On α_s variations in MC generators

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(from October 1st 2014 → Monash University, Melbourne)

Disclaimer: these slides are rather basic.
Transcribed from blackboard notes for Collider Cross Talk presentation

Where is α_s ?

$\alpha_s(M_Z)$

HEP MC GENERATOR

- ISR (0.137)
- ME (0.1265)
- FSR (0.1383)
- MPI (0.127)

(PYTHIA 8 DEFAULTS)

- PDF
- PDG : 0.1185(6)

What is α_s ?

$$\mu^2 \frac{d\alpha_s}{d\mu^2} = \frac{d\alpha_s}{d\ln\mu^2} = \beta(\alpha_s) \qquad \beta(\alpha_s) = -\alpha_s^2(b_0 + b_1\alpha_s + b_2\alpha_s^2 + \ldots)$$

$$\alpha_s(\mu^2) = \alpha_s(M_Z^2)\frac{1}{1 + b_0\alpha_s(M_Z^2)\ln(\mu^2/M_Z^2) + \ldots}$$

$$b_0 = \frac{11C_A - 4T_R n_F}{12\pi} \qquad b_1 = \frac{17C_A^2 - 10T_R C_A n_F - 6T_R C_F n_F}{24\pi^2} = \frac{153 - 19n_F}{24\pi^2}$$

$$\alpha_s^{(1)}(\mu^2) = \frac{1}{b_0\ln(\mu^2/\Lambda^2)}$$

$\Lambda \sim 200\text{MeV}$

$$\alpha_s^{(2)}(\mu^2) = \frac{1}{b_0\ln(\mu^2/\Lambda^2)} - \frac{b_1}{b_0}\frac{\ln\ln(\mu^2/\Lambda^2)}{\ln^2(\mu^2/\Lambda^2)}$$

Main Point:

Choose $\alpha_s(M_Z)$?
Choose Λ?
Choose k in $\alpha_s(k\mu)$?

All Equivalent

What is α_s ?

PDG: 0.119
ME : 0.127
PS: 0.138

$\alpha_s(M_Z)$

Different MC codes use different choices to parametrize

E.g., one code may ask you to specify Λ

Another may ask you to give the effective value of $\alpha_s(M_Z)$

And/or you may specify a pre-factor, k, in $\alpha_s(k\mu)$

Use eqs on previous slide to translate

Examples:

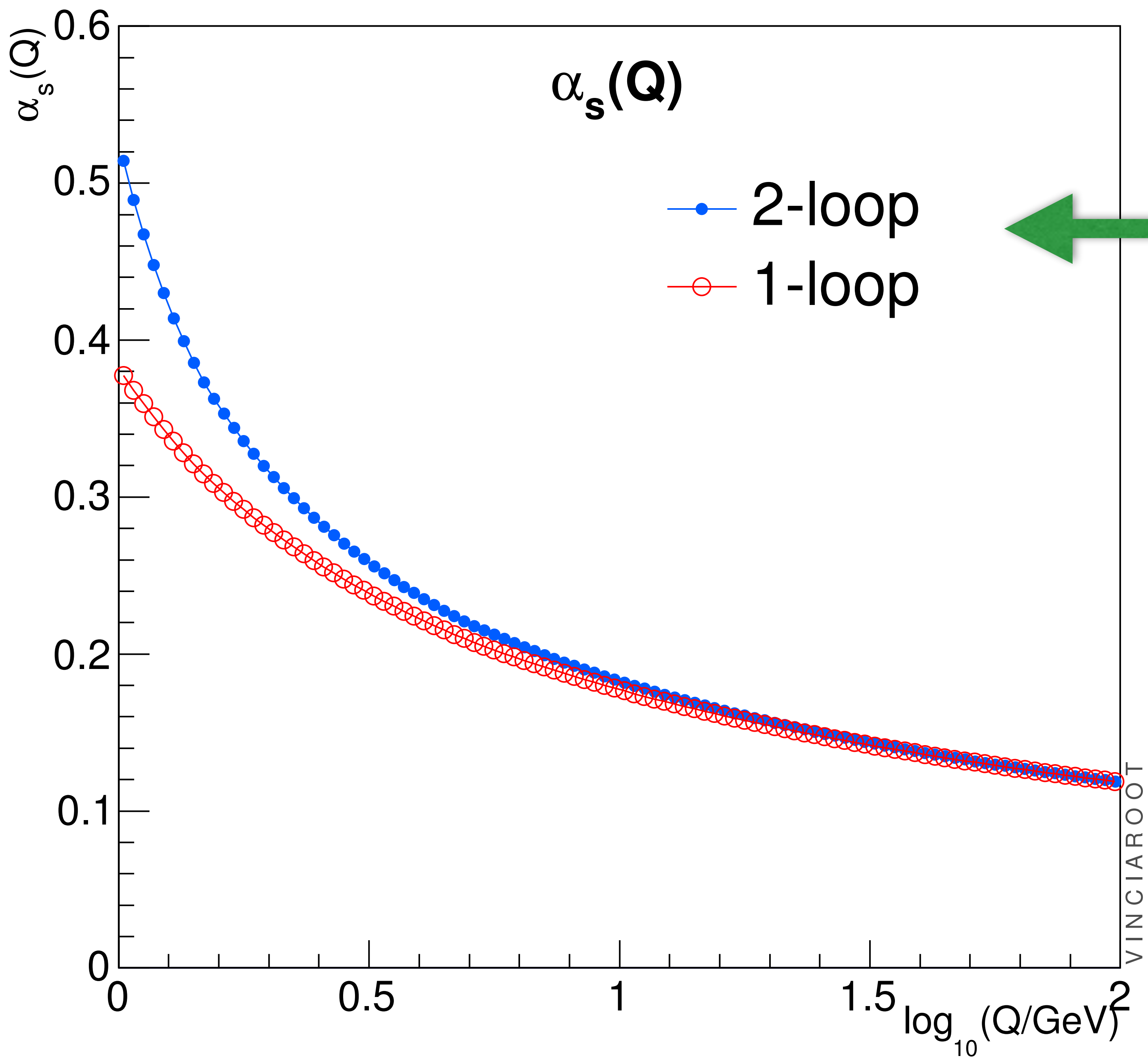
$$k = 0.38 \implies \alpha_s(kM_Z) = 0.14 \text{ for } \alpha_s^{(1)}(M_Z) = 0.12$$

$$k = 0.69 \implies \alpha_s(kM_Z) = 0.127 \text{ for } \alpha_s^{(1)}(M_Z) = 0.12$$

PDG: 0.119
ME : 0.127
PS: 0.138

1-Loop vs 2-Loop running

2-loop running is faster than 1-loop running



Larger $\Lambda^{(2)}$ for given $\alpha_s(M_Z)$

Smaller $\alpha_s^{(2)}(M_Z)$ for given Λ

Skands, TASI Lectures, arXiv:1207.2389

From $\overline{\mathrm{MS}}$ to MC

PDG: 0.119
ME : 0.127
PS: 0.138

$\alpha_s(M_Z)$

CMW Nucl Phys B 349 (1991) 635 : Drell-Yan and DIS processes

$A^{(1)}$

$$P(\alpha_s, z) = \frac{\alpha_s}{2\pi} C_F \frac{1+z^2}{1-z} + \left(\frac{\alpha_s}{\pi}\right)^2 \frac{A^{(2)}}{1-z}$$

Eg Analytic resummation (in Mellin space): General Structure

for DIS

$$\propto \exp\left[\int_0^1 dz \frac{z^{N-1}-1}{1-z}\left[\int \frac{dp_\perp^2}{p_\perp^2}\left(A(\alpha_s) + B(\alpha_s)\right)\right]\right]$$

$$B^{(1)} = -3C_F/2$$

$$A(\alpha_s) = A^{(1)}\frac{\alpha_s}{\pi} + A^{(2)}\left(\frac{\alpha_s}{\pi}\right)^2 + \ldots$$

$$A^{(2)} = \frac{1}{2} C_F \left(C_A\left(\frac{67}{18} - \frac{1}{6}\pi^2\right) - \frac{5}{9}N_F\right) = \frac{1}{2} C_F K_{\mathrm{CMW}}$$

Replace
(for z→1: soft gluon limit):

$$P_i(\alpha_s, z) = \frac{C_i \frac{\alpha_s}{\pi}\left(1 + K_{\mathrm{CMW}}\frac{\alpha_s}{2\pi}\right)}{1-z}$$

From $\overline{\text{MS}}$ to MC

PDG: 0.119
ME : 0.127
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CMW Nucl Phys B 349 (1991) 635 : Drell-Yan and DIS processes

$$P(\alpha_s, z) = \frac{\alpha_s}{2\pi} \overset{A^{(1)}}{C_F} \frac{1+z^2}{1-z} + \left(\frac{\alpha_s}{\pi}\right)^2 \frac{A^{(2)}}{1-z}$$

