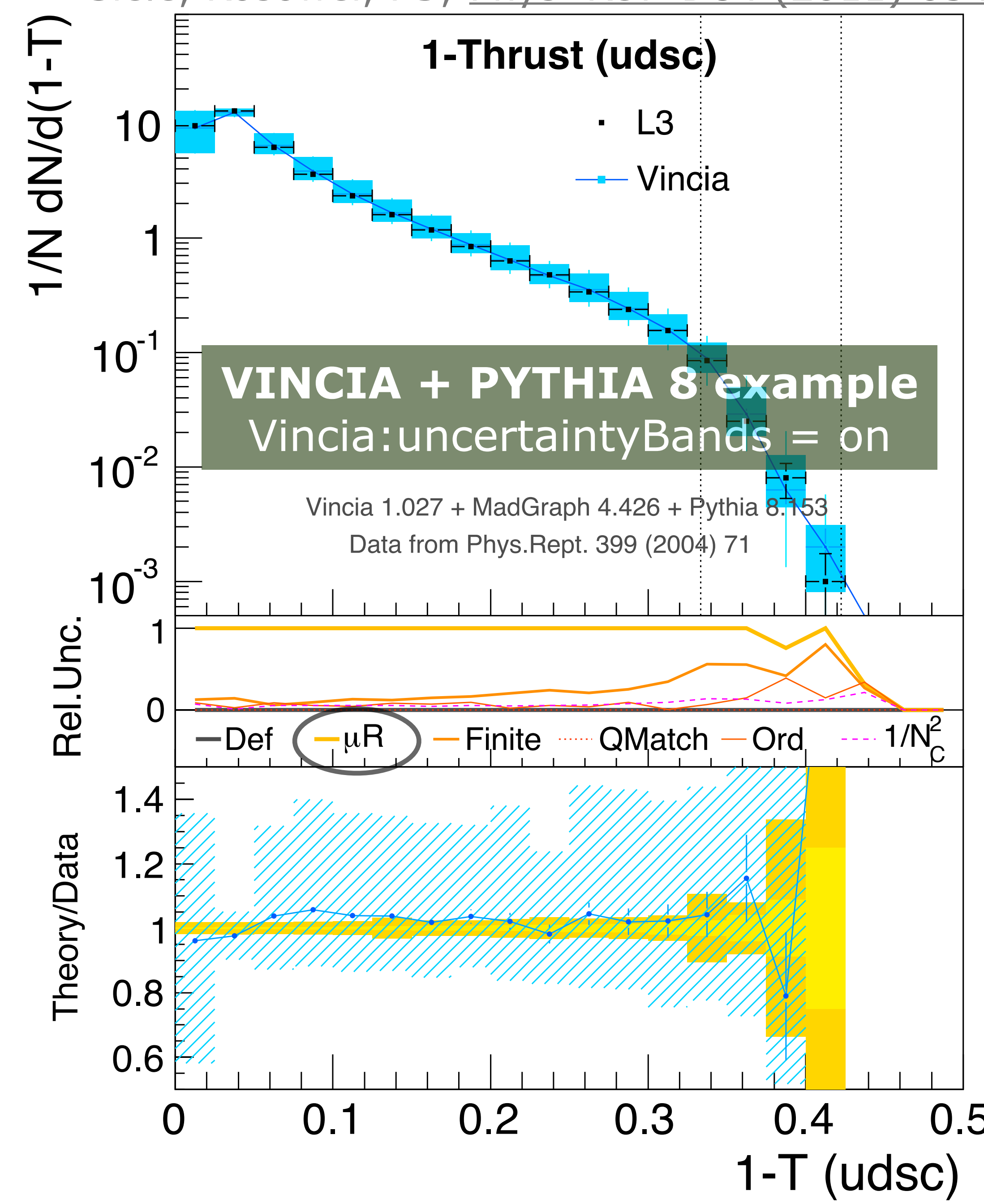
a) Authors provide specific "tune variations"
Run once for each variation→ envelope

PS, Phys. Rev. D82 (2010) 074018



b) **One** shower run
+ unitarity-based uncertainties → envelope

Giele, Kosower, PS; Phys. Rev. D84 (2011) 054003



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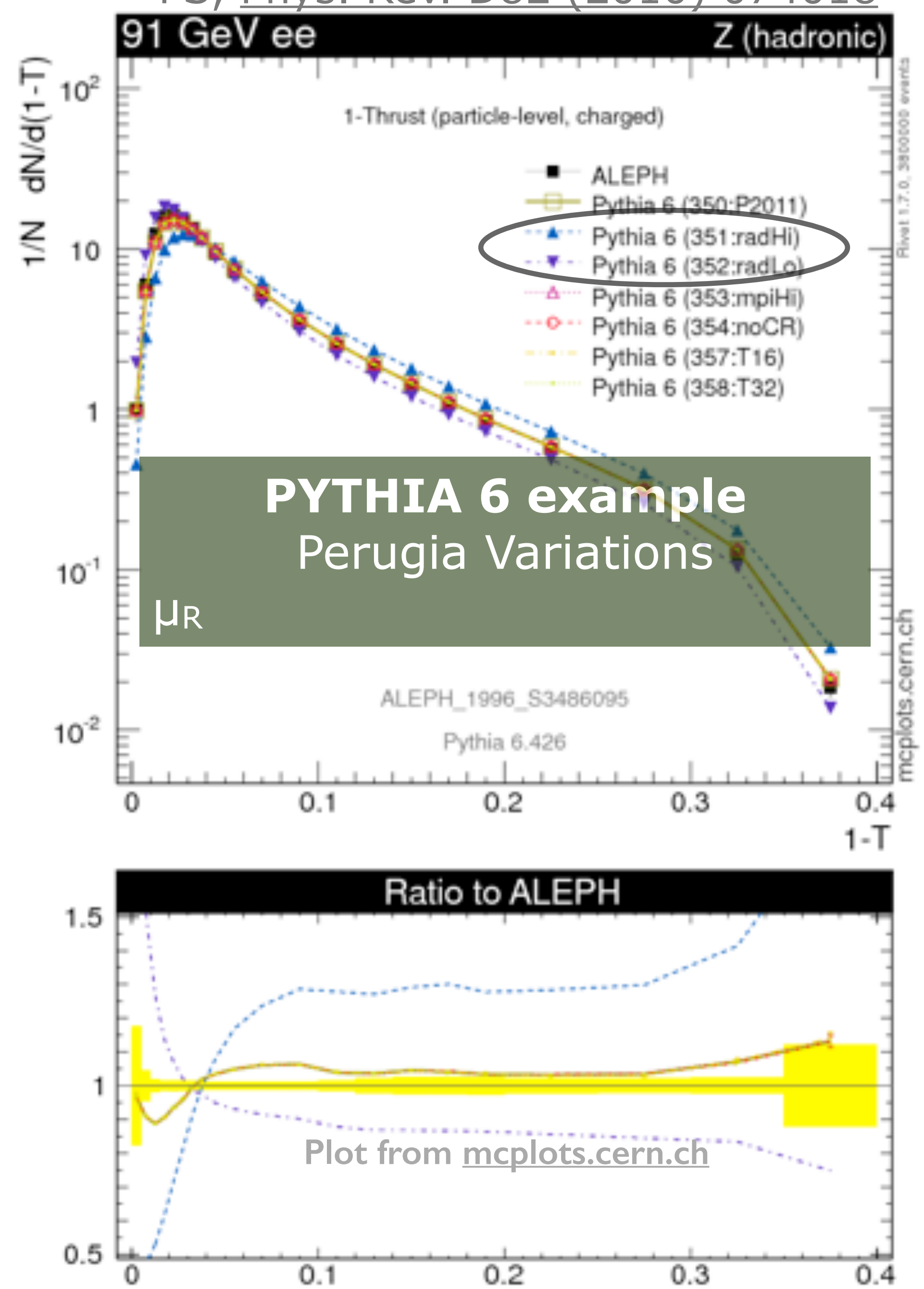