Replace
(for z→1: soft gluon limit):

$$P_i(\alpha_s, z) = \frac{C_i \frac{\alpha_s}{\pi} \left(1 + K_{\text{CMW}} \frac{\alpha_s}{2\pi}\right)}{1-z}$$

$$\alpha_s^{(\text{MC})} = \alpha_s^{(\overline{\text{MS}})} \left(1 + K_{\text{CMW}} \frac{\alpha_s^{(\overline{\text{MS}})}}{2\pi}\right)$$

$$\Lambda_{\text{MC}} = \Lambda_{\overline{\text{MS}}} \exp\left(\frac{K_{\text{CMW}}}{4\pi\beta_0}\right) \sim 1.57 \Lambda_{\overline{\text{MS}}}$$

(for nF=5)

Note also: used $\mu^2 = p_T^2 = (1-z)Q^2$
Amati, Bassetto, Ciafaloni, Marchesini, Veneziano, 1980

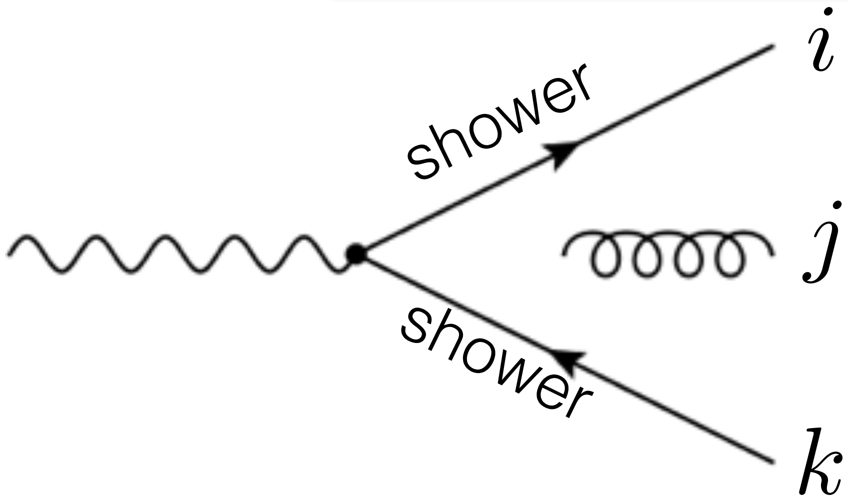
Main Point:

Doing an uncompensated scale variation actually ruins this result

Hartgring, Laenen, Skands, arXiv:1303.4974

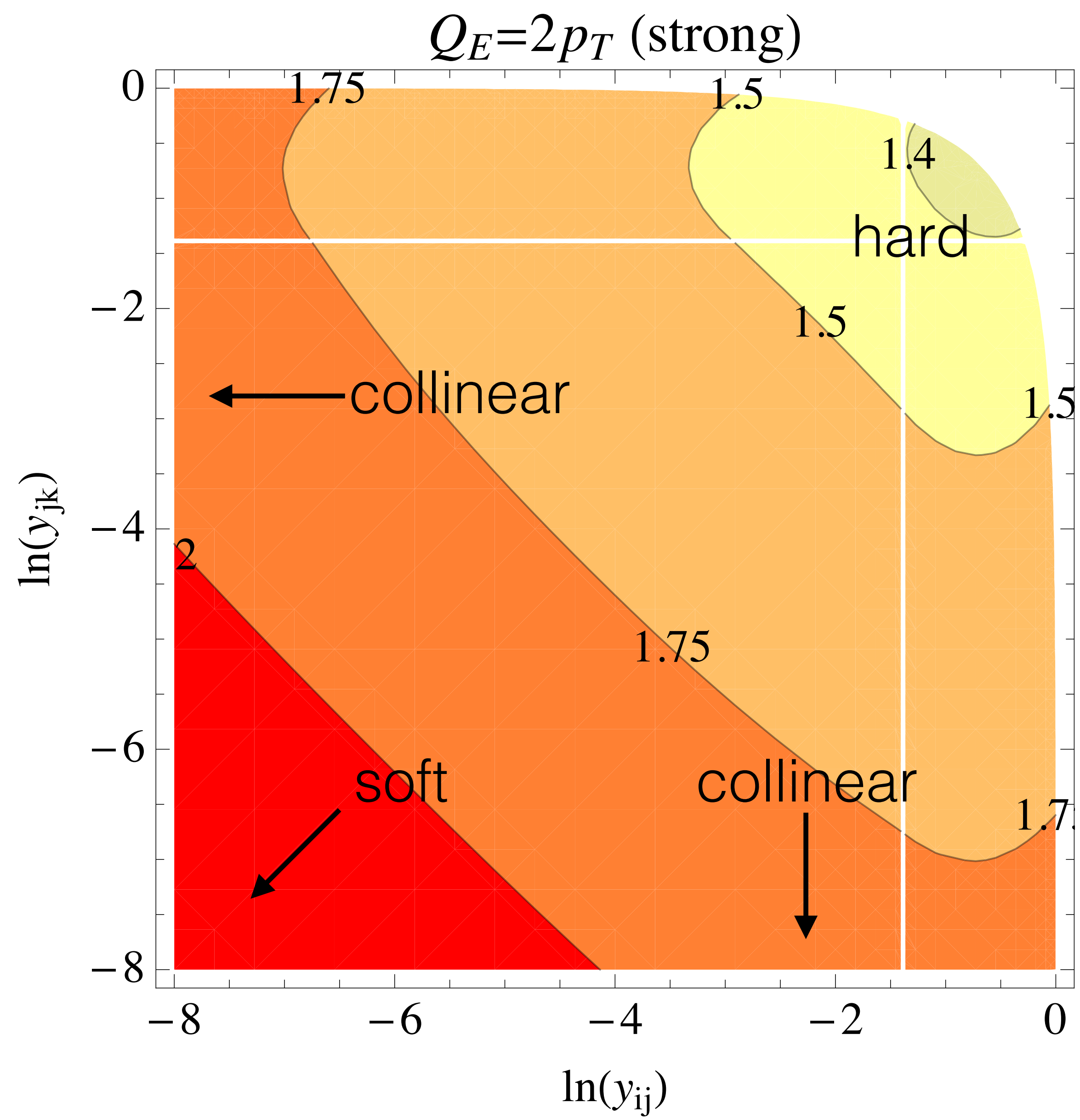
Z → 3 Jets

Size of NLO "K" factor over phase space

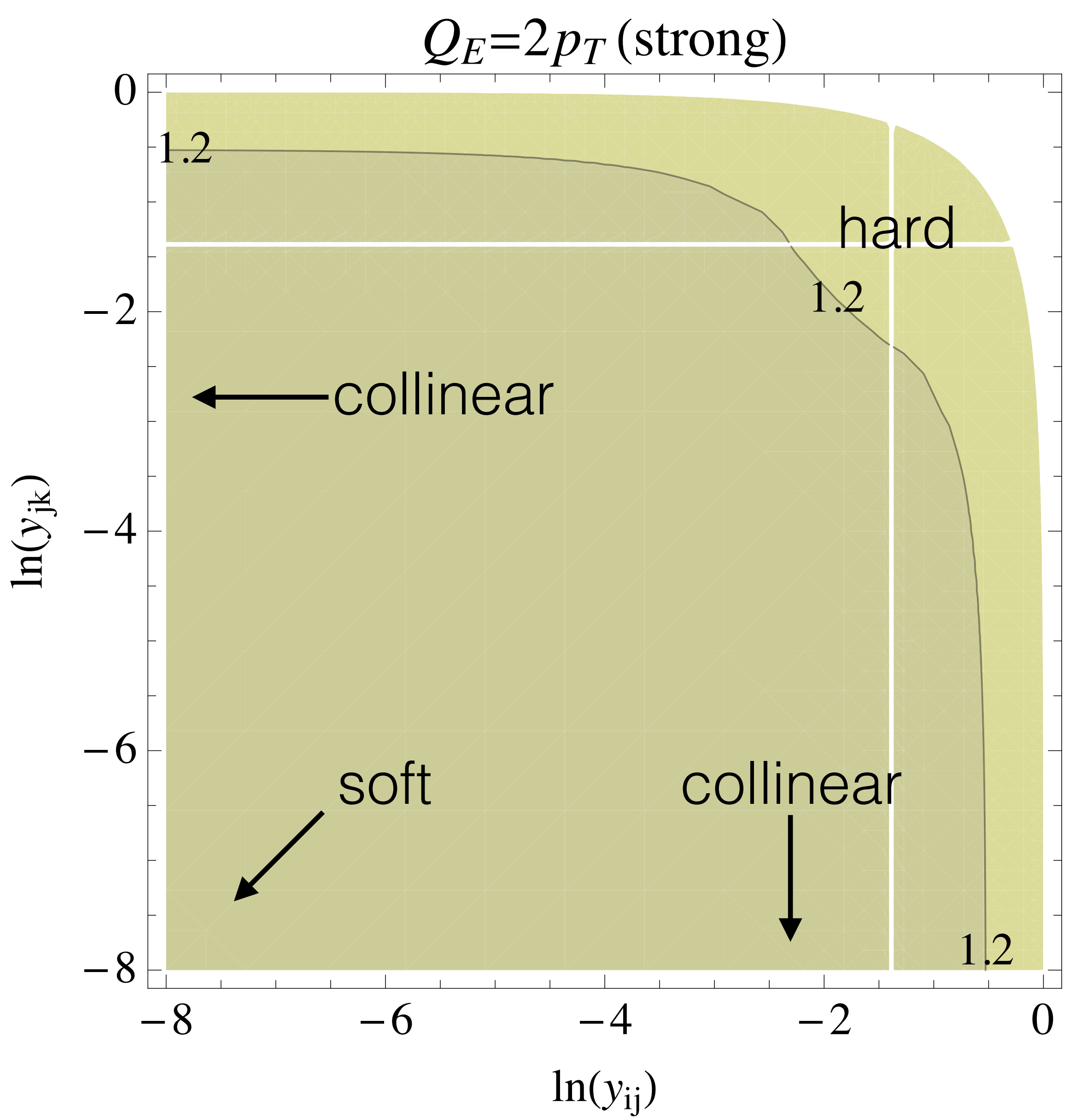


$$y_{ij} = \frac{s_{ij}}{M_Z^2}$$

$$y_{jk} = \frac{s_{jk}}{M_Z^2}$$



(a) $\mu_{\mathrm{PS}} = \sqrt{s}$



(b) $\mu_{\mathrm{PS}} = p_\perp$

Hartgring, Laenen, Skands, arXiv:1303.4974

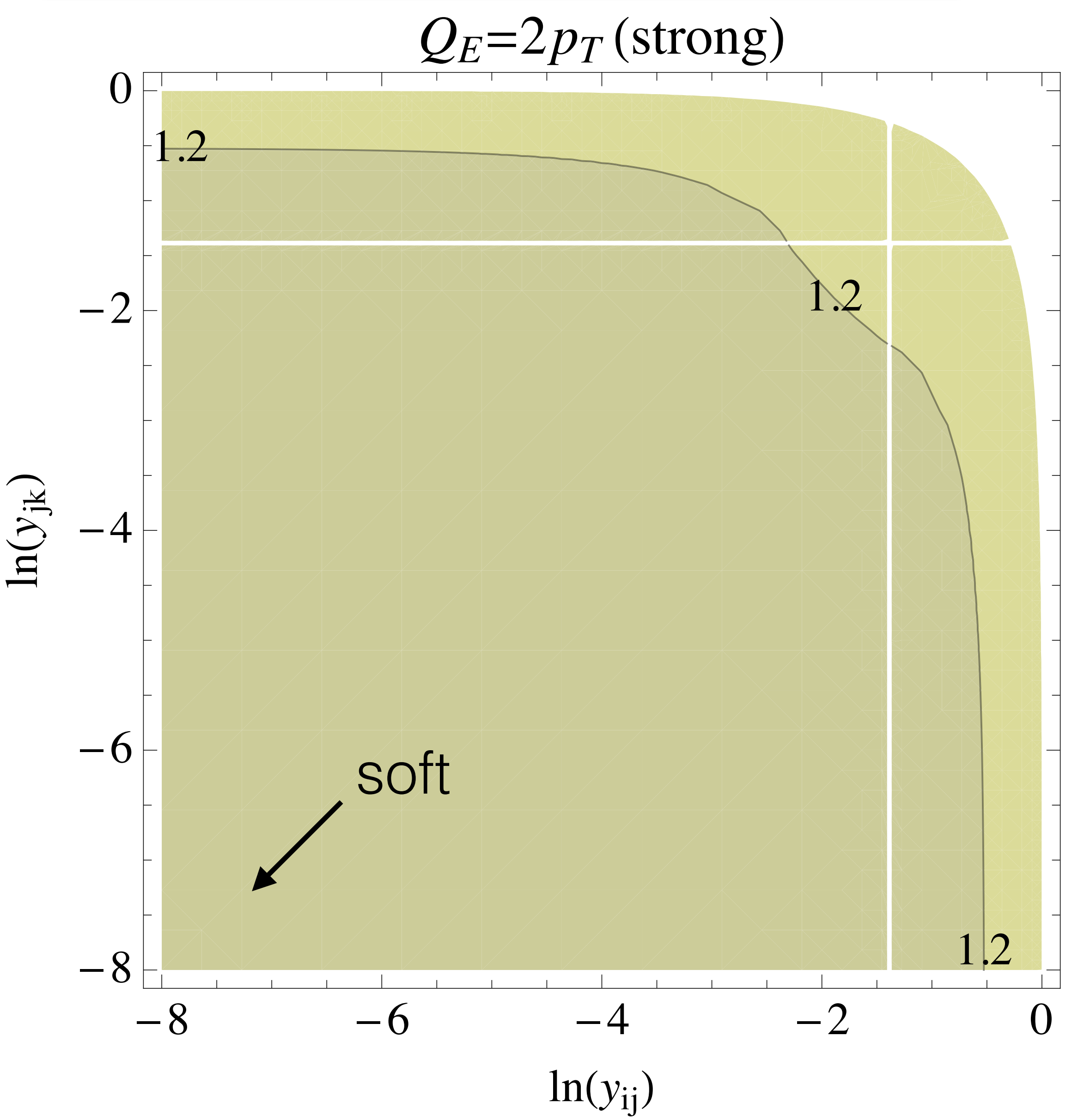
$Z \to 3$ Jets   Size of NLO "K" factor over phase space

The "CMW" factor : Constant shift by

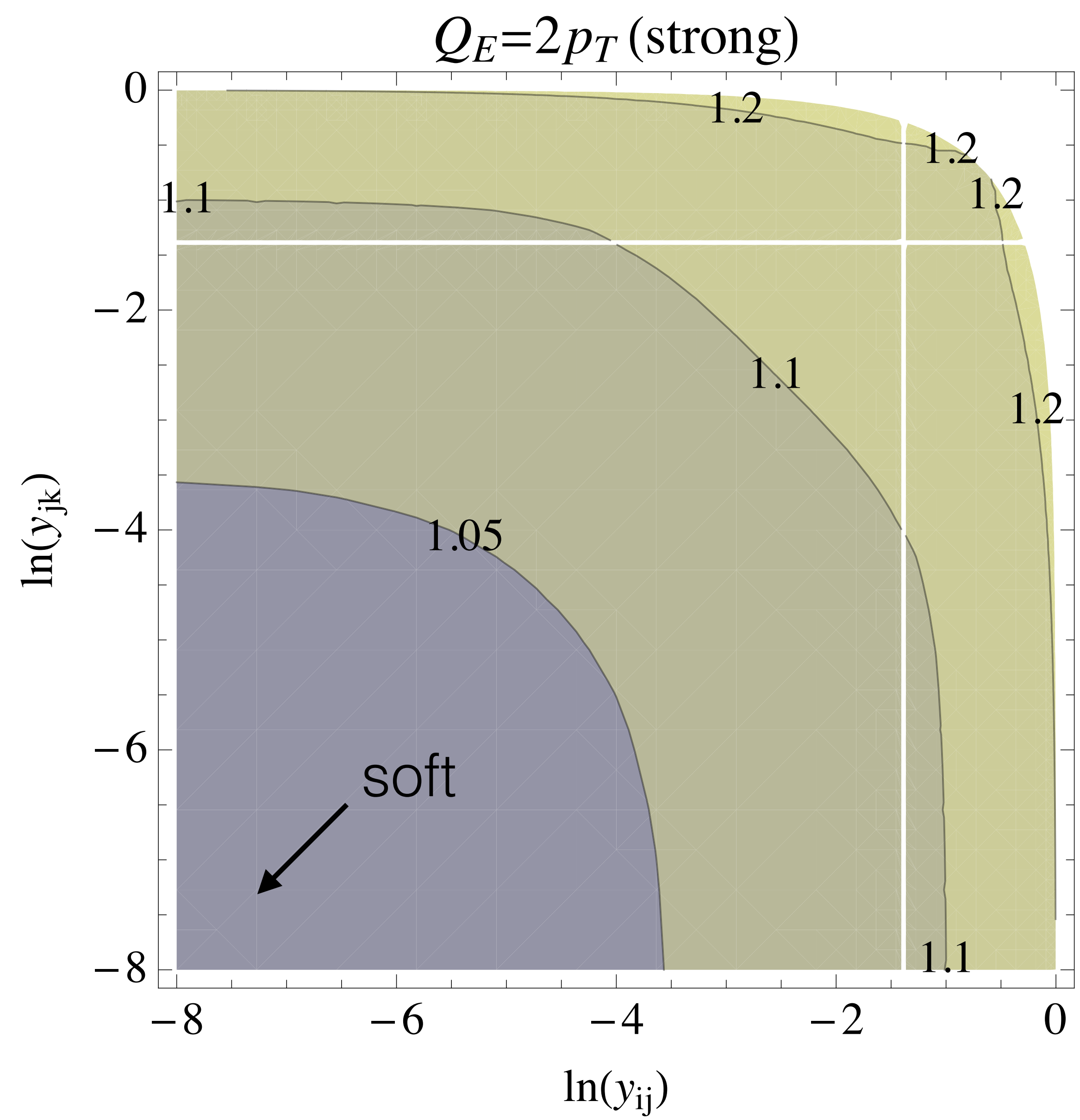
$$k_{\mathrm{CMW}} = \exp\left(\frac{67 - 3\pi^2 - 10n_F/3}{2(33 - 2n_F)}\right) = \begin{cases} 1.513 & n_F = 6 \\ 1.569 & n_F = 5 \\ 1.618 & n_F = 4 \\ 1.661 & n_F = 3 \end{cases}$$

$$\frac{\alpha_s}{2\pi}\frac{\beta_0}{2}\ln\left(k_{\mathrm{CMW}}^2\right) \sim 0.07$$

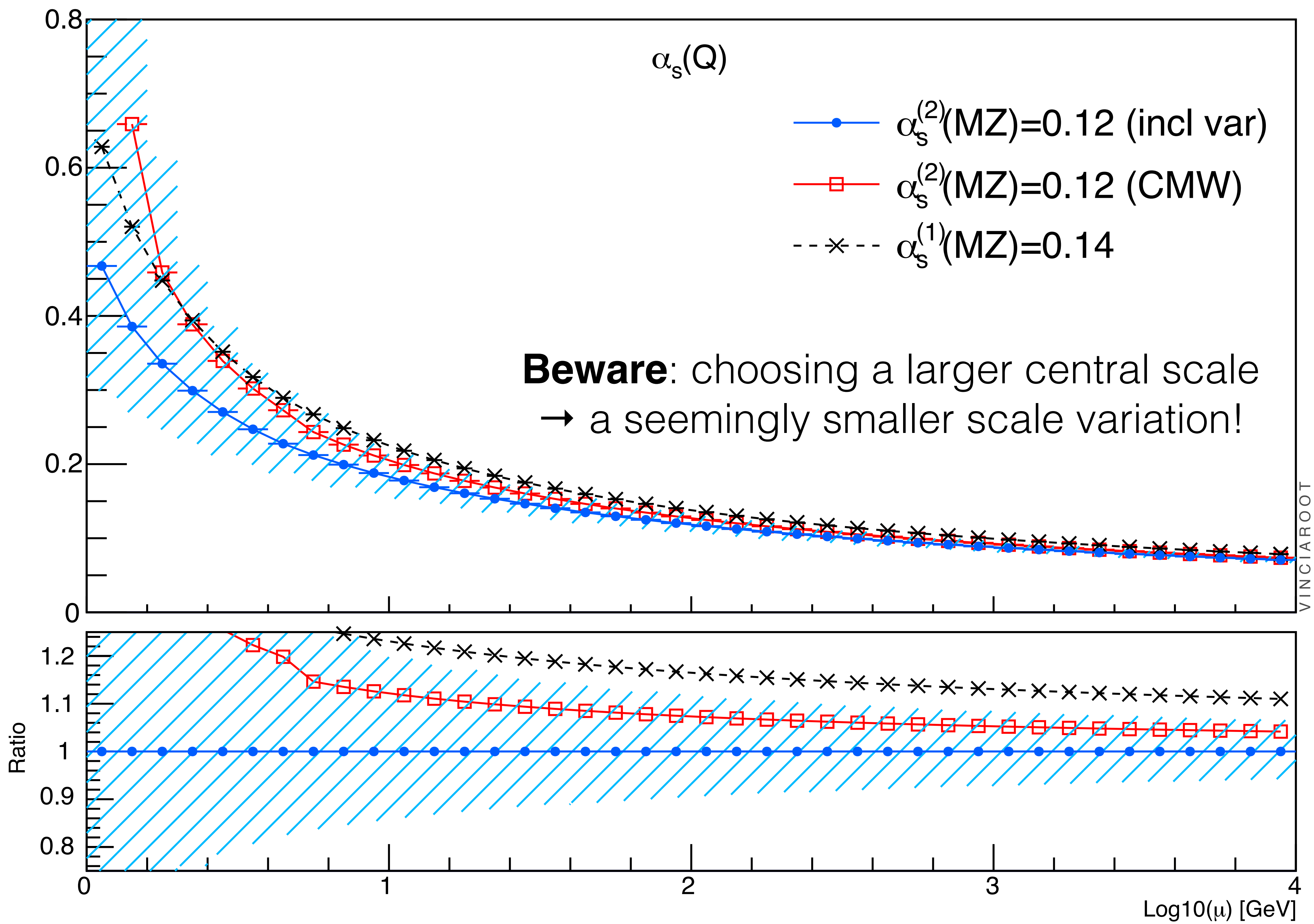
Catani, Marchesini, Webber, NPB349 (1991) 635



(b) $\mu_{\mathrm{PS}} = p_\perp$



$\mu_{\mathrm{PS}} = p_\perp$, with CMW



2 Loop: $\alpha_s(M_Z)=0.12$ $\Lambda_3 = 0.37$ $\Lambda_4 = 0.32$ $\Lambda_5 = 0.23$

1 Loop: $\alpha_s(M_Z)=0.14$ $\Lambda_3 = 0.37$ $\Lambda_4 = 0.33$ $\Lambda_5 = 0.26$

(In all cases, 5-flavor running is still used above mt)

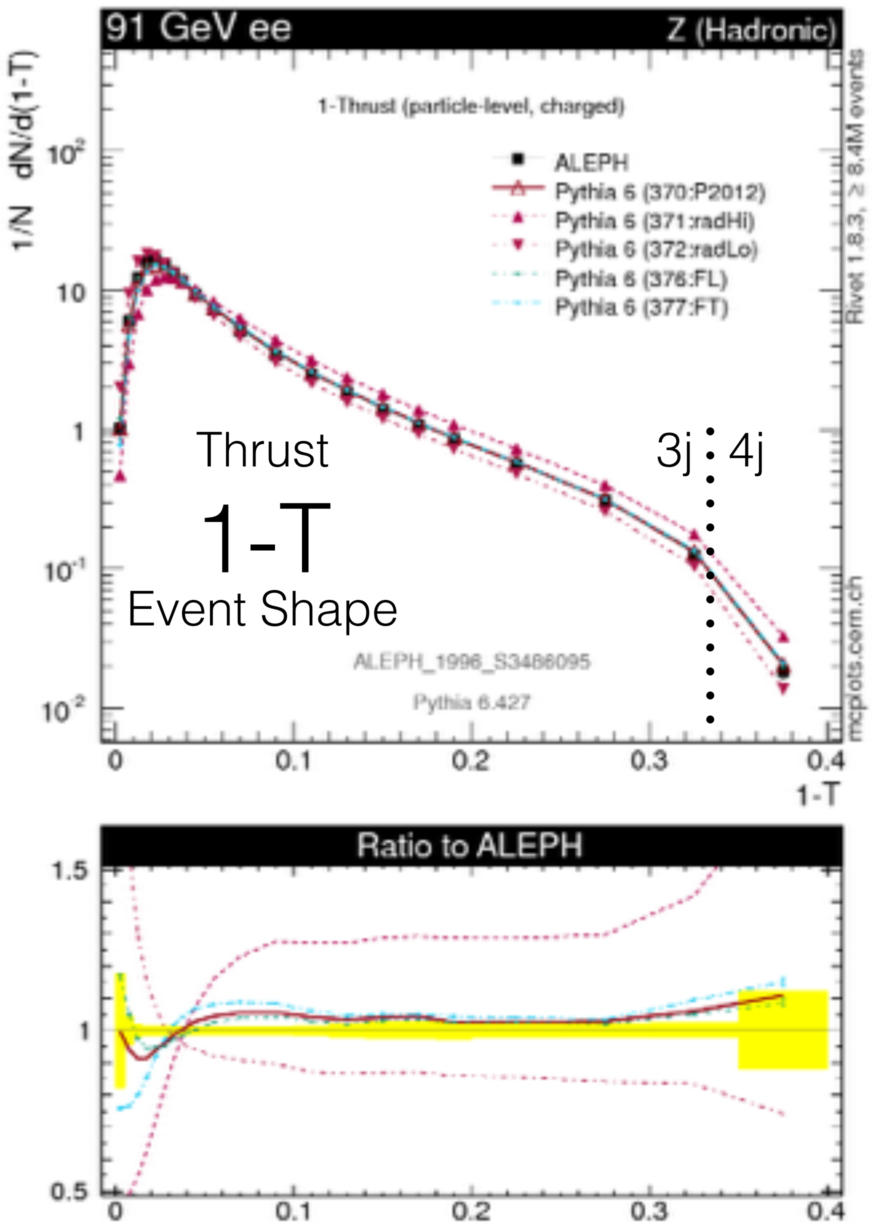
Variations in e^+e^-

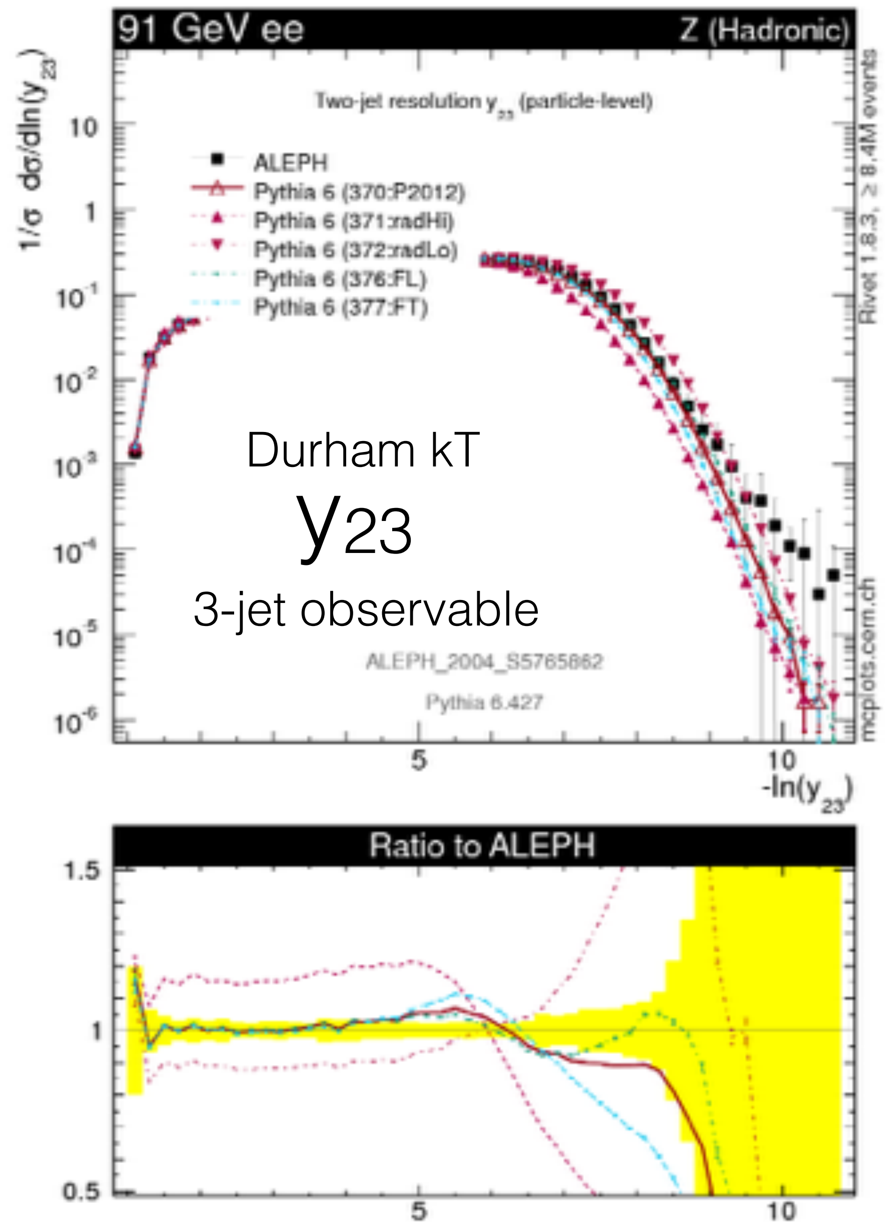
μ_R by factor 2 in either direction

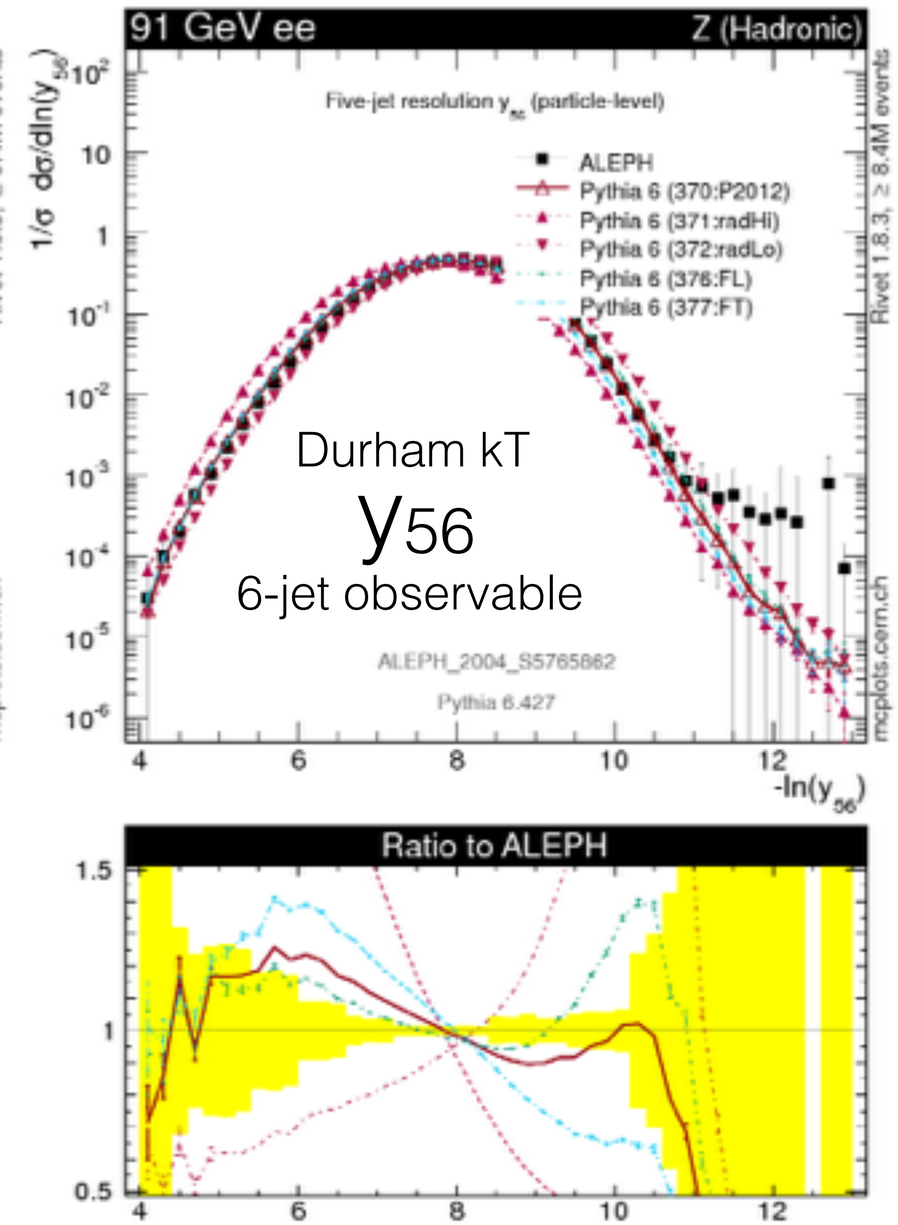
Pythia 6 “Perugia 2012 : Variations”

(with central choice $\mu_R = p_T$, and $\alpha_s(M_Z)^{(1)} \sim 0.14$)

Skands, arXiv:1005.3457







$\propto \alpha_s^1$

$\propto \alpha_s^4$

Beware! α_s pileup

→ Factor 2 looks pretty extreme?

See mcplots.cern.ch

Karneyeu et al, arXiv:1306.3436

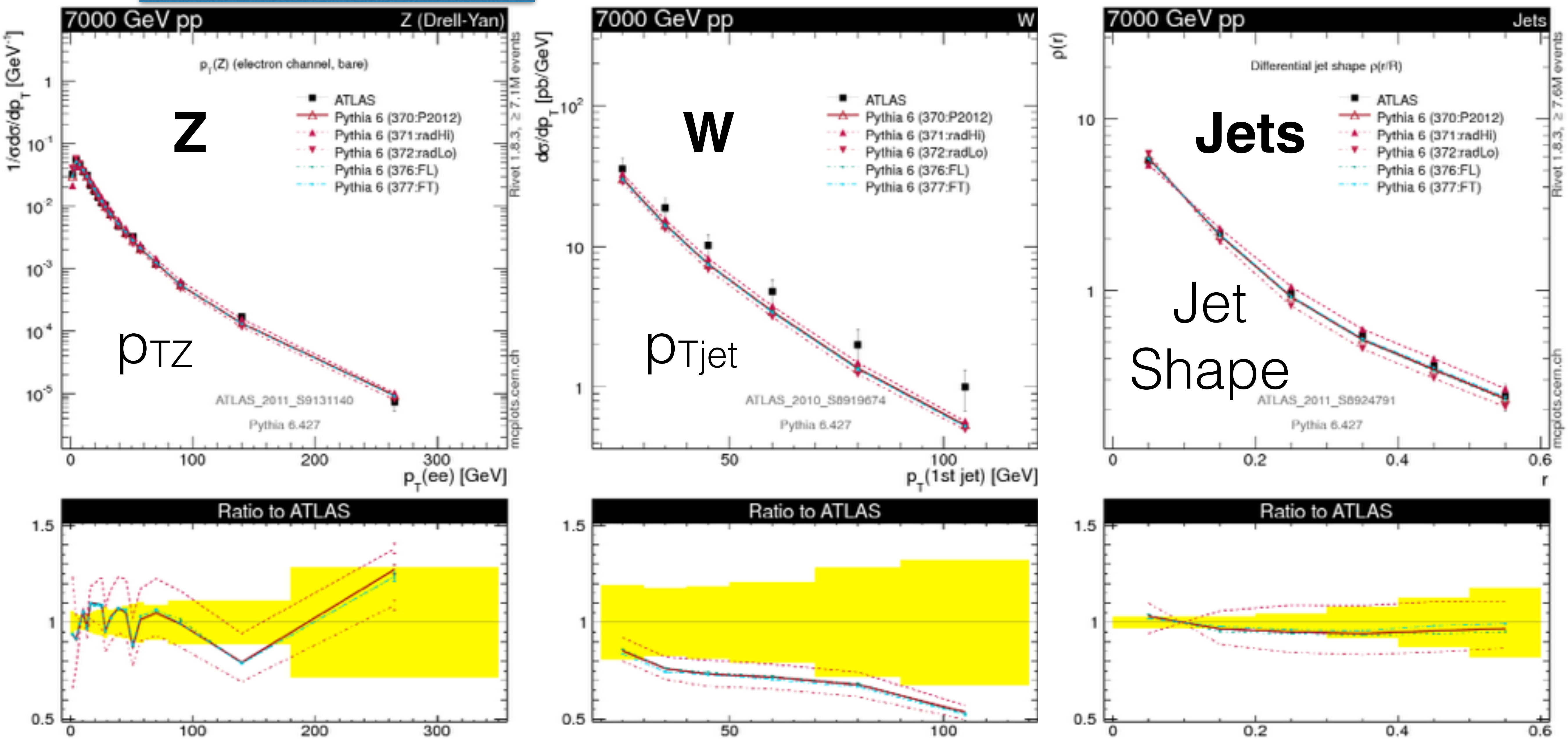
Variations in pp

μ_R by factor 2 in either direction

Pythia 6 "Perugia 2012 : Variations"

(with central choice $\mu_R = p_T$, and $\alpha_s(M_Z)^{(1)} \sim 0.14$)

Skands, arXiv:1005.3457



$1/\sigma \, d\sigma/dp_T$

"normalized"

$d\sigma/dp_T$

"dimensionful"

→ Factor 2 looks reasonable?

See mcplots.cern.ch

Karneyeu et al, arXiv:1306.3436

Cooper et al., arXiv:1109.5295

Matrix Element Matching

$\alpha_s^{\rm ME} \to$ Real

$\alpha_s^{\rm PS} \to$ Virtual

Different Codes?
Different Parameters?

$$\sigma_{F+1}^{\rm incl} = \int_{Q_F^2}^{s} \mathrm{d}\Phi_{F+1}\ \alpha_s^{\rm MG}\ |M_{F+1}|^2$$

$$\sigma_F^{\rm excl} = \sigma_F^{\rm incl} - \int \mathrm{d}\Phi_F \int_{Q_F^2}^{s} \frac{\mathrm{d}Q^2}{Q^2}\ \mathrm{d}z \sum_i \frac{\alpha_s^{\rm SG}}{2\pi} P_i(z)\ |M_F|^2\ +\ \mathcal{O}(\alpha_s^2)$$

Different Λ values

$$\alpha_s^{\rm MG} \neq \alpha_s^{\rm SG} \implies \alpha_s^2\ b_0 \ln\left(\frac{\Lambda_{\rm MG}^2}{\Lambda_{\rm SG}^2}\right) \frac{\mathrm{d}Q^2}{Q^2} \sum_i P_i(z)\ |M_F|^2$$

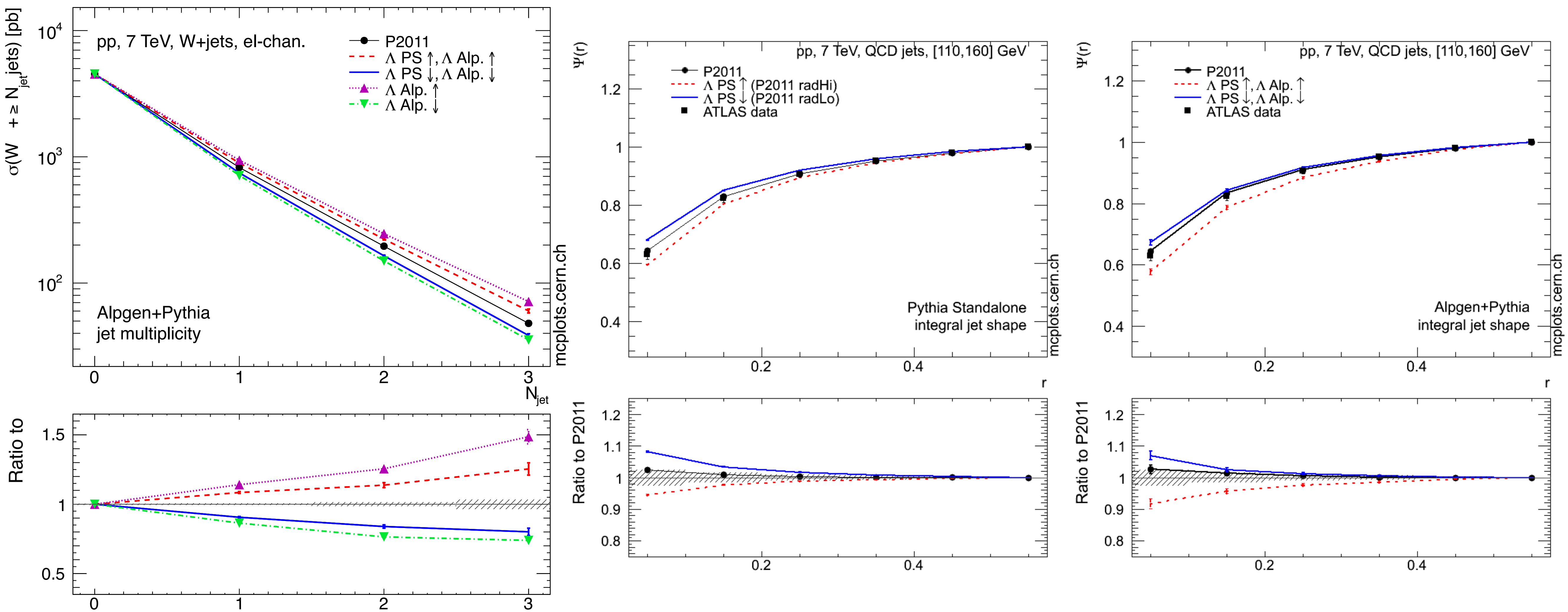
Different running orders:

$$\mathcal{O}(\alpha_s^3 \ln(p_{\rm T}^2/\Lambda^2))\ \frac{\mathrm{d}Q^2}{Q^2} \sum_i P_i(z)\ |M_F|^2$$

(so using same $\alpha_s(M_Z)$ is better than using same Λ since shower anyway takes over at low scales)

Cooper et al., arXiv:1109.5295

Matrix Elements (E.g., AlpGen/MadGraph + Herwig/Pythia) W+jets



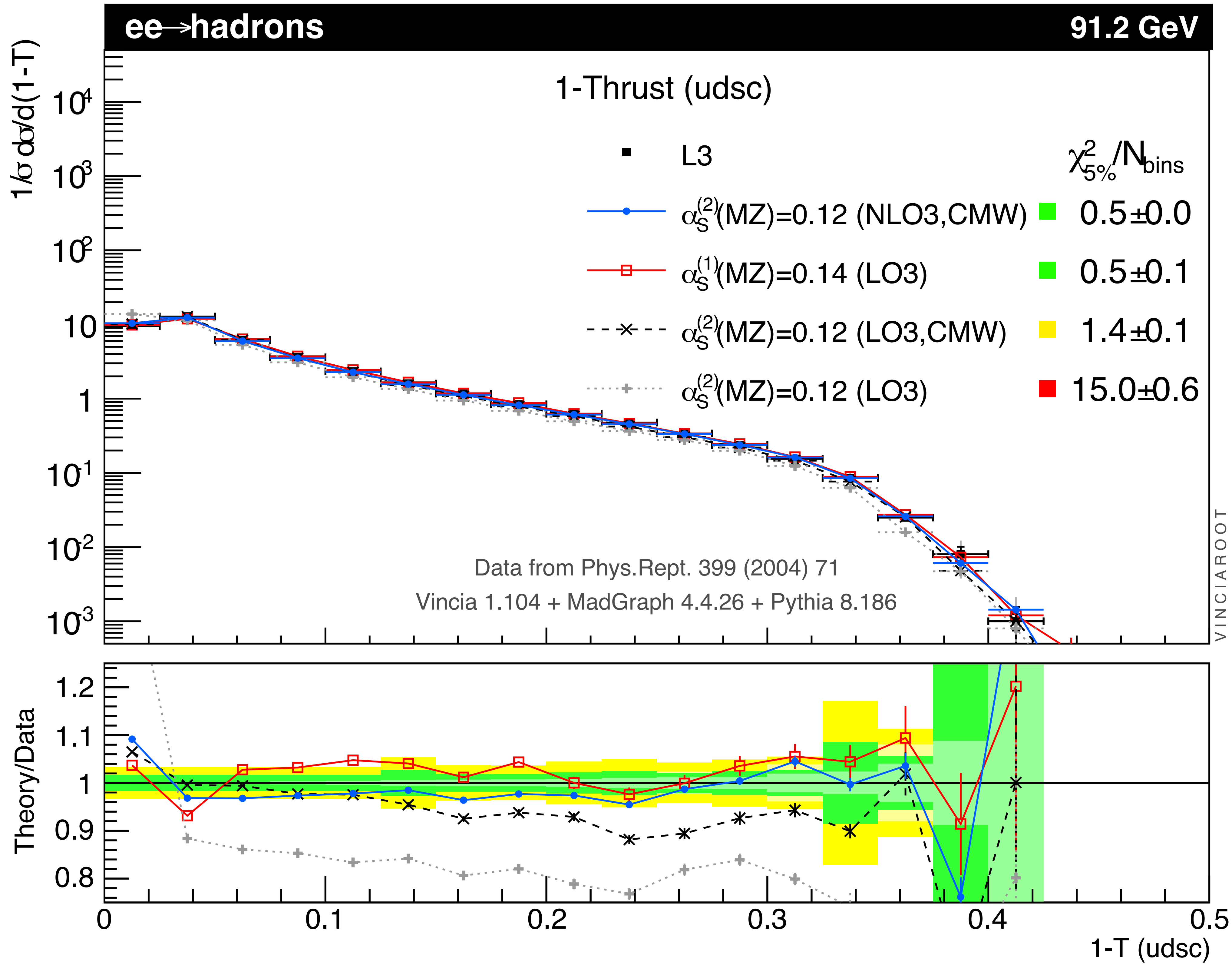
NJets **Jet Shape PS** **Jet Shape ME+PS**

NJets: dominated by ME (+Sudakov from PS)

Jet Shapes: dominated by PS

From multi-leg LO to multi-leg NLO

Hartgring, Laenen, Skands, arXiv:1303.4974



Multi-Scale Exercise

Skands, TASI Lectures, arXiv:1207.2389

If needed, can convert from multi-scale to single-scale

$$
\begin{aligned}
\alpha_s(\mu_1)\alpha_s(\mu_2)\cdots\alpha_s(\mu_n) &= \prod_{i=1}^{n}\alpha_s(\mu)\left(1+b_0\,\alpha_s\ln\left(\frac{\mu^2}{\mu_i^2}\right)+\mathcal{O}(\alpha_s^2)\right) \\
&= \alpha_s^n(\mu)\left(1+b_0\,\alpha_s\ln\left(\frac{\mu^{2n}}{\mu_1^2\mu_2^2\cdots\mu_n^2}\right)+\mathcal{O}(\alpha_s^2)\right)
\end{aligned}
$$

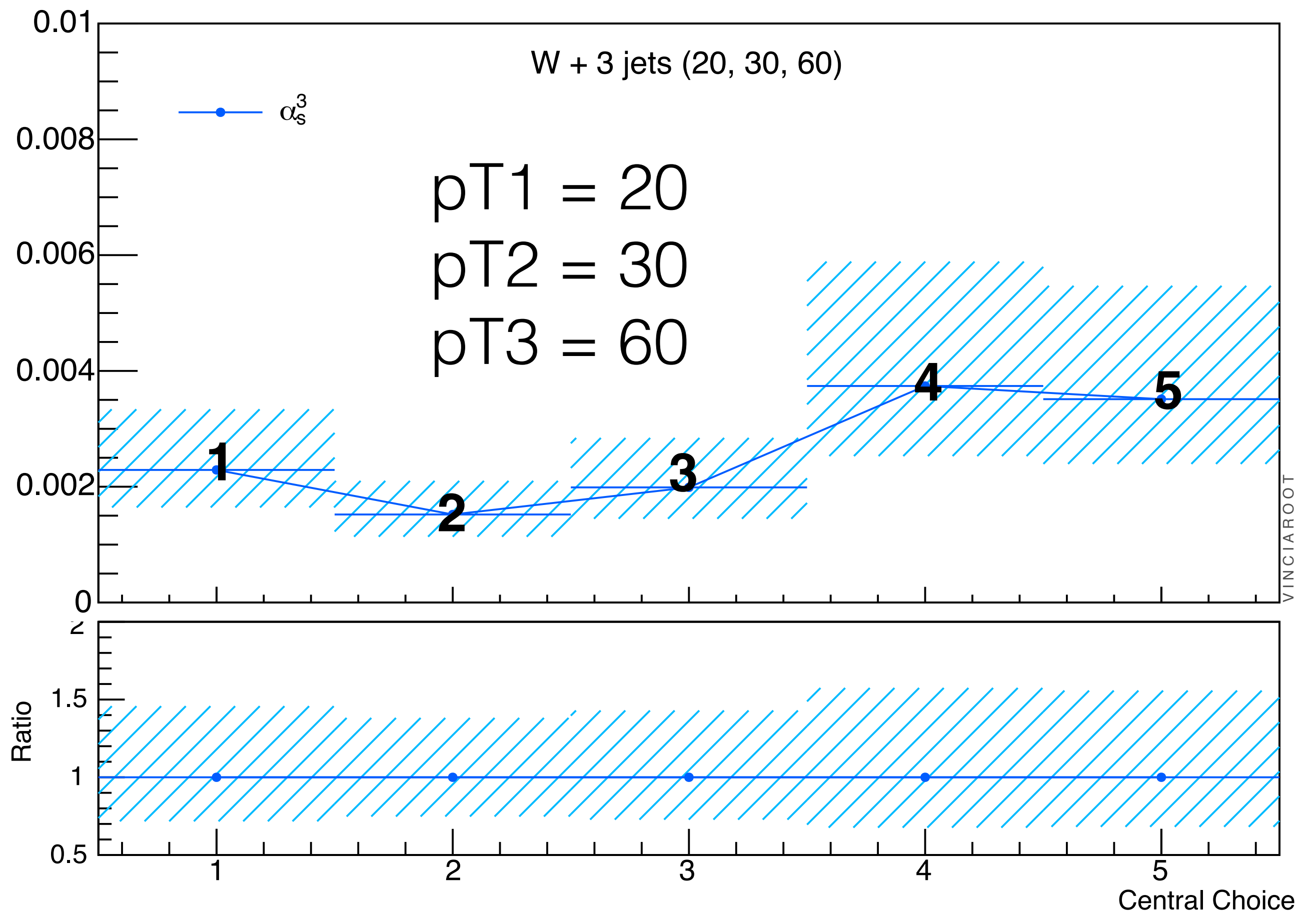
by taking geometric mean of scales

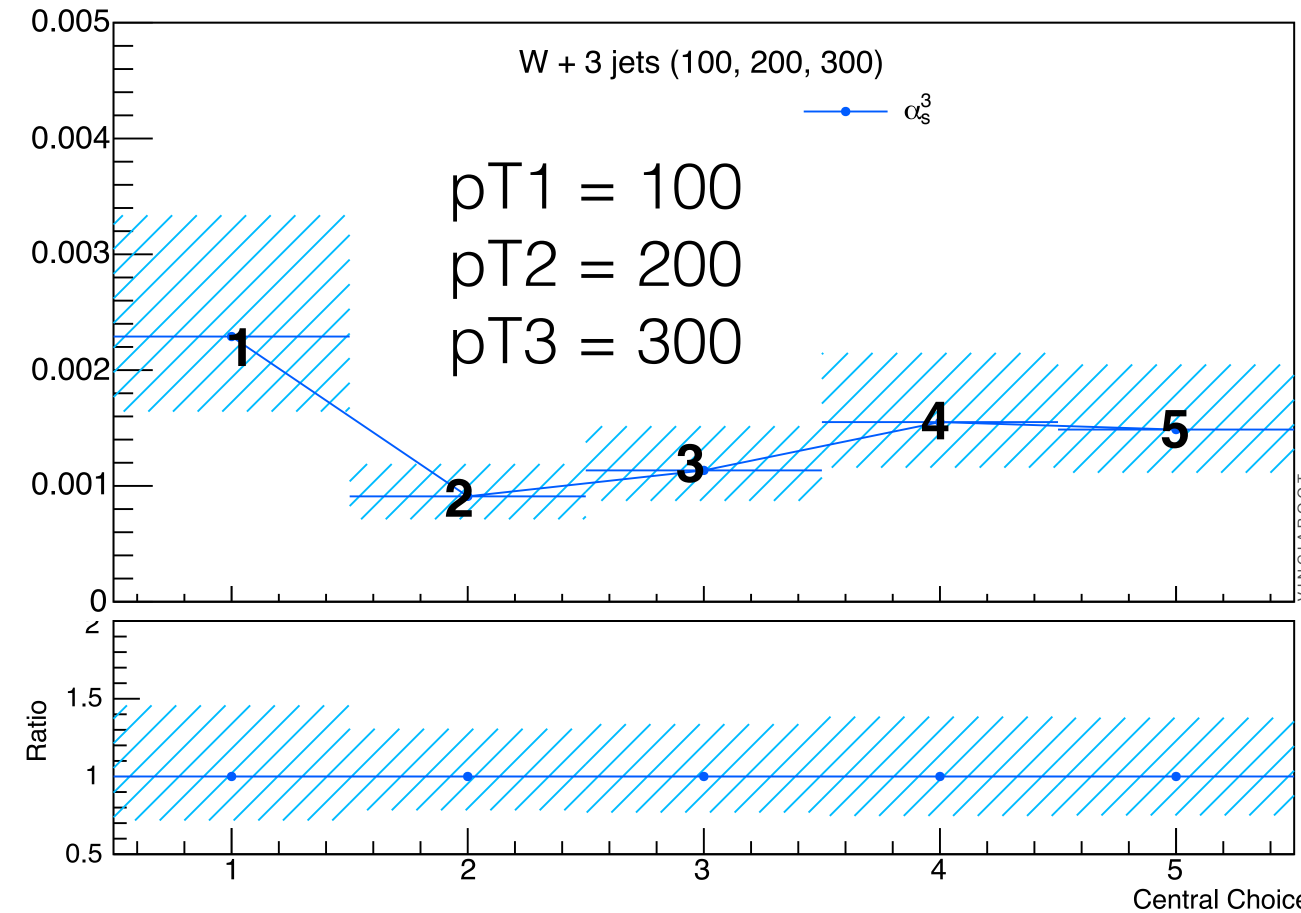
Warning: fixed order misses Sudakovs: partially compensated for by large scale choices?
(must break down eventually; Sudakovs generate double logs, scale variations only single)

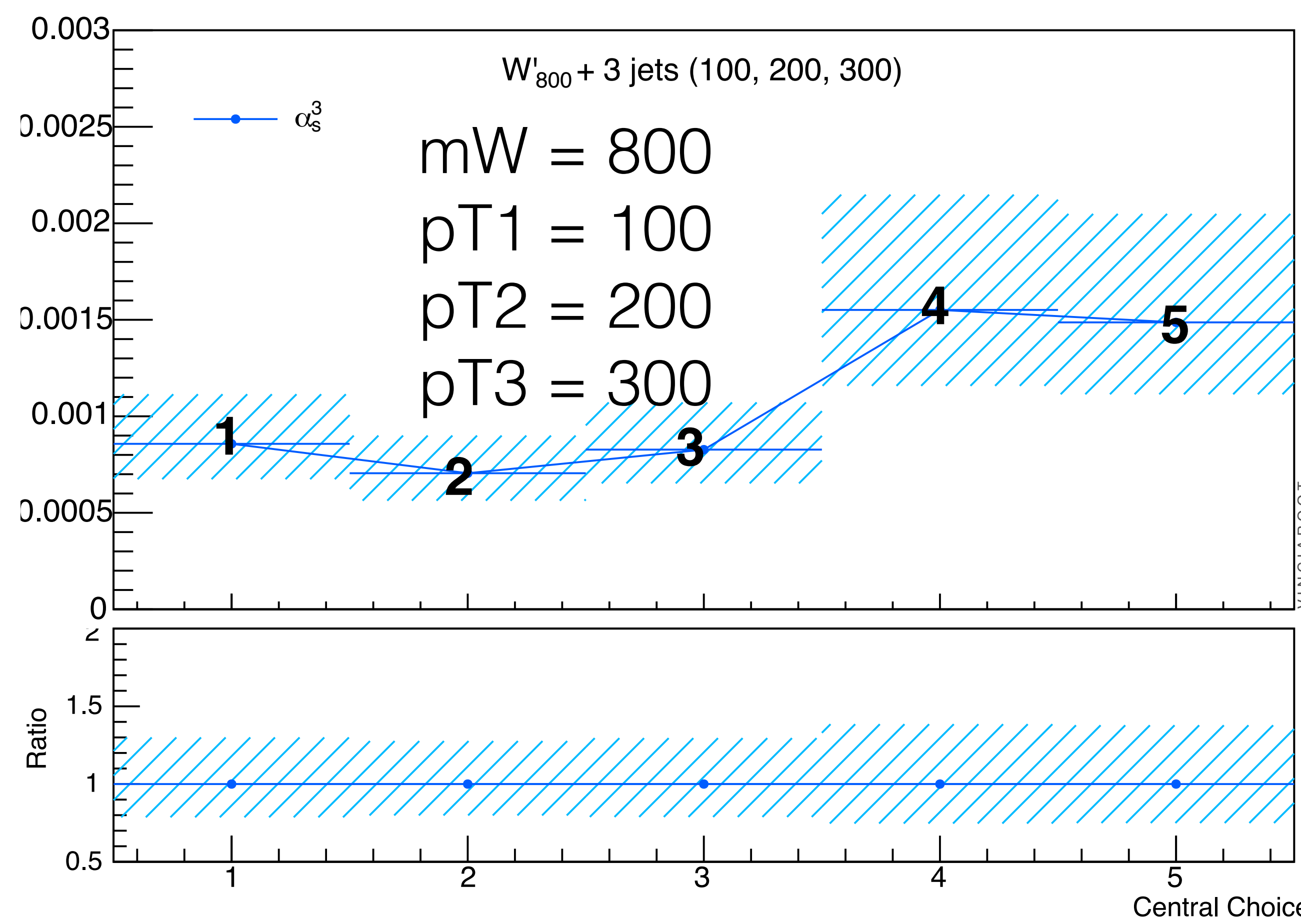
Multi-scale problems

E.g., in context of ME matching with many legs

Example: W+3







1: MW
2: MW + Sum(|pT|)
3: -“- (quadratically)
4: Geometric mean pT (~PS)
5: Arithmetic mean pT