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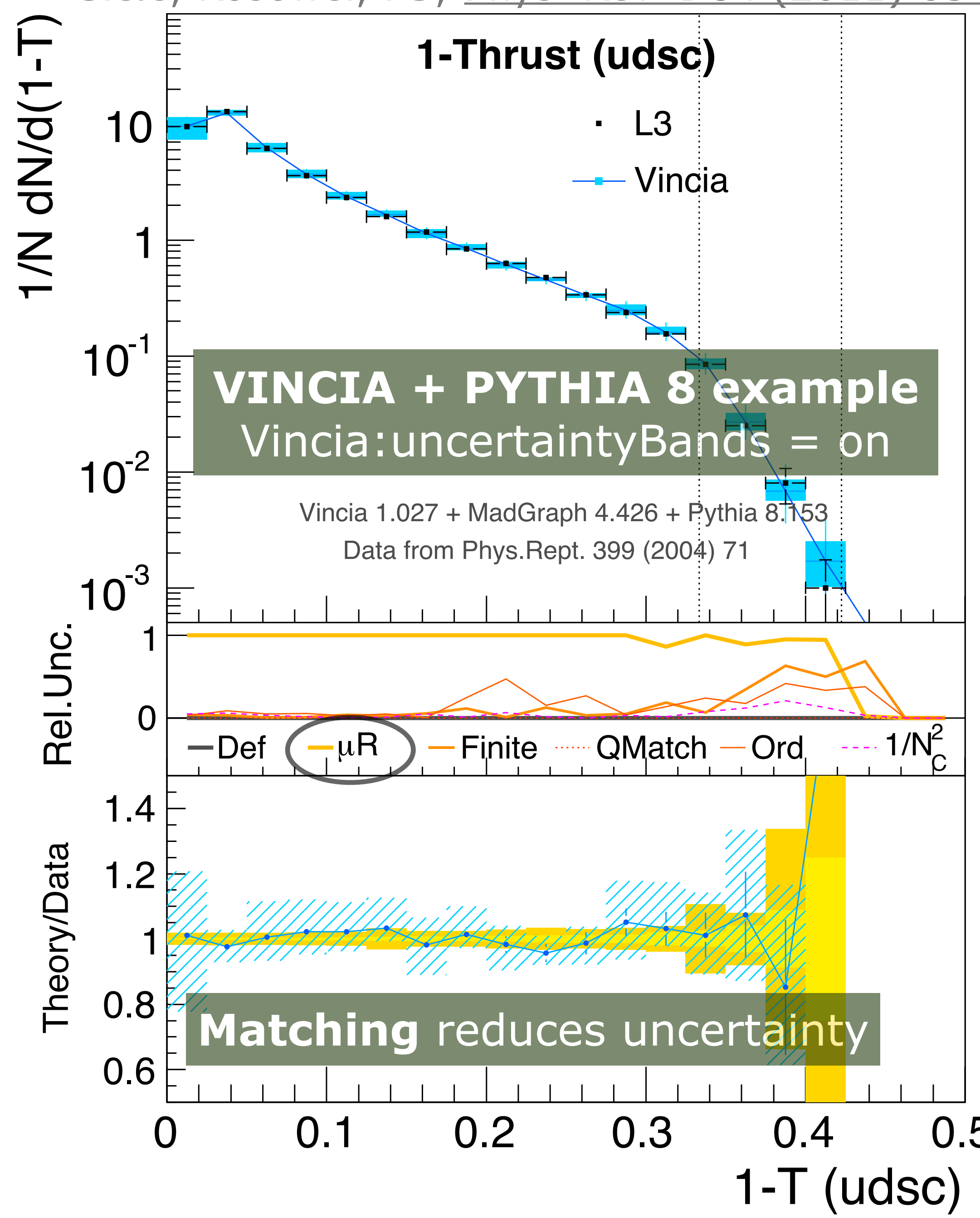
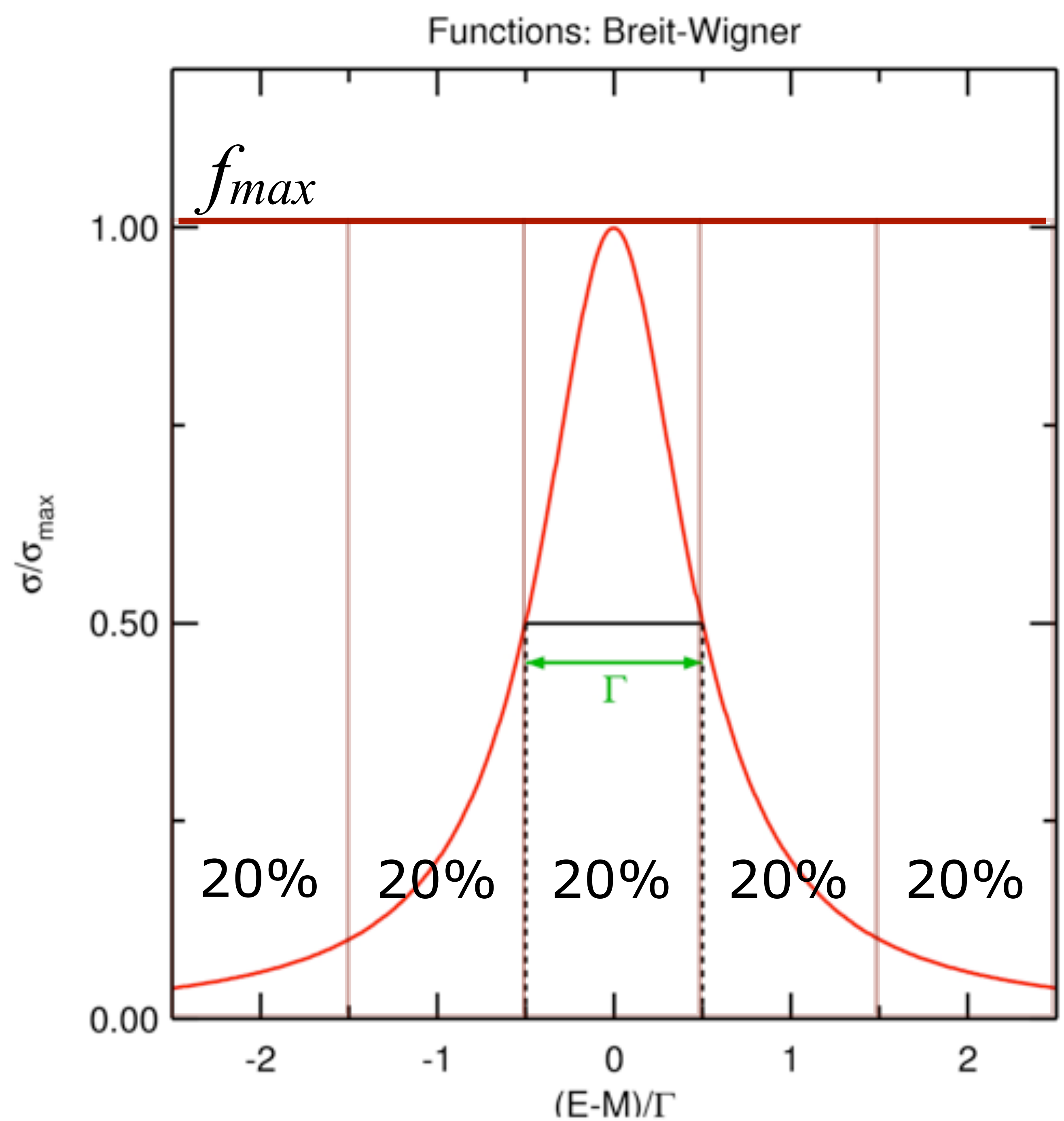


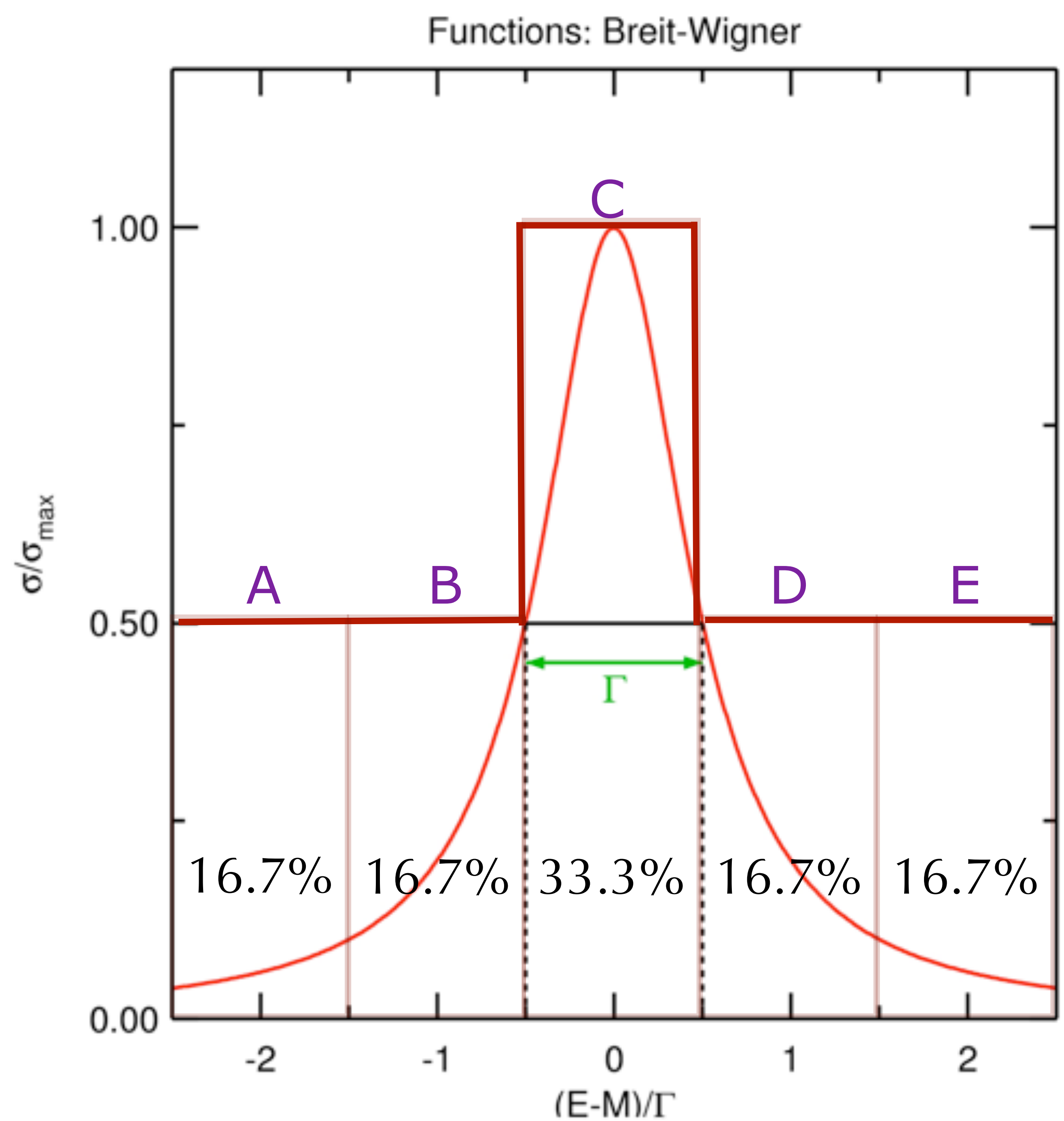
Peaked Functions



Precision on integral dominated by the points with $f \approx f_{max}$ *(i.e., peak regions)*

→ slow convergence if high, narrow peaks

Stratified Sampling



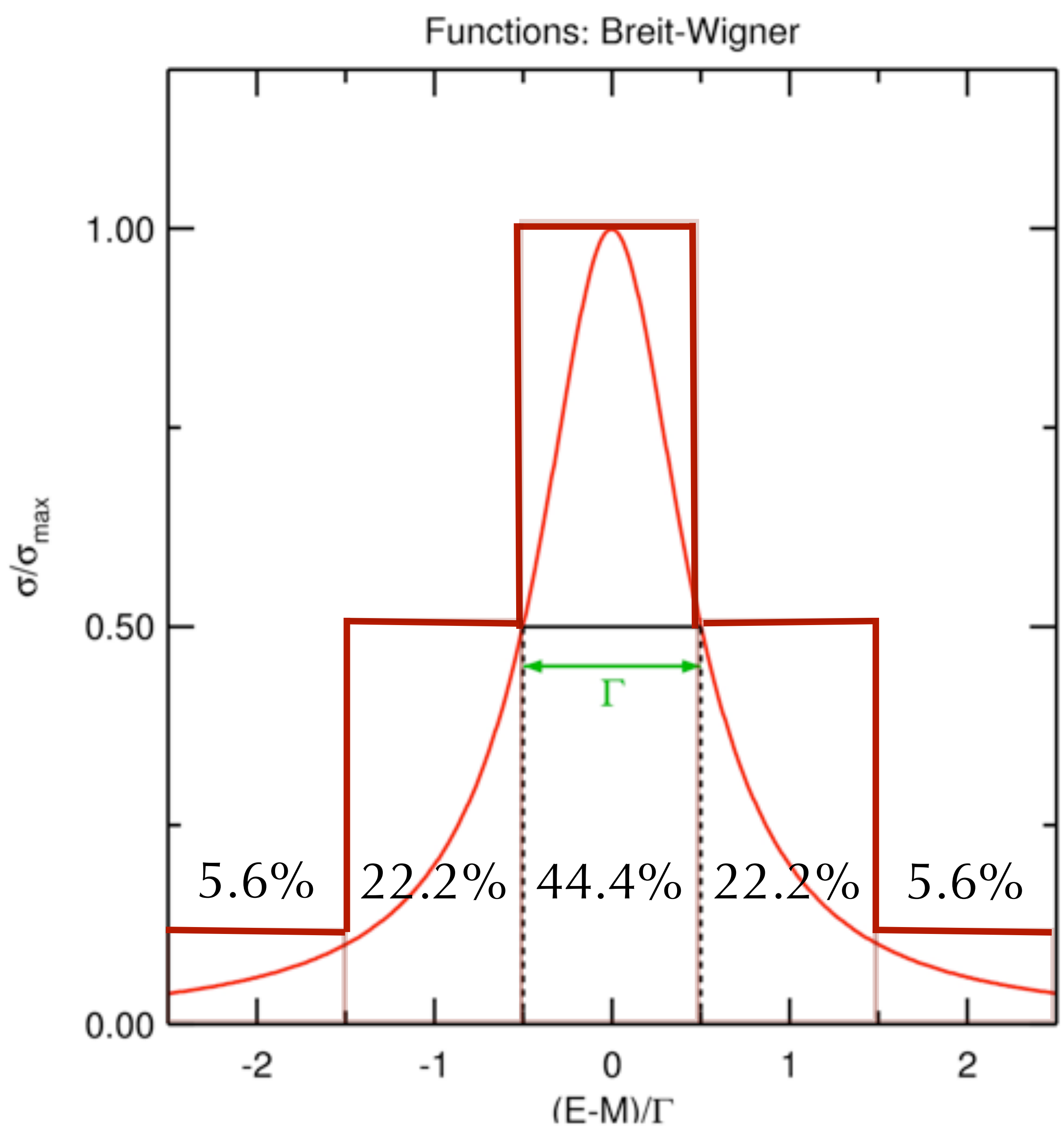
→ Make it twice as likely to throw points in the peak

Choose:

For: $6 * R_1 \in$
- [0,1] → Region A
- [1,2] → Region B
- [2,4] → Region C
- [4,5] → Region D
- [5,6] → Region E

→ faster convergence for same number of function evaluations

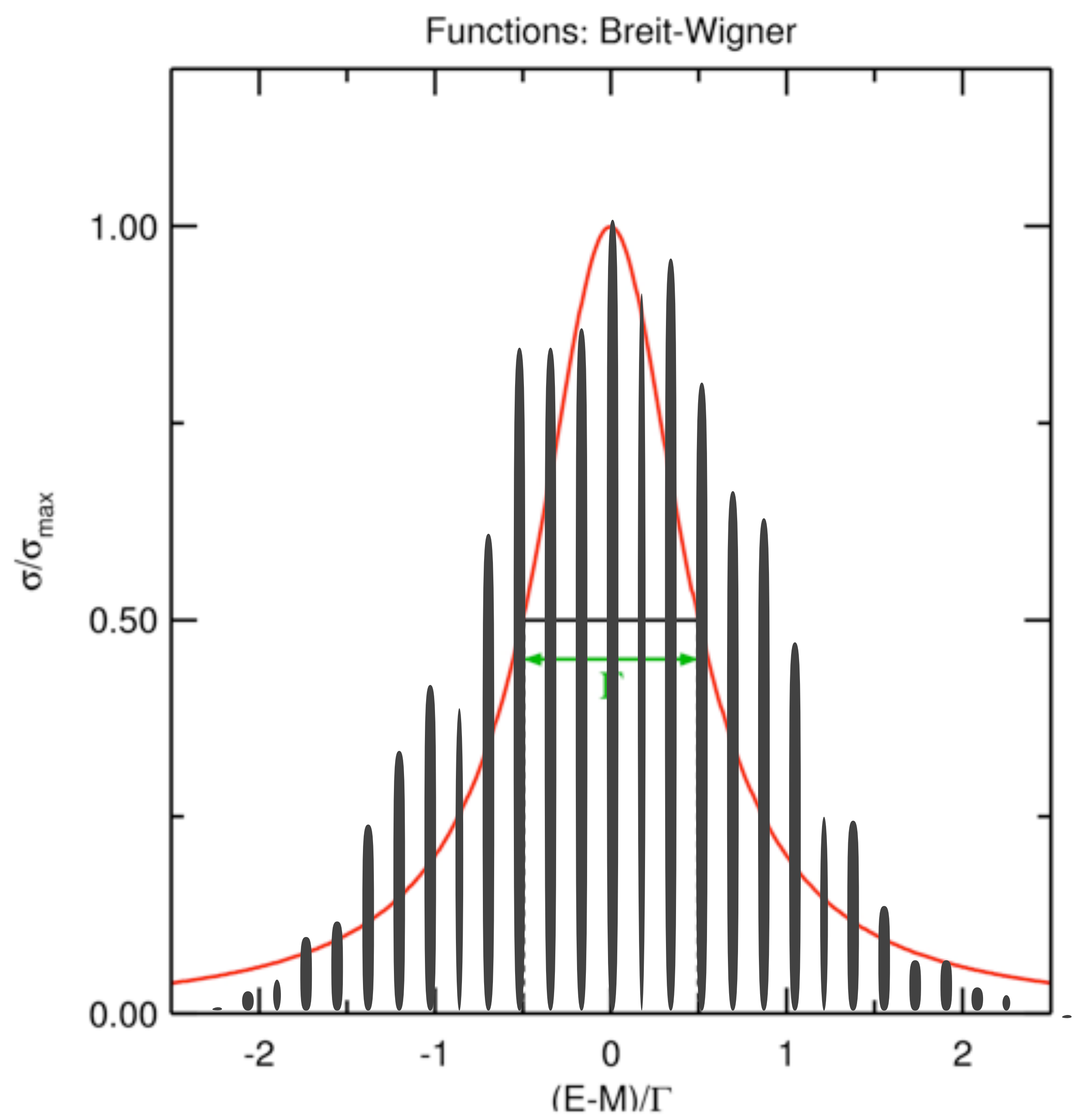
Adaptive Sampling



→ Can even design algorithms to do this automatically as they run (not covered here)

→ Adaptive sampling

Importance Sampling



→ or throw points according to some smooth peaked function for which you have, or can construct, a random number generator
(here: Gauss)

E.g., VEGAS algorithm, by G. Lepage

Why does this work?

1) You are inputting knowledge: obviously need to know where the peaks are to begin with … (say you know, e.g., the location and width of a resonance)

2) Stratified sampling increases efficiency by combining fixed-grid methods with the MC method, with further gains from adaptation

3) Importance sampling:

$$\int_a^b f(x)\mathrm{d}x = \int_a^b \frac{f(x)}{g(x)}\mathrm{d}G(x)$$

Effectively does flat MC with changed integration variables

Fast convergence if $f(x)/g(x) \approx 1$

How we do Monte Carlo

Take your system

Set of radioactive nuclei

Set of hard scattering processes

Set of resonances that are going to decay

Set of particles coming into your detector

Set of cosmic photons traveling across the galaxy

Set of molecules

…

How we do Monte Carlo

Take your system

Generate a "trial" (event/decay/interaction/…)

Not easy to generate random numbers distributed according to exactly the right distribution?

May have complicated dynamics, interactions …

→ use a simpler "trial" distribution

Flat with some stratification

Or importance sample with simple overestimating function (for which you can generate random #s)

How we do Monte Carlo

Take your system

Generate a “trial” (event/decay/interaction/…)

Accept trial with probability f(x)/g(x)

f(x) contains all the complicated dynamics

g(x) is the simple trial function

If accept: replace with new system state

If reject: keep previous system state

no dependence on g in final result - only affects convergence rate

And keep going: generate next trial …

How we do Monte Carlo

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Sounds deceptively simple, but …

with it, you can integrate
arbitrarily complicated functions *(in particular chains of nested functions)*, over arbitrarily complicated regions, in arbitrarily many dimensions …

And keep going